## Series and Parallel Piping



## 1. SERIES PIPING

In the previous chapters, we assumed the pipeline to have the same diameter throughout its length. Pipes are said to be in series if different lengths of pipes of different diameters are joined end to end with the entire flow passing through all pipes.

Consider a pipeline consisting of two different lengths and pipe diameters joined together in series. A pipeline, 1000 ft long, 16 -in diameter, connected in series with a pipeline, 500 ft long and 14 -in diameter would be an example of a series pipeline. At the connection point, we will need to have a fitting known as a reducer, that will join the 16 -in pipe with the smaller 14 -in pipe. This fitting will be a $16-$ in $\times 14$-in reducer. The reducer causes transition in the pipe diameter smoothly from 16 in to 14 in. We can calculate the total pressure drop through this 16 -in/14-in pipeline system by adding the individual pressure drops in the 16 -in and the 14-in pipe segment and accounting for the pressure loss in the 16 -in $\times 14$-in reducer.

There are situations where a pipeline may consist of different pipe diameter connected together in series to transport different volumes of fluid as shown in Figure 11.1.

In Figure 11.1, pipe section AB with a diameter of 16 -in is used to transport natural gas volume of 100 million standard cubic feet per day (MMSCFD) and after making a delivery of 20 MMSCFD at B , the remainder of 80 MMSCFD flows through the 14 -in diameter pipe BC.


Figure 11.1 Series piping.

At C, a delivery of 30 MMSCFD is made and the balance volume of 50 MMSCFD is delivered to the terminus $D$ through a 12-in pipeline CD.

It is clear that the pipe section AB flows the largest volume ( 100 MMSCFD), whereas the pipe segment CD transports the least volume ( 50 MMSCFD ). Therefore, segments AB and CD for reasons of economy should be of different pipe diameters as indicated in the Figure 11.1. If we maintained the same pipe diameter of 16 -in from $A$ to $D$ it would be a waste of pipe material and therefore cost. Constant diameter is used only when the same flow that enters the pipeline is also delivered at the end of the pipeline, with no intermediate injections or deliveries.

However, in reality there is no way of determining ahead what the future delivery volumes would be along the pipeline. Hence it is difficult to determine initially the different pipe sizes for each segment. Therefore, in many cases you will find that the same diameter pipe is used throughout the entire length of the pipeline even though there are intermediate deliveries. Even with the same nominal pipe diameter, different pipe sections may have different wall thicknesses; therefore, we have different pipe inside diameters for each pipe segment. Such wall thickness changes are made to compensate for varying pressures along the pipeline. The subject of pipe strength and its relation to pipe diameter and wall thickness were discussed in Chapter 5.

The pressure required to transport gas or liquid in a series pipeline from point A to point D in Figure11.1 is calculated by considering each pipe segment such as $\mathrm{AB}, \mathrm{BC}$, etc., and applying the appropriate pressure drop equation for each segment.

Another approach to calculating the pressures in series piping system is using the equivalent length concept. This method can be applied when the same uniform flow exists throughout the pipeline, with no intermediate deliveries or injections. We will explain this method of calculation for a series piping system with the same flow rate Q through all pipe segments. Suppose the first pipe segment has an inside diameter $\mathrm{D}_{1}$ and length $\mathrm{L}_{1}$ followed by the second segment of inside diameter $\mathrm{D}_{2}$ and length $\mathrm{L}_{2}$ and so on. We calculate the equivalent length of the second pipe segment based on the diameter $\mathrm{D}_{1}$ such that the pressure drop in the equivalent length matches that in the original pipe segment of diameter $\mathrm{D}_{2}$.

Pressure drop in diameter $\mathrm{D}_{2}$ and Length $\mathrm{L}_{2}=$
Pressure drop in diameter $\mathrm{D}_{1}$ and equivalent Length $\mathrm{Le}_{2}$

Thus the second segment can be replaced with a piece of pipe of length $\mathrm{Le}_{2}$ and diameter $\mathrm{D}_{1}$ Similarly, the third pipe segment with diameter $\mathrm{D}_{3}$ and length $L_{3}$ will be replaced with a piece of pipe of $\mathrm{Le}_{3}$ and diameter $D_{1}$. Thus we have converted the three segments of pipe in terms of diameter $D_{1}$ as follows

Segment 1 - diameter $\mathrm{D}_{1}$ and length $\mathrm{L}_{1}$
Segment 2 - diameter $\mathrm{D}_{1}$ and length $\mathrm{Le}_{2}$
Segment 3 - diameter $\mathrm{D}_{1}$ and length $\mathrm{Le}_{3}$
For convenience, we picked the diameter $\mathrm{D}_{1}$ of segment 1 as the base diameter to use, to convert from the other pipe sizes. We now have the series piping system reduced to one constant diameter $\left(\mathrm{D}_{1}\right)$ pipe of total equivalent length given by

$$
\begin{equation*}
\mathrm{Le}=\mathrm{L}_{1}+\mathrm{Le}_{2}+\mathrm{Le}_{3} \tag{11.1}
\end{equation*}
$$

The pressure required at the inlet of this series piping system can then be calculated based on diameter $\mathrm{D}_{1}$ and length Le. We will now explain how the equivalent length is calculated.

### 1.1 Equivalent Length of Pipes: Gas Pipelines

Upon examining the general flow equation, it can be seen that for the same flow rate and gas properties, neglecting elevation effects the pressure difference $\left(\mathrm{P}_{1}^{2}-\mathrm{P}_{2}^{2}\right)$ is inversely proportional to the fifth power of the pipe diameter and directly proportional to the pipe length. Therefore, we can state that approximately

$$
\begin{equation*}
\Delta \mathrm{P}_{\mathrm{sq}}=\frac{\mathrm{CL}}{\mathrm{D}^{5}} \tag{11.2}
\end{equation*}
$$

where
$\Delta \mathrm{P}_{\mathrm{sq}}=$ difference in square of pressures $\left(\mathrm{P}_{1}^{2}-\mathrm{P}_{2}^{2}\right)$ for pipe segment.
$\mathrm{C}=\mathrm{a}$ constant
$\mathrm{L}=$ pipe length
$\mathrm{D}=$ pipe inside diameter
The value of C depends on the flow rate, gas properties, gas temperature, base pressure, and base temperature. Therefore, C will be the same for all pipe segments in a series pipeline with constant flow rate.

From Eqn (11.2), we conclude that the equivalent length for the same pressure drop is proportional to the fifth power of the diameter. Therefore,
in the series piping example discussed earlier, the equivalent length of the second pipe segment of diameter $D_{2}$ and length $L_{2}$ is

$$
\begin{equation*}
\frac{\mathrm{CL}_{2}}{\mathrm{D}_{2}^{5}}=\frac{\mathrm{CLe}_{2}}{\mathrm{D}_{1}^{5}} \tag{11.3}
\end{equation*}
$$

Simplifying, we get

$$
\begin{equation*}
\mathrm{Le}_{2}=\mathrm{L}_{2}\left(\frac{\mathrm{D}_{1}}{\mathrm{D}_{2}}\right)^{5} \tag{11.4}
\end{equation*}
$$

Similarly, for the third pipe segment of diameter $D_{3}$ and length $L_{3}$, the equivalent length is

$$
\begin{equation*}
\mathrm{Le}_{3}=\mathrm{L}_{3}\left(\frac{\mathrm{D}_{1}}{\mathrm{D}_{3}}\right)^{5} \tag{11.5}
\end{equation*}
$$

Therefore, the total equivalent length Le for all three pipe segments in terms of diameter $D_{1}$ is

$$
\begin{equation*}
\mathrm{Le}=\mathrm{L}_{1}+\mathrm{L}_{2}\left(\frac{\mathrm{D}_{1}}{\mathrm{D}_{2}}\right)^{5}+\mathrm{L}_{3}\left(\frac{\mathrm{D}_{1}}{\mathrm{D}_{3}}\right)^{5} \tag{11.6}
\end{equation*}
$$

It can be seen from Eqn (11.6) that if $D_{1}=D_{2}=D_{3}$, the total equivalent length becomes $\left(\mathrm{L}_{1}+\mathrm{L}_{2}+\mathrm{L}_{3}\right)$ as expected.

We can now calculate the pressure drop for the series piping system considering a single pipe of length Le and uniform diameter $\mathrm{D}_{1}$ flowing a constant volume Q . A problem will illustrate the equivalent length method.

## Problem 11.1: Gas Pipeline

A series piping system, shown in Figure 11.2, consists of 12 miles of Nominal Pipe Size (NPS) 16, 0.375-in wall thickness connected to 24 miles of NPS $14,0.250$-in wall thickness and 8 miles of NPS $12,0.250$-in wall thickness pipes. Calculate the inlet pressure required at the origin A of this pipeline system for a gas flow rate of 100 MMSCFD. Gas is delivered to the terminus $B$ at a de-livery pressure of 500 psig. The gas gravity and viscosity are 0.6 and $0.000008 \mathrm{lb} / \mathrm{ft}-\mathrm{s}$. The gas temperature is assumed constant at $60^{\circ} \mathrm{F}$. Use a compressibility factor of 0.90 and the general flow equation with Darcy


Figure 11.2 Series piping in a gas pipeline.
friction factor $=0.02$. The base temperature and base pressure are $60^{\circ} \mathrm{F}$ and 14.7 psia, respectively.

Compare results using the equivalent length method and with the more detailed method of calculating pressure for each pipe segment separately.

## Solution

Inside diameter of first pipe segment $=16-2 \times 0.375=15.25$ in
Inside diameter of second pipe segment $=14-2 \times 0.250=13.50$ in
Inside diameter of third pipe segment $=12.75-21 \times 10.250=12.25$ in
Using Eqn (11.6), we calculate the equivalent length of the pipeline, considering NPS 16 as the base diameter.

$$
\mathrm{Le}=12+24 \times\left(\frac{15.25}{13.5}\right)^{5}+8 \times\left(\frac{15.25}{12.25}\right)^{5}
$$

or

$$
\mathrm{Le}=12+44.15+23.92=80.07 \mathrm{mi}
$$

Therefore, we will calculate the inlet pressure $\mathrm{P}_{1}$ considering a single pipe from A to B having a length of 80.07 miles and inside diameter of 15.25 in.

Outlet pressure $=500+14.7=514.7$ psia
Using the general flow, neglecting elevation effects and substituting given values, we get

$$
100 \times 10^{6}=77.54\left(\frac{1}{\sqrt{0.02}}\right)\left(\frac{520}{14.7}\right)\left[\frac{\left(P_{1}^{2}-514.7^{2}\right)}{0.6 \times 520 \times 80.07 \times 0.9}\right]^{0.5} 15.25^{2.5}
$$

Transposing and simplifying, we get

$$
\mathrm{P}_{1}^{2}-514.7^{2}=724,642.99
$$

Finally, solving for the inlet pressure $\mathrm{P}_{1}$, we get

$$
\mathrm{P}_{1}=994.77 \mathrm{psia}=980.07 \mathrm{psig}
$$

Next we will compare the preceding result, using the equivalent length method, with the more detailed calculation of treating each pipe segment separately and adding the pressure drops.

Consider the 8 -mile pipe segment 3 first because we know the outlet pressure at B is 500 psig . Therefore, we can calculate the pressure at the beginning of the segment 3 using the general flow equation as follows.

$$
100 \times 10^{6}=77.54\left(\frac{1}{\sqrt{0.02}}\right)\left(\frac{520}{14.7}\right)\left[\frac{\left(P_{1}^{2}-514.7^{2}\right)}{0.6 \times 520 \times 8 \times 0.9}\right]^{0.5} 12.25^{2.5}
$$

Solving for the pressure $\mathrm{P}_{1}$, we get

$$
\mathrm{P}_{1}=693.83 \mathrm{psia}=679.13 \mathrm{psig}
$$

This is the pressure at the beginning of the pipe segment 3 , which is also the end of pipe segment 2 .

Next consider pipe segment 2 ( 24 miles of NPS 14 pipe) and calculate the upstream pressure $\mathrm{P}_{1}$ required for a downstream pressure of 679.13 psig calculated in the preceding section. Using the general flow equation for pipe segment 2 , we get

$$
100 \times 10^{6}=77.54\left(\frac{1}{\sqrt{0.02}}\right)\left(\frac{520}{14.7}\right)\left[\frac{\left(P_{1}^{2}-693.83^{2}\right)}{0.6 \times 520 \times 24 \times 0.9}\right]^{0.5} 13.5^{2.5}
$$

Solving for the pressure $\mathrm{P}_{1}$, we get

$$
\mathrm{P}_{1}=938.58 \mathrm{psia}=923.88 \mathrm{psig}
$$

This is the pressure at the beginning of the pipe segment 2 , which is also the end of pipe segment 1 .

Next we calculate the inlet pressure $\mathrm{P}_{1}$ of pipe segment 1 (12 miles of NPS 16 pipe) for an outlet pressure of 923.88 psig, we just calculated. Using the general flow equation for pipe segment 1 , we get
$100 \times 10^{6}=77.54\left(\frac{1}{\sqrt{0.02}}\right)\left(\frac{520}{14.7}\right)\left[\frac{\left(P_{1}^{2}-938.58^{2}\right)}{0.6 \times 520 \times 12 \times 0.9}\right]^{0.5} 15.25^{2.5}$
Solving for the pressure $\mathrm{P}_{1}$, we get

$$
\mathrm{P}_{1}=994.75 \mathrm{psia}=980.05 \mathrm{psig}
$$

This compares well with the pressure of 980.07 psig we calculated earlier using the equivalent length method.

## Problem 11.2

A natural gas pipeline consists of three different pipe segments connected in series, pumping the same uniform flow rate of $3.0 \mathrm{Mm}^{3} /$ day at $20^{\circ} \mathrm{C}$. The first segment, DN 500 with a $12-\mathrm{mm}$ wall thickness is $20-\mathrm{km}$ long. The second segment is DN 400, with a $10-\mathrm{mm}$ wall thickness and $25-\mathrm{km}$ long. The last segment is DN 300 , with a $6-\mathrm{mm}$ wall thickness and 10 km long. The inlet pressure is 8500 kPa . Assuming flat terrain, calculate the delivery pressure, using the general flow equation and Colebrook friction factor of 0.02 . Gas gravity $=0.65$. Viscosity $=0.000119$ poise. Compressibility factor $\mathrm{Z}=0.9$. Base temperature $=15^{\circ} \mathrm{C}$ and base pressure $=101 \mathrm{kPa}$. Compare results using the equivalent length method as well as the method using individual pipe segment pressure drops.

## Solution

Inside diameter of first pipe segment $=500-2 \times 12=476 \mathrm{~mm}$.
Inside diameter of second pipe segment $=400-2 \times 10=380 \mathrm{~mm}$.
Inside diameter of last pipe segment $=300-2 \times 6=288 \mathrm{~mm}$.
Equivalent length method:
Using Eqn (11.6), we calculate the total equivalent length of the pipeline system based on the first segment diameter DN 500 as follows.

$$
\mathrm{Le}=20+25 \times\left(\frac{500-2 \times 12}{400-2 \times 10}\right)^{5}+10 \times\left(\frac{500-2 \times 12}{300-2 \times 6}\right)^{5}
$$

or

$$
\mathrm{Le}=20+77.10+123.33=220.43 \mathrm{~km}
$$

Thus the given pipeline system can be considered equivalent to a single pipe DN 500, 12-mm wall thickness, 220.43-km long.

The outlet pressure $\mathrm{P}_{2}$ is calculated using the general flow equation as follows

$$
\begin{aligned}
3 \times 10^{6}= & 1.1494 \times 10^{-3}\left(\frac{15+273}{101}\right) \\
& \times\left[\frac{\left(8500^{2}-P_{2}^{2}\right)}{0.65 \times 293 \times 0.9 \times 0.02 \times 220.43}\right]^{0.5}(476)^{2.5}
\end{aligned}
$$

Solving for $\mathrm{P}_{2}$ we get

$$
8500^{2}-\mathrm{P}_{2}^{2}=25,908,801
$$

or

$$
\mathrm{P}_{2}=6807 \mathrm{kPa} \text { (absolute) }
$$

We have assumed that given inlet pressure is in absolute value.
Therefore, the delivery pressure is 6807 kPa (absolute).
Next we calculate the delivery pressure considering the three pipe segments treated separately. For the first pipe segment, $20-\mathrm{km}$ long, we calculate the outlet pressure $P_{2}$ at the end of the first segment as follows. Using the general flow equation, we get

$$
\begin{aligned}
3 \times 10^{6}= & 1.1494 \times 10^{-3}\left(\frac{15+273}{101}\right) \\
& \times\left[\frac{\left(8500^{2}-P_{2}^{2}\right)}{0.65 \times 293 \times 0.9 \times 0.02 \times 20}\right]^{0.5}(476)^{2.5}
\end{aligned}
$$

Solving for $\mathrm{P}_{2}$, we get

$$
\mathrm{P}_{2}=8361 \mathrm{kPa} \text { (absolute) }
$$

Thus, the pressure at the end of the first pipe segment or the beginning of the second segment is 8361 kPa (absolute).

Next we repeat the calculation for the second pipe segment DN 400, 25-km long using
$\mathrm{P}_{1}=8361 \mathrm{kPa}$ (absolute), to calculate $\mathrm{P}_{2}$

$$
\begin{aligned}
3 \times 10^{6}= & 1.1494 \times 10^{-3}\left(\frac{15+273}{101}\right) \\
& \times\left[\frac{\left(8361^{2}-P_{2}^{2}\right)}{0.65 \times 293 \times 0.9 \times 0.02 \times 25}\right]^{0.5}(380)^{2.5}
\end{aligned}
$$

Solving for $\mathrm{P}_{2}$ we get

$$
\mathrm{P}_{2}=7800 \mathrm{kPa} \text { (absolute) }
$$

This is the pressure at the end of the second pipe segment, which is also the inlet pressure for the third pipe segment.

Finally, we calculate the outlet pressure of the last pipe segment (DN $300,10 \mathrm{~km}$ ) using $\mathrm{P}_{1}=7800 \mathrm{kPa}$ (absolute) as follows

$$
\begin{aligned}
3 \times 10^{6}= & 1.1494 \times 10^{-3}\left(\frac{15+273}{101}\right) \\
& \times\left[\frac{\left(7800^{2}-P_{2}^{2}\right)}{0.65 \times 293 \times 0.9 \times 0.02 \times 10}\right]^{0.5}(288)^{2.5}
\end{aligned}
$$

Solving for $\mathrm{P}_{2}$ we get.

$$
\mathrm{P}_{2}=6808 \mathrm{kPa} \text { (absolute) }
$$

Therefore the delivery pressure is 6808 kPa (absolute).
This compares favorably with the values of 6807 kPa we calculated earlier using the equivalent length approach.

### 1.2 Equivalent Length of Pipes: Liquid Pipelines

A pipe is equivalent to another pipe or a pipeline system, when the same pressure loss from friction occurs in the equivalent pipe compared with that of the other pipe or pipeline system. Because the pressure drop can be caused by an infinite combination of pipe diameters and pipe lengths, we must specify a particular diameter to calculate the equivalent length.

Suppose a pipe $A$ of length $L_{A}$ and internal diameter $D_{A}$ is connected in series with a pipe $B$ of length $L_{B}$ and internal diameter $D_{B}$, If we were to replace this two-pipe system with a single pipe of length $\mathrm{L}_{\mathrm{E}}$ and diameter
$D_{\mathrm{E}}$, we have what is known as the equivalent length of pipe. This equivalent length of pipe may be based on one of the two diameters $\left(D_{A}\right.$ or $\left.D_{B}\right)$ or a totally different diameter $\mathrm{D}_{\mathrm{E}}$.

The equivalent length $\mathrm{L}_{\mathrm{E}}$ in terms of pipe diameter $\mathrm{D}_{\mathrm{E}}$ can be written as

$$
\begin{equation*}
\mathrm{L}_{\mathrm{E}} /\left(\mathrm{D}_{\mathrm{E}}\right)^{5}=\mathrm{L}_{\mathrm{A}} /\left(\mathrm{D}_{\mathrm{A}}\right)^{5}+\mathrm{L}_{\mathrm{B}} /\left(\mathrm{D}_{\mathrm{B}}\right)^{5} \tag{11.7}
\end{equation*}
$$

This formula for equivalent length is based on the premise that the total friction loss in the two-pipe system exactly equals that of the single equivalent pipe.

Because a pressure drop per unit length is inversely proportional to the fifth power of the diameter. If we refer to the diameter $\mathrm{D}_{\mathrm{A}}$ as the basis, this equation becomes, after setting $\mathrm{D}_{\mathrm{E}}=\mathrm{D}_{\mathrm{A}}$.

$$
\begin{equation*}
\mathrm{L}_{\mathrm{E}}=\mathrm{L}_{\mathrm{A}}+\mathrm{L}_{\mathrm{B}}\left(\mathrm{D}_{\mathrm{A}} / \mathrm{D}_{\mathrm{B}}\right)^{5} \tag{11.8}
\end{equation*}
$$

Thus, we have an equivalent length $\mathrm{L}_{\mathrm{E}}$ that will be based on diameter $\mathrm{D}_{\mathrm{A}}$. This length $\mathrm{L}_{\mathrm{E}}$ of pipe diameter $\mathrm{D}_{\mathrm{A}}$ will produce the same amount of frictional pressure drop as the two lengths $\mathrm{L}_{\mathrm{A}}$ and $\mathrm{L}_{\mathrm{B}}$ in a series. We have thus simplified the problem by reducing it to one single pipe length of uniform diameter $\mathrm{D}_{\mathrm{A}}$.

The equivalent length method discussed previously is only approximate. Furthermore, if elevation changes are involved, it becomes more complicated, unless there are no controlling elevations along the pipeline system.

An example will illustrate this concept of equivalent pipe length.
Consider a pipeline 16 -in $\times 0.281$-in wall thickness pipeline, 20 miles long installed in series with a 14 -in $\times 0.250$-in wall thickness pipeline 10 miles long. The equivalent length of this pipeline is

$$
20+10 \times(16-0.562)^{5} /(14-0.50)^{5}=39.56 \text { miles of } 16 \text { inch pipe. }
$$

The actual physical length of 30 miles of 16 -in and 14 -in pipes is replaced with a single 16 -in pipe 39.56 miles long, for pressure drop calculations. Note that we have left out the pipe fitting that would connect the 16 -in pipe with the 14 -in pipe. This would be a $16 \times 14$ reducer which would have its own equivalent length. To be precise, we should determine the equivalent length of the reducer from the table in Appendix A. 10 and add it to the above length to obtain the total equivalent length, including the fitting.

After the equivalent length pipe is determined, we can calculate the pressure drop based on this pipe size.

## Problem 11.3: Liquid Pipeline

A refined products pipeline consists of three pipe segments connected in series, pumping the same uniform flow rate of 60,000 barrels/day of diesel. The first segment, NPS 20, 0.500-in wall thickness is 20 miles long. The second segment is NPS 16, 0.250-in wall thickness and 15 miles long. The last segment is NPS 14, 0.250-in wall thickness and 10 miles long. The inlet pressure is 1400 psig. Assuming flat terrain, calculate the delivery pressure using the Colebrook-White equation for pressure drop and 0.002 -in absolute pipe roughness throughout. Diesel gravity $=0.85$. Viscosity $=5.0 \mathrm{cSt}$. Compare results using the equivalent length method as well as the method using individual pipe segment pressure drops.

## Solution

Calculate the Reynolds number as follows:

$$
R=\left(\frac{92.24 \times 60000}{5 \times(20-1.0)}\right)=58,257 \quad \text { For a } 20=\text { in section }
$$

and

$$
R=\left(\frac{92.24 \times 60000}{5 \times 15.5)}\right)=71,412 \quad \text { For a } 16=\text { in section }
$$

Finally, $R=81,991$ for a $14=$ in section
Next, calculate the friction factor $f$ for each pipe size.
For a 20 -in pipe section, using the Colebrook-White equation, we get $f$ as follows:

$$
\frac{1}{\sqrt{f}}=-2 \log _{10}\left(\frac{0.002}{3.7 \times 19}+\frac{2.51 \times 1}{58257 f^{0.5}}\right)
$$

Solving for $f$, we get $f=0.02$ (corresponding $F=14.14$ )
Similarly, for a 16 -in pipe section

$$
\frac{1}{\sqrt{f}}=-2 \log _{10}\left(\frac{0.002}{3.7 \times 15.5}+\frac{2.51 \times 1}{71412 f^{0.5}}\right)
$$

and $f=0.0198(F=14.21)$
Finally, for a 14 -in pipe section, $f=0.0194(F=14.36)$
Next we calculate the Pressure drop per mile Pm for 20 inch, 16 inch and 14 inch diameter pipe sizes as follows:

For 20-in pipe

$$
P m=0.2421\left(\frac{60000}{14.14}\right)^{2.0} \times \frac{0.85}{(19)^{5}}=1.5 \mathrm{psi} / \mathrm{mi}
$$

For a 16-in pipe

$$
P m=0.2421\left(\frac{60000}{14.21}\right)^{2.0} \times \frac{0.85}{(15.5)^{5}}=4.10 \mathrm{psi} / \mathrm{mi}
$$

For a 14 -in pipe

$$
P m=0.2421\left(\frac{60000}{14.36}\right)^{2.0} \times \frac{0.85}{(13.5)^{5}}=8.36 \mathrm{psi} / \mathrm{mi}
$$

The total pressure drop in the entire pipeline is

$$
(1.5 \times 20)+(4.1 \times 15)+(8.36 \times 10)=175.1 \mathrm{psi}
$$

Therefore delivery pressure at the end of the pipe is $1400-175.1=$ 1224.9 psi.

Next we will calculate the results using the equivalent length method:
Converting each of the segment in terms of the base diameter of 20 in , we get the equivalent length of the middle segment $(16 \mathrm{in})$ as $((20-1)$ / $(15.5))^{5} \times 15=41.5$ milesand similarly:

The equivalent length of the third segment (14 in) as ( $20-1$ )/ $(13.5))^{5} \times 10=55.22$ miles.

Therefore, the total equivalent length of the entire pipeline is

$$
20.0+41.5+55.22=116.72 \mathrm{mi}
$$

The total pressure drop is then $116.72 \times 1.5=175.08 \mathrm{psi}$, which is the same as before.

## 2. PARALLEL PIPING

Sometimes two or more pipes are connected such that the fluid flow splits among the branch pipes and eventually combine downstream into a single pipe as illustrated in Figure 11.3. Such a piping system is referred to as parallel pipes. It is also called a looped piping system, where each parallel pipe is known as a loop. The reason for installing parallel pipes or loops is to reduce pressure drop in a certain section of the pipeline because of pipe pressure limitation or for increasing the flow rate in a bottleneck section. By


Figure 11.3 Parallel pipes.
installing a pipe loop from B to E in Figure 11.3, we are effectively reducing the overall pressure drop in the pipeline from $A$ to $F$, because between $B$ and E , the flow is split through two pipes.

In Figure 11.3, fluid flows through pipe AB and at point B , part of the flow branches off into pipe BCE, whereas the remainder flows through the pipe BDE . At point E , the flows recombine to the original value and the liquid flows through the pipe EF. We will assume that the entire pipeline system is in the horizontal plane with no changes in pipe elevations.

To solve for the pressures and flow rates in a parallel piping system, such as the one depicted in Figure 11.3, we use the following two principles of pipes in parallel.

1. Conservation of total flow.
2. Common pressure loss across each parallel pipe.

According to the principle (1), the total flow entering each junction of pipe must equal the total flow leaving the junction.

Therefore,

$$
\text { Total Inflow }=\text { Total Outflow }
$$

Thus, in Figure 11.3, all flows entering and leaving the junction B must satisfy the above principle. If the flow into the junction $B$ is $Q$, the flow in branch BCE is $\mathrm{Q}_{\mathrm{BC}}$ and flow in the branch BDE is $\mathrm{Q}_{\mathrm{BD}}$, we have from above conservation of total flow

$$
\begin{equation*}
\mathrm{Q}=\mathrm{Q}_{\mathrm{BC}}+\mathrm{Q}_{\mathrm{BD}} \tag{11.9}
\end{equation*}
$$

The second principle of parallel pipes, defined as (2), requires that the pressure drop across the branch BCE must equal the pressure drop across the branch BDE. This is simply because point B represents the common upstream pressure for each of these branches, whereas the pressure at point E represents the common downstream pressure. Referring to these pressures as $P_{B}$ and $P_{E}$, we can state

$$
\begin{align*}
& \text { Pressure drop in branch } \mathrm{BCE}=\mathrm{P}_{\mathrm{B}}-\mathrm{P}_{\mathrm{E}}  \tag{11.10}\\
& \text { Pressure drop in branch } \mathrm{BDE}=\mathrm{P}_{\mathrm{B}}-\mathrm{P}_{\mathrm{E}} \tag{11.11}
\end{align*}
$$

Assuming that the flow $\mathrm{Q}_{\mathrm{BC}}$ and $\mathrm{Q}_{\mathrm{BD}}$ are in the direction of BCE and BDE , respectively. If we had a third pipe branch between B and E , such as that shown by the dashed line BE in Figure 11.3, we can state that the common pressure drop $\mathrm{P}_{\mathrm{B}}-\mathrm{P}_{\mathrm{E}}$ would be applicable to the third parallel pipe between B and E also.

We can rewrite Eqns (11.9) and (11.10) as follows for the three parallel pipe system.

$$
\begin{equation*}
\mathrm{Q}=\mathrm{Q}_{\mathrm{BC}}+\mathrm{Q}_{\mathrm{BD}}+\mathrm{Q}_{\mathrm{BE}} \tag{11.12}
\end{equation*}
$$

and

$$
\begin{equation*}
\Delta \mathrm{P}_{\mathrm{BCE}}=\Delta \mathrm{P}_{\mathrm{BDE}}=\Delta \mathrm{P}_{\mathrm{BE}} \tag{11.13}
\end{equation*}
$$

where $\Delta \mathrm{P}$ is the pressure drop in respective parallel pipes.
Similar to the equivalent length concept in series piping, we can calculate an equivalent pipe diameter for pipes connected in parallel.

### 2.1 Equivalent Diameter of Pipes: Liquid Pipelines

Because each of the parallel pipes in Figure 11.3 has a common pressure drop, we can replace all the parallel pipes between B and E with one single pipe of length $L_{E}$ and diameter $\mathrm{D}_{\mathrm{E}}$ so that the pressure drop through the single pipe at flow Q equals that of the individual pipes as follows:

Pressure drop in equivalent single pipe length $\mathrm{L}_{\mathrm{E}}$ and diameter $\mathrm{D}_{\mathrm{E}}$ at flow rate $\mathrm{Q}=\Delta \mathrm{P}_{\mathrm{BCE}}$

Assuming now that we have only the two parallel pipes BCE and BDE in Figure 11.3, ignoring the dashed line BE , we can state that

$$
\begin{equation*}
\mathrm{Q}=\mathrm{Q}_{\mathrm{BC}}+\mathrm{Q}_{\mathrm{BD}} \tag{11.14}
\end{equation*}
$$

and

$$
\begin{equation*}
\Delta \mathrm{P}_{\mathrm{EQ}}=\Delta \mathrm{P}_{\mathrm{BCE}}=\Delta \mathrm{P}_{\mathrm{BDE}} \tag{11.15}
\end{equation*}
$$

The pressure drop $\Delta \mathrm{P}_{\mathrm{EQ}}$ for the equivalent pipe can be written as follows, using Eqn (11.15)

$$
\begin{equation*}
\Delta \mathrm{P}_{\mathrm{EQ}}=\mathrm{K}\left(\mathrm{~L}_{\mathrm{E}}\right)(\mathrm{Q})^{2} / \mathrm{D}_{\mathrm{E}}^{5} \tag{11.16}
\end{equation*}
$$

where K is a constant, that depends on the liquid properties.
Eqn (11.16) will then become

$$
\begin{equation*}
\mathrm{K} \mathrm{~L}_{\mathrm{E}} \mathrm{Q}^{2} / \mathrm{D}_{\mathrm{E}}^{5}=\mathrm{K} \mathrm{~L}_{\mathrm{BC}} \mathrm{Q}_{\mathrm{BC}}^{2} / \mathrm{D}_{\mathrm{BC}}^{5}=\mathrm{K} \mathrm{~L}_{\mathrm{BD}} \mathrm{Q}_{\mathrm{BD}}^{2} / \mathrm{D}_{\mathrm{BD}}^{5} \tag{11.17}
\end{equation*}
$$

Simplifying, we get

$$
\begin{equation*}
\mathrm{L}_{\mathrm{E}} \mathrm{Q}^{2} / \mathrm{D}_{\mathrm{E}}^{5}=\mathrm{L}_{\mathrm{BC}} \mathrm{Q}_{\mathrm{BC}}^{2} / \mathrm{D}_{\mathrm{BC}}^{5}=\mathrm{L}_{\mathrm{BD}} \mathrm{Q}_{\mathrm{BD}}^{2} / \mathrm{D}_{\mathrm{BD}}^{5} \tag{11.18}
\end{equation*}
$$

Further simplifying the problem by assuming each loop to be the same length as the equivalent length

$$
\begin{equation*}
\mathrm{L}_{\mathrm{BC}}=\mathrm{L}_{\mathrm{BD}}=\mathrm{L}_{\mathrm{E}} \tag{11.19}
\end{equation*}
$$

We get

$$
\begin{equation*}
\mathrm{Q}^{2} / \mathrm{D}_{\mathrm{E}}^{5}=\mathrm{Q}_{\mathrm{BC}}^{2} / \mathrm{D}_{\mathrm{BC}}^{5}=\mathrm{Q}_{\mathrm{BD}}^{2} / \mathrm{D}_{\mathrm{BD}}^{5} \tag{11.20}
\end{equation*}
$$

Substituting for $\mathrm{Q}_{\mathrm{BD}}$ in terms for $\mathrm{Q}_{\mathrm{BC}}$ from Eqn (11.20), we get

$$
\begin{equation*}
\mathrm{Q}^{2} / \mathrm{D}_{\mathrm{E}}^{5}=\mathrm{Q}_{\mathrm{BC}}^{2} / \mathrm{D}_{\mathrm{BC}}^{5} \tag{11.21}
\end{equation*}
$$

and

$$
\begin{equation*}
\mathrm{Q}_{\mathrm{BC}}^{2} / \mathrm{D}_{\mathrm{BC}}^{5}=\left(\mathrm{Q}-\mathrm{Q}_{\mathrm{BC}}\right)^{2} / \mathrm{D}_{\mathrm{BD}}^{5} \tag{11.22}
\end{equation*}
$$

From Eqns (11.21) and (11.22), we can solve for the two flows $\mathrm{Q}_{\mathrm{BC}}$, $\mathrm{Q}_{\mathrm{BD}}$, and the equivalent diameter $\mathrm{D}_{\mathrm{E}}$ in terms of the known quantities $\mathrm{Q}, \mathrm{D}_{\mathrm{BC}}$, and $\mathrm{D}_{\mathrm{BC}}$.

The following problem will illustrate the equivalent diameter approach in parallel piping systems.

## Problem 11.4: Parallel Pipes in a Liquid Pipeline

A parallel pipe system, transporting water, similar to the one shown in Figure 11.3 is located in a horizontal plane with the following data.

$$
\text { Flow rate } \mathrm{Q}=2000 \mathrm{gal} / \mathrm{min}
$$

Pipe segment $\mathrm{AB}=15.5$ in inside diameter, 4000 ft
Pipe segment $\mathrm{BCE}=12$ in inside diameter, 8000 ft
Pipe segment $\mathrm{BDE}=10$ in inside diameter, 6500 ft
Pipe segment $\mathrm{EF}=15.5$ in inside diameter, 3000 ft

1. Calculate the flow rate through each parallel pipe and the equivalent diameter of a single pipe 5000 -ft-long between B and E to that will replace the two parallel pipes.
2. Determine the pressure required at the origin A to provide a delivery pressure of 50 psig at the terminus F. Use the Hazen-Williams equation with a C factor $=120$.

## Solution

$$
\begin{gathered}
\text { (a) } \mathrm{Q}_{1}+\mathrm{Q}_{2}=2000 \\
\mathrm{Q}_{1}^{2} \mathrm{~L}_{1} / \mathrm{D}_{1}^{5}=\mathrm{Q}_{2}^{2} \mathrm{~L}_{2} / \mathrm{D}_{2}^{5}
\end{gathered}
$$

where suffix 1 and 2 refer to the two branches BCE and BDE, respectively.

$$
\begin{aligned}
\left(\mathrm{Q}_{2} / \mathrm{Q}_{1}\right)^{2} & =\left(\mathrm{D}_{2} / \mathrm{D}_{1}\right)^{5}\left(\mathrm{~L}_{1} / \mathrm{L}_{2}\right) \\
& =(10 / 12)^{5} \times(8000 / 6500) \\
\mathrm{Q}_{2} / \mathrm{Q}_{1} & =0.7033
\end{aligned}
$$

Solving, we get

$$
\begin{aligned}
& \mathrm{Q}_{1}=1174 \mathrm{gal} / \mathrm{min} \\
& \mathrm{Q}_{2}=826 \mathrm{gal} / \mathrm{min}
\end{aligned}
$$

The equivalent pipe diameter for a single pipe 5000 ft long is calculated as follows

$$
\begin{aligned}
(2000)^{2}(5000) / \mathrm{D}_{\mathrm{E}}^{5} & =(1174)^{2} \times 8000 /(12)^{5} \\
\text { or } \quad \mathrm{D}_{\mathrm{E}} & =13.52 \mathrm{in}
\end{aligned}
$$

Therefore, a 13.52 -in-diameter pipe, 5000 ft long between B and E will replace the two parallel pipes.
(b) To determine the pressure required at the origin A, we will first calculate the pressure required at E for the pipe segment EF to provide a delivery pressure of 50 psig at the terminus F .

### 2.2 Parallel Pipes in Gas Pipelines

Similar to the parallel pipes in liquid pipelines discussed in the previous section, we can perform an analysis of the parallel pipes or loops in a gas pipeline system. In calculating the pressure drop through each parallel pipe in a gas pipeline we use a slightly different approach.

According to the second principle of parallel pipes, the pressure drop in pipe branch BCE must equal the pressure drop in pipe branch BDE. This is because both pipe branches have a common starting point (B) and common ending point ( E ). Therefore, the pressure drop in the branch pipe BCE and branch pipe BDE are each equal to $\left(\mathrm{P}_{\mathrm{B}}-\mathrm{P}_{\mathrm{E}}\right)$ where $\mathrm{P}_{\mathrm{B}}$ and $\mathrm{P}_{\mathrm{E}}$ are the pressures at junctions B and E , respectively.

Therefore, we can write

$$
\begin{equation*}
\Delta \mathrm{P}_{\mathrm{BCE}}=\Delta \mathrm{P}_{\mathrm{BDE}}=\mathrm{P}_{\mathrm{B}}-\mathrm{P}_{\mathrm{E}} \tag{11.23}
\end{equation*}
$$

$\Delta \mathrm{P}$ represents pressure drop and $\Delta \mathrm{P}_{\mathrm{BCE}}$ is a function of the diameter and length of branch BCE and the flow rate $\mathrm{Q}_{1}$. Similarly, $\Delta \mathrm{P}_{\mathrm{BDE}}$ is a function of the diameter and length of branch BCE and the flow rate $\mathrm{Q}_{2}$.

To calculate the pressure drop in parallel pipes, we must first determine the flow split at junction $B$. We know that the sum of the two flow rates $Q_{1}$ and $Q_{2}$ must equal the given inlet flow rate Q . If both pipe loops $B C E$ and BDE are equal in lengths and pipe inside diameters, we can infer that the flow rate will be split equally between the two branches.

Thus, for identical pipe loops:

$$
\begin{equation*}
\mathrm{Q}_{1}=\mathrm{Q}_{2}=\frac{\mathrm{Q}}{2} \tag{11.24}
\end{equation*}
$$

In this case, the pressure drop from B to E can be calculated assuming a flow rate of $\frac{\mathrm{Q}}{2}$ flowing through one of the pipe loops.

To illustrate this further, suppose we are interested in determining the pressure at $A$ for the given flow rate Q and a specified delivery pressure $\left(\mathrm{P}_{\mathrm{F}}\right)$ at the pipe terminus F . We start with the last pipe segment EF and calculate the pressure required at E for a flow rate of Q to deliver gas at F at a pressure $\mathrm{P}_{\mathrm{F}}$. We could use the general flow equation for this and substitute $\mathrm{P}_{\mathrm{E}}$ for upstream pressure. $\mathrm{P}_{1}$ and $\mathrm{P}_{\mathrm{F}}$ for downstream pressure $\mathrm{P}_{2}$. Having calculated $\mathrm{P}_{\mathrm{E}}$, we can now consider one of the pipe loops such as BCE and calculate the upstream pressure $P_{B}$ required for a flow rate of $\frac{Q}{2}$ through $B C E$ for a downstream pressure of $\mathrm{P}_{\mathrm{E}}$. In the general flow equation, the upstream pressure $\mathrm{P}_{1}=\mathrm{P}_{\mathrm{B}}$ and the downstream pressure $\mathrm{P}_{2}=\mathrm{P}_{\mathrm{E}}$.

This is correct only for identical pipe loops. Otherwise, the flow rate $\mathrm{Q}_{1}$ and $Q_{2}$ through the pipe branches $B C E$ and $B D E$ will be unequal. From the calculated value of $\mathrm{P}_{\mathrm{E}}$, we can now determine the pressure required at A by applying the general flow equation to pipe segment AB that has a gas flow rate of Q . The upstream pressure $\mathrm{P}_{1}$ will be calculated for a downstream pressure $\mathrm{P}_{2}=\mathrm{P}_{\mathrm{E}}$.

Consider now a situation in which the pipe loops are not identical. This means that the pipes BCE and BDE may have different lengths and different diameters. In this case, we must determine the flow split between these two branches by equating the pressure drops through each of the branches. Because $Q_{1}$ and $Q_{2}$ are two unknowns, we will use the flow conservation principle and the common pressure drop principle to determine the values of $\mathrm{Q}_{1}$ and $\mathrm{Q}_{2}$. From the general flow equation, we can state the following.

The pressure drop from friction in branch BCE can be calculated from

$$
\begin{equation*}
\left(\mathrm{P}_{B}^{2}-\mathrm{P}_{E}^{2}\right)=\frac{\mathrm{K}_{1} \mathrm{~L}_{1} \mathrm{Q}_{1}^{2}}{\mathrm{D}_{1}^{5}} \tag{11.25}
\end{equation*}
$$

where
$\mathrm{K}_{1}=$ a parameter that depends on gas properties, gas temperature, etc.
$\mathrm{L}_{1}=$ length of pipe branch BCE
$\mathrm{D}_{1}=$ inside diameter of pipe branch BCE
$\mathrm{Q}_{1}=$ flow rate through pipe branch BCE
Other symbols are as defined previously.
$\mathrm{K}_{1}$ is a parameter that depends on the gas properties, gas temperature, base pressure, and base temperature that will be the same for both pipe branches BCE and BDE in a parallel pipeline system. Hence we regard this as a constant from branch to branch.

Similarly, the pressure drop because of friction in branch BDE is calculated from

$$
\begin{equation*}
\left(\mathrm{P}_{B}^{2}-\mathrm{P}_{E}^{2}\right)=\frac{\mathrm{K}_{2} \mathrm{~L}_{2} \mathrm{Q}_{2}^{2}}{\mathrm{D}_{2}^{5}} \tag{11.26}
\end{equation*}
$$

where
$\mathrm{K}_{2}=$ a constant like $\mathrm{K}_{1}$
$\mathrm{L}_{2}=$ length of pipe branch BDE
$\mathrm{D}_{2}=$ inside diameter of pipe branch BDE
$\mathrm{Q}_{2}=$ flow rate through pipe branch BDE
Other symbols are as defined earlier.
In Eqns (11.25) and (11.26), the constants $\mathrm{K}_{1}$ and $\mathrm{K}_{2}$ are equal because of they do not depend on the diameter or length of the branch pipes BCE and BDE. Combining both equations, we can state the following for common pressure drop through each branch.

$$
\begin{equation*}
\frac{\mathrm{L}_{1} \mathrm{Q}_{1}^{2}}{\mathrm{D}_{1}^{5}}=\frac{\mathrm{L}_{2} \mathrm{Q}_{2}^{2}}{\mathrm{D}_{2}^{5}} \tag{11.27}
\end{equation*}
$$

Simplifying further, we get the following relationship between the two flow rates $Q_{1}$ and $Q_{2}$.

$$
\begin{equation*}
\frac{\mathrm{Q}_{1}}{\mathrm{Q}_{2}}=\left(\frac{\mathrm{L}_{2}}{\mathrm{~L}_{1}}\right)^{0.5}\left(\frac{\mathrm{D}_{1}}{\mathrm{D}_{2}}\right)^{2.5} \tag{11.28}
\end{equation*}
$$

Combining Eqn (11.27) with Eqn (11.28), we can solve for the flow rates $Q_{1}$ and $Q_{2}$.

To illustrate this, consider the inlet flow $Q=100$ MMSCFD and the pipe branches as follows

$$
\begin{array}{ll}
\mathrm{L}_{1}=10 \mathrm{mi} & \mathrm{D}_{1}=15.5 \mathrm{in} . \\
\mathrm{L}_{2}=15 \mathrm{mi} & \mathrm{D}_{2}=13.5 \mathrm{in} . \\
\text { for branch } \mathrm{BCE}
\end{array}
$$

From Eqn (11.24) for flow conservation, we get

$$
\mathrm{Q}_{1}+\mathrm{Q}_{2}=100
$$

From Eqn (11.28), we get the ratio of flow rates as

$$
\frac{\mathrm{Q}_{1}}{\mathrm{Q}_{2}}=\left(\frac{15}{10}\right)^{0.5}\left(\frac{15.5}{13.5}\right)^{2.5}=1.73
$$

Solving these two equations in $\mathrm{Q}_{1}$ and $\mathrm{Q}_{2}$, we get

$$
\begin{aligned}
& \mathrm{Q}_{1}=63.37 \mathrm{MMSCFD} \\
& \mathrm{Q}_{2}=36.63 \mathrm{MMSCFD}
\end{aligned}
$$

Once we know the values of $\mathrm{Q}_{1}$ and $\mathrm{Q}_{2}$, we can easily calculate the common pressure drop in the branch pipes BCE and BDE. A problem will be used to illustrate this method.

Another method of calculating pressure drops in parallel pipes is using the equivalent diameter. In this method, we replace the pipe loops BCE and BDE with a certain length of an equivalent diameter pipe that has the same pressure drop as one of the branch pipes. The equivalent diameter pipe can be calculated using the general flow equation as explained next. The equivalent pipe with the same $\Delta \mathrm{P}$ that will replace both branches will have a diameter $D_{e}$ and a length equal to one of the branch pipes, say $L_{1}$.

Because of the pressure drop in the equivalent diameter pipe, which flows the full volume $Q$, is the same as that in any of the branch pipes, we can state the following:

$$
\begin{equation*}
\left(\mathrm{P}_{\mathrm{B}}^{2}-\mathrm{P}_{\mathrm{E}}^{2}\right)=\frac{\mathrm{k}_{\mathrm{e}} \mathrm{~L}_{\mathrm{e}} \mathrm{Q}^{2}}{\mathrm{D}_{\mathrm{e}}^{5}} \tag{11.29}
\end{equation*}
$$

where $\mathrm{Q}=\mathrm{Q}_{1}+\mathrm{Q}_{2}$ from Eqn (11.24) and $\mathrm{K}_{\mathrm{e}}$ represents the constant for the equivalent diameter pipe of length $\mathrm{L}_{\mathrm{e}}$ flowing the full volume Q . Equating the value of $\left(\mathrm{P}_{\mathrm{B}}^{2}-\mathrm{P}_{\mathrm{E}}^{2}\right)$ to the corresponding values considering each branch separately, we get

$$
\begin{equation*}
\frac{\mathrm{K}_{1} \mathrm{~L}_{1} \mathrm{Q}_{1}^{2}}{\mathrm{D}_{1}^{5}}=\frac{\mathrm{K}_{2} \mathrm{~L}_{2} \mathrm{Q}_{2}^{2}}{\mathrm{D}_{2}^{5}}=\frac{\mathrm{K}_{e} \mathrm{~L}_{e} \mathrm{Q}^{2}}{\mathrm{D}_{e}^{5}} \tag{11.30}
\end{equation*}
$$

Also setting $\mathrm{K}_{1}=\mathrm{K}_{2}=\mathrm{K}_{\mathrm{e}}$ and $\mathrm{L}_{\mathrm{e}}=\mathrm{L}_{1}$, we simplify Eqn (11.30) as follows.

$$
\begin{equation*}
\frac{\mathrm{L}_{1} \mathrm{Q}_{1}^{2}}{\mathrm{D}_{1}^{5}}=\frac{\mathrm{L}_{2} \mathrm{Q}_{2}^{2}}{\mathrm{D}_{2}^{5}}=\frac{\mathrm{L}_{1} \mathrm{Q}^{2}}{\mathrm{D}_{e}^{5}} \tag{11.31}
\end{equation*}
$$

Using Eqn (11.30) in conjunction with Eqn (11.31), we solve for the equivalent diameter $D_{e}$ as

$$
\begin{equation*}
\mathrm{De}=\mathrm{D}_{1}\left[\left(\frac{1+\text { Const1 }}{\text { Const1 }}\right)^{2}\right]^{1 / 5} \tag{11.32}
\end{equation*}
$$

where

$$
\begin{equation*}
\text { Const1 }=\sqrt{\left(\frac{D_{1}}{D_{2}}\right)^{5}\left(\frac{L_{2}}{L_{1}}\right)} \tag{11.33}
\end{equation*}
$$

And the individual flow rates $Q_{1}$ and $Q_{2}$ are calculated from

$$
\begin{equation*}
\mathrm{Q}_{1}=\frac{\mathrm{Q} \text { Const } 1}{1+\text { Const } 1} \tag{11.34}
\end{equation*}
$$

and

$$
\begin{equation*}
\mathrm{Q}_{2}=\frac{\mathrm{Q}}{1+\text { Const1 }} \tag{11.35}
\end{equation*}
$$

To illustrate the equivalent diameter method, consider the inlet flow $\mathrm{Q}=100 \mathrm{MMSCFD}$ and the pipe loops as follows

$$
\begin{aligned}
& \mathrm{L}_{1}=10 \mathrm{mi} \quad \mathrm{D}_{1}=15.5 \mathrm{in} . \quad \text { for branch BCE } \\
& \mathrm{L}_{2}=15 \mathrm{mi} \quad \mathrm{D}_{2}=13.5 \mathrm{in} . \quad \text { for branch BDE }
\end{aligned}
$$

From Eqn (11.35)

$$
\text { Const1 }=\sqrt{\left(\frac{15.5}{13.5}\right)^{5}\left(\frac{15}{10}\right)}=1.73
$$

Using Eqn (11.32), the equivalent diameter is

$$
\mathrm{De}=15.5\left[\left(\frac{1+1.73}{1.73}\right)^{2}\right]^{1 / 5}=18.60 \mathrm{in}
$$

Thus the NPS 16 and NPS 14 pipes in parallel can be replaced with an equivalent pipe having an inside diameter of 18.6 in.

Next we calculate the flow rates in the two parallel pipes as follows

$$
\mathrm{Q}_{1}=\frac{100 \times 1.73}{1+1.73}=63.37 \mathrm{MMSCFD}
$$

and

$$
\mathrm{Q}_{2}=36.63 \mathrm{MMSCFD}
$$

Having calculated an equivalent diameter De , we can now calculate the common pressure drop in the parallel branches by considering the entire flow Q flowing through the equivalent diameter pipe.

## Problem 11.5: Gas Pipeline

A gas pipeline consists of two parallel pipes, as shown in Figure 11.3. It is operated at a flow rate of 100 MMSCFD. The first pipe segment $A B$ is 12 miles long and consists of NPS 16, 0.250-in wall thickness pipe. The loop BCE is 24 miles long and consists of NPS 14, 0.250-in wall thickness pipe. The loop BDE is 16 miles long and consists of NPS 12, 0.250 -in wall thickness pipe. The last segment EF is 20 miles long, NPS 16, 0.250-in wall thickness pipe. Assuming a gas gravity of 0.6, calculate the outlet pressure at F and the pressures at the beginning and the end of the pipe loops and the flow rates through them. The inlet pressure at $\mathrm{A}=1200 \mathrm{psig}$. Gas flowing temperature $=80^{\circ} \mathrm{F}$, base temperature $=60^{\circ} \mathrm{F}$, and base pressur$\mathrm{e}=14.73$ psia. Compressibility factor $\mathrm{Z}=0.92$. Use the general flow equation with Colebrook friction factor $f=0.015$.

## Solution

From Eqn (11.28), the ratio of the flow rates through the two pipe loops is given by

$$
\frac{\mathrm{Q}_{1}}{\mathrm{Q}_{2}}=\left(\frac{16}{24}\right)^{0.5}\left(\frac{14-2 \times 0.25}{12.75-2 \times 0.25}\right)^{2.5}=1.041
$$

And from Eqn (11.24)

$$
\mathrm{Q}_{1}+\mathrm{Q}_{2}=100
$$

Solving for $\mathrm{Q}_{1}$ and $\mathrm{Q}_{2}$, we get
$\mathrm{Q}_{1}=51.0 \mathrm{MMSCFD}$ and $\mathrm{Q}_{2}=49.0 \mathrm{MMSCFD}$.
Next considering the first pipe segment AB , we will calculate the pressure at B based on the inlet pressure of 1200 psig at A , using the general flow equation as follows.
$100 \times 10^{6}=77.54\left(\frac{1}{\sqrt{0.015}}\right)\left(\frac{520}{14.73}\right)\left[\frac{\left(1214.73^{2}-P_{2}^{2}\right)}{0.6 \times 540 \times 12 \times 0.92}\right]^{0.5} 15.5^{2.5}$
Solving for the pressure at B , we get

$$
\mathrm{P}_{2}=1181.33 \mathrm{psia}=1166.6 \mathrm{psig}
$$

This is the pressure at the beginning of the looped section at B. Next we calculate the outlet pressure at E of pipe branch BCE considering a flow rate of 51 MMSCFD through the NPS 14 pipe, starting at a pressure of 1181.33 psia at B.

Using the general flow equation, we get

$$
51 \times 10^{6}=77.54\left(\frac{1}{\sqrt{0.015}}\right)\left(\frac{520}{14.73}\right)\left[\frac{\left(1181.33^{2}-P_{2}^{2}\right)}{0.6 \times 540 \times 24 \times 0.92}\right]^{0.5} 13.5^{2.5}
$$

Solving for the pressure at E , we get

$$
\mathrm{P}_{2}=1145.63 \mathrm{psia}=1130.9 \mathrm{psig}
$$

We will now calculate the pressures using the equivalent diameter method.

From Eqn (11.35)

$$
\text { Const } 1=\sqrt{\left(\frac{13.5}{12.25}\right)^{5}\left(\frac{16}{24}\right)}=1.041
$$

From Eqn (11.32), the equivalent diameter is

$$
\mathrm{De}=13.5\left[\left(\frac{1+1.041}{1.041}\right)^{2}\right]^{1 / 5}=17.67 \mathrm{in}
$$

Thus we can replace the two branch pipes between $B$ and $E$ with a single piece of pipe 24 miles long having an inside diameter of 17.67 in flowing 100 MMSCFD.

The pressure at B was calculated earlier as

$$
\mathrm{P}_{\mathrm{B}}=1181.33 \mathrm{psia}
$$

Using this pressure, we can calculate the downstream pressure at E for the equivalent pipe diameter as follows

$$
100 \times 10^{6}=77.54\left(\frac{1}{\sqrt{0.015}}\right)\left(\frac{520}{14.73}\right)\left[\frac{\left(1181.33^{2}-P_{2}^{2}\right)}{0.6 \times 540 \times 24 \times 0.92}\right]^{0.5} 17.67^{2.5}
$$

Solving for the outlet pressure at E, we get

$$
\mathrm{P}_{2}=1145.60 \mathrm{psia}
$$

which is almost the same as what we calculate before.
The pressure at F will therefore be the same as what we calculated before.
Therefore, using the equivalent diameter method the parallel pipes BCE and BDE can be replaced with a single pipe 24 miles long having an inside diameter of 17.67 in .

## Problem 11.6: Gas Pipeline (SI Units)

A natural gas pipeline DN 500 with a $12-\mathrm{mm}$ wall thickness is 60 km long. The gas flow rate is $5.0 \mathrm{Mm}^{3} /$ day at $20^{\circ} \mathrm{C}$. Calculate the inlet pressure required for a delivery pressure of 4 MPa (absolute), using the general flow equation with the modified Colebrook-White friction factor. Pipe roughness $=0.015 \mathrm{~mm}$. To increase the flow rate through the pipeline, the entire line is looped with a DN 500 pipeline, 12 -mm-wall thickness. Assuming the same delivery pressure, calculate the inlet pressure at the new flow rate of $8 \mathrm{Mm}^{3} /$ day. Gas gravity $=0.65$. Viscosity $=$ 0.000119 poise. Compressibility factor $\mathrm{Z}=0.88$. Base temperature $=$ $15^{\circ} \mathrm{C}$ and base pressure $=101 \mathrm{kPa}$. If the inlet and outlet pressures are held the same as before, what length of the pipe should be looped to achieve the increased flow?

## Solution

Pipe inside diameter $\mathrm{D}=500-2 \times 12=476 \mathrm{~mm}$.
Flow rate $\mathrm{Q}=5.0 \times 10^{6} \mathrm{~m}^{3} /$ day.
Base temperature $\mathrm{T}_{\mathrm{b}}=15+273=288 \mathrm{~K}$.
Gas flow temperature $\mathrm{T}_{\mathrm{f}}=20+273=293 \mathrm{~K}$.
Delivery pressure $\mathrm{P}_{2}=4 \mathrm{MPa}$.
Calculate the Reynolds number as follows:

$$
R=0.5134\left(\frac{101}{288}\right)\left(\frac{0.65 \times 5 \times 10^{6}}{0.000119 \times 476}\right)=10,330,330
$$

From the modified Colebrook-White equation, the transmission factor is

$$
F=-4 \log _{10}\left(\frac{0.015}{3.7 \times 476}+\frac{1.4125 F}{10,330,330}\right)
$$

Solving by successive iteration, we get

$$
F=19.80
$$

Using the general flow equation, the inlet pressure is calculated next.

$$
\begin{aligned}
5 \times 10^{6}= & 5.747 \times 10^{-4} \\
& \times 19.80\left(\frac{273+15}{101}\right)\left[\frac{\mathrm{P}_{1}^{2}-4000^{2}}{0.65 \times 293 \times 60 \times 0.88}\right]^{0.5} \times(476)^{2.5}
\end{aligned}
$$

Solving for the inlet pressure, we get

$$
\left.\mathrm{P}_{1}=5077 \mathrm{kPa}(\text { absolute })=5.08 \mathrm{MPa} \text { (absolute }\right)
$$

Therefore, the inlet pressure required at $5 \mathrm{Mm}^{3} /$ day flow rate is 5.08 MPa .

Next, at $8 \mathrm{Mm}^{3} /$ day flow rate, we calculate the new inlet pressure with the entire $60-\mathrm{km}$ length looped with an identical DN 500 pipe. Because the
loop is the same size as the main line, each parallel branch will carry half the total flow rate or $4 \mathrm{Mm}^{3} /$ day .

We calculate the Reynolds number for flow through one of the loops.

$$
R=0.5134\left(\frac{101}{288}\right)\left(\frac{0.65 \times 4 \times 10^{6}}{0.000119 \times 476}\right)=8,264,264
$$

From the modified Colebrook-White equation, the transmission factor is

$$
F=-4 \log _{10}\left(\frac{0.015}{3.7 \times 476}+\frac{1.4125 F}{8,264,264}\right)
$$

Solving by successive iteration, we get

$$
F=19.70
$$

Keeping the delivery pressure the same as before ( 4 MPa ), using the general flow equation, we calculate the inlet pressure required as follows.

$$
\begin{aligned}
4 \times 10^{6}= & 5.747 \times 10^{-4} \\
& \times 19.70\left(\frac{273+15}{101}\right)\left[\frac{\mathrm{P}_{1}^{2}-4000^{2}}{0.65 \times 293 \times 60 \times 0.88}\right]^{0.5} \times(476)^{2.5}
\end{aligned}
$$

Solving for the inlet pressure, we get

$$
\left.\mathrm{P}_{1}=4724 \mathrm{kPa}(\text { absolute })=4.72 \mathrm{MPa} \text { (absolute }\right)
$$

Therefore, for the fully looped pipeline at $8 \mathrm{Mm}^{3}$ /day flow rate the inlet pressure required is

$$
4.72 \mathrm{MPa} .
$$

Next, keeping the inlet and outlet pressures the same at 5077 and 4000 kPa , respectively, at the new flow rate of $8 \mathrm{Mm}^{3} /$ day we assume L km of the pipe from the inlet is looped. We will calculate the value of L by first calculating the pressure at the point where the loop ends. Because each parallel pipe carries $4 \mathrm{Mm}^{3} /$ day, we use the Reynolds number and transmission factor calculated earlier.

$$
R=8,264,264 \quad \text { and } \quad F=19.70
$$

Using the general flow equation, we calculate the outlet pressure at the end of the loop of length L km as follows.

$$
\begin{aligned}
4 \times 10^{6}= & 5.747 \times 10^{-4} \\
& \times 19.70\left(\frac{273+15}{101}\right)\left[\frac{5077^{2}-P_{2}^{2}}{0.65 \times 293 \times L \times 0.88}\right]^{0.5} \times(476)^{2.5}
\end{aligned}
$$

Solving for pressure in terms of the loop length L , we get

$$
\begin{equation*}
\mathrm{P}_{2}^{2}=5077^{2}-105,291.13 \mathrm{~L} \tag{11.36}
\end{equation*}
$$

Next we apply the general flow equation for the pipe segment of length $(60-\mathrm{L}) \mathrm{km}$, that carries the full $8 \mathrm{Mm}^{3} /$ day flow rate. The inlet pressure is $\mathrm{P}_{2}$ and the outlet pressure is 4000 kPa .

The Reynolds number at $8 \mathrm{Mm}^{3} /$ day is

$$
R=0.5134\left(\frac{101}{288}\right)\left(\frac{0.65 \times 8 \times 10^{6}}{0.000119 \times 476}\right)=16,528,528
$$

From the modified Colebrook-White equation, the transmission factor is

$$
F=-4 \log _{10}\left(\frac{0.015}{3.7 \times 476}+\frac{1.4125 F}{16,528,528}\right)
$$

Solving by successive iteration, we get

$$
F=19.96
$$

Using the general flow equation, we calculate the inlet pressure for the pipe segment of length (60-L) km as follows.

$$
\begin{aligned}
8 \times 10^{6}= & 5.747 \times 10^{-4} \\
& \times 19.96\left(\frac{273+15}{101}\right)\left[\frac{P_{2}^{2}-4000^{2}}{0.65 \times 293 \times(60-\mathrm{L}) \times 0.88}\right]^{0.5} \\
& \times(476)^{2.5}
\end{aligned}
$$

Simplifying, we get

$$
\begin{equation*}
\mathrm{P}_{2}^{2}=4000^{2}+410,263.77(60-\mathrm{L}) \tag{11.37}
\end{equation*}
$$

From Eqns (11.36) and (11.37), eliminating $\mathrm{P}_{2}$, we solve for L as follows.

$$
5077^{2}-105,291.13 \mathrm{~L}=4000^{2}+410,263.77(60-\mathrm{L})
$$

Therefore

$$
\mathrm{L}=48.66 \mathrm{~km}
$$

Thus 48.66 km of the $60-\mathrm{km}$ pipeline length will have to be looped starting at the pipe inlet so that at $8 \mathrm{Mm}^{3} /$ day both inlet and outlet pressures will be the same as before at $5 \mathrm{Mm}^{3} /$ day.

What will be the effect if the loop was installed starting at the downstream end of the pipeline and proceeding towards the upstream end? Will the results be the same? In the next section, we will explore the best location to install the pipe loop.

## 3. LOCATING PIPE LOOP: GAS PIPELINES

In the preceding example, we looked at looping an entire pipeline to reduce pressure drop and increase the flow rate. We also explored looping a portion of the pipe, beginning at the upstream end. How do we determine where the loop should be placed for optimum results? Should it be located upstream, downstream, or in a mid-section of the pipe? We will analyze this as follows.

Three looping scenarios are presented in Figure 11.4.
In case (1), a pipeline of length $L$ is shown looped with $X$ miles of pipe, beginning at the upstream end $A$. In case (2), the same length $X$ of pipe is looped, but it is located on the downstream end B. Case (3) shows the mid-section of the pipeline being looped. For most practical purposes, we can say that the cost of all three loops will be the same as long as the loop length is the same.

To determine which of these cases are optimum, we must analyze how the pressure drop in the pipeline varies with distance from the pipe inlet to outlet. It is found that, if the gas temperature is constant throughout, at locations near the upstream end the pressure drops at a slower rate than at the downstream end. Therefore, there is more pressure drop in the downstream section compared to that in the upstream section. Hence, to reduce the overall pressure drop, the loop must be installed towards the downstream end of the pipe. This argument is valid only if the gas temperature is constant
(a)


Upstream loop
(b)


Downstream loop
(c)


Figure 11.4 Different looping scenarios.
throughout the pipeline. In reality, because of heat transfer between the flowing gas and the surrounding soil (buried pipe) or the outside air (above-ground pipe), the gas temperature will change along the length of the pipeline. If the gas temperature at the pipe inlet is higher than that of the surrounding soil (buried pipe), the gas will lose heat to the soil and the temperature will drop from the pipe inlet to the pipe outlet. If the gas is compressed at the inlet using a compressor, then the gas temperature will be a lot higher than that of the soil immediately downstream of the compressor. The hotter gas will cause higher pressure drops (examine the general flow equation and see how the pressure varies with the gas flow temperature). Hence, in this case, the upstream segment will have a larger pressure drop compared with the downstream segment. Therefore, considering heat transfer effects, the pipe loop should be installed in the upstream portion for maximum benefit. The installation of the pipe loop in the mid-section of the pipeline as in case (3) in Figure 11.4 will not be the optimum location based on the preceding discussion. It can therefore be concluded that if the gas temperature is fairly constant along the pipeline, the loop should be installed toward the downstream end as in case (2). If heat transfer is taken into account and the gas temperature varies along the pipeline, with the hotter gas being upstream, the better location for the pipe loop will be on the upstream end as in case (1).

Looping pipes will be explored more in Chapter 14, where we discuss several case studies and pipeline economics.

