SCHEDULING/COST OPTIMIZATION AND NEURAL DYNAMICS MODEL FOR CONSTRUCTION

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ABSTRACT: A general mathematical formulation is presented for the scheduling of construction projects and is applied to the problem of highway construction scheduling. Repetitive and nonrepetitive tasks, work continuity constraints, multiple-crew strategies, and the effects of varying job conditions on the performance of a crew can be modeled. An optimization formulation is presented for the construction project scheduling problem, with the goal of minimizing the direct construction cost. The nonlinear optimization is then solved by the neural dynamics model developed recently by Adeli and Park. For any given construction duration, the model yields the optimum construction schedule for minimum construction cost automatically. By varying the construction duration, one can solve the cost-duration trade-off problem and obtain the global optimum schedule and the corresponding minimum construction cost. The new construction scheduling model provides the capabilities of both the critical path method (CPM) and linear scheduling method (LSM) approaches. In addition, it provides features desirable for repetitive projects, such as highway construction, and allows schedulers greater flexibility. It is particularly suitable for studying the effects of change order on the construction cost. This research provides the mathematical foundation for development of a new generation of more general, flexible, and accurate construction scheduling systems.

INTRODUCTION

Most construction projects involve a combination of repetitive and nonrepetitive tasks. A typical example is highway construction in which tasks such as clearing and grubbing are performed repeatedly over the length of the highway, and tasks such as site office construction are carried out only once. Presently, traditional network scheduling methods such as critical path method (CPM) and program evaluation and review technique are used for the scheduling and monitoring of such projects. Despite their extensive use, these methods have a number of shortcomings:

- Network methods do not guarantee continuity of work in time, which may result in crews being idle.
- Multiple-crew strategies are difficult to implement in the network methods.
- The network diagram is not suitable for monitoring the progress of a project.
- Network methods do not provide an efficient structure for the representation of repetitive tasks. All tasks are represented similarly, and there is no consideration of the location of work in the scheduling.

To overcome these shortcomings, new approaches have been proposed in the literature, particularly for repetitive projects. Fig. 1 presents a linear planning chart that is a graph of location (distance) versus time for the work to be carried out. Such a planning chart represents the progress of a task and can be used to monitor a project. The linear planning chart (also called LSM diagram) motivated the development of the linear scheduling method (LSM). Selinger (1980) presented equations for the lines in a linear planning chart assuming noninterference of crews and continuity of work. Johnston (1981) showed that the LSM is flexible and can be used to model most situations encountered in highway construction projects. Using optimal control theory, Handa and Barcia (1986) formulated the problem as an optimization one that minimizes the project duration. These early LSM models had limitations such as constant rate of production for each task, binding continuity constraints, and no provisions for the use of multiple crews.

Russell and Caselton (1988) presented a dynamic programming formulation to minimize the project duration. Their formulation can accommodate variable production rates for each task and nonbinding work continuity constraints. Russell and Wong (1993) described and showed the use of a general scheduling model developed by incorporating the capabilities of CPM and LSM. In their model each task is defined by a set of attributes that are then linked together using general precedence conditions to form a schedule.

Highway construction projects are large projects in terms of capital requirement. Minimizing cost is therefore a primary goal in the planning and scheduling of such projects. Cost, however, is closely related to time. In general, direct project cost increases with a decrease in project duration, and this trade-off problem is complicated by the number of variables involved. A computer model to automate the process of project direct cost minimization is therefore highly desirable.

In the recent literature, direct cost optimization systems have been presented for LSM and CPM. Reda's (1990) LSM model assumes constant production rate for each task and binding constraints on work continuity. The cost-duration relationship for each task is assumed to be linear. Liang et al. (1995) present a hybrid linear/integer programming approach for handling a combination of discrete and linearly continuous cost-duration relationships for tasks.

In this paper a general mathematical formulation is presented for the scheduling of construction projects. Various scheduling constraints are expressed mathematically. The construction scheduling is posed as an optimization problem in which project direct cost is minimized for a given project duration assuming any combination of linear and nonlinear task cost-duration relationships. The robust neural dynamics model developed recently by Adeli and Park (1995a) is adapted for optimization.

COST-DURATION RELATIONSHIP OF PROJECT

The major cost of a project consists of direct and indirect costs. The resources allocated to each task of a project deter-

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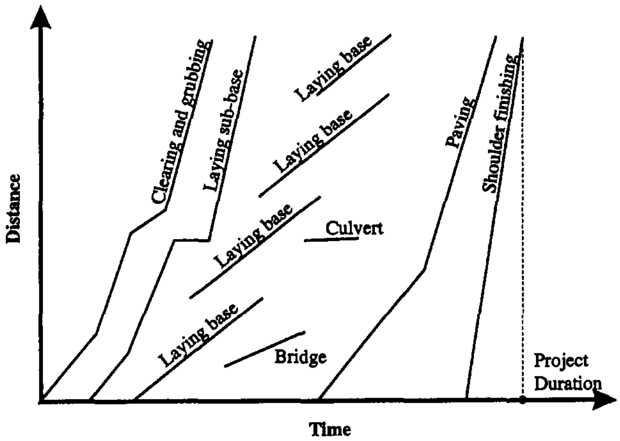

FIG. 1. Linear Planning Chart

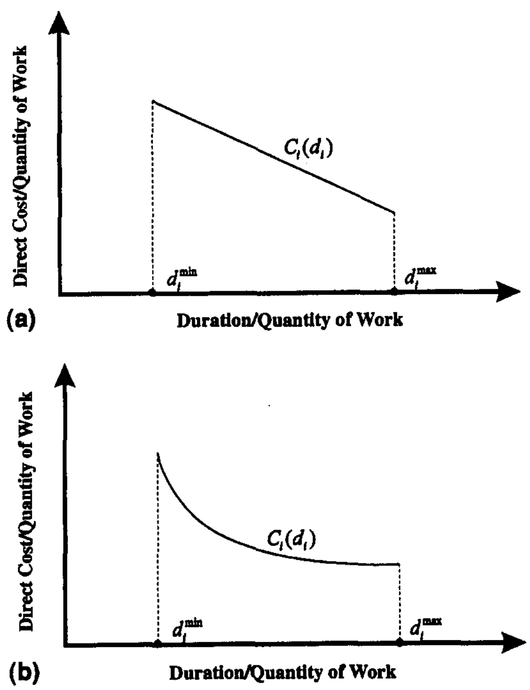

FIG. 2. Direct Cost-Duration Curve for Unit Quantity of Task *i*: (a) Linear; (b) Nonlinear

mine the direct cost. Indirect costs are overhead costs. Additional costs may be incurred by the contractor in the form of damages, if the project is not completed on time. The duration of a project is obtained by sequencing individual tasks whose durations are estimated from a knowledge of the resources allocated to each task and the job conditions. Thus, cost and duration are intricately related. Both of these parameters are of great importance to the contractor who strives to minimize cost while satisfying the contractual requirements, the most important of which is the completion deadline. Assuming the sequencing constraints are not changed, the direct cost, in general, has an inverse relationship with the duration of a construction project. The indirect cost increases with an increase in the duration of the project. The total cost is the sum of these two costs and can increase or decrease with duration. The goal is to obtain the global optimum solution for the scheduling problem.

There is also an inverse relationship between direct cost and the duration of an individual task. A scheduler estimates the time required to complete a task from the resources allocated to it. This time is based on assumed labor and equipment productivity rates ignoring the effects of varying job conditions. Depending on the options and the availability of resources the scheduler has for each task, a cost-duration curve can be constructed. This curve can be continuous or discrete. For efficient mathematical formulation, the discrete relationship is approximated by a continuous linear [Fig. 2(a)] or nonlinear [Fig. 2(b)] curve. In this way a continuous variable optimization technique can be used to solve the construction time-cost trade-off problem.

In highway construction it is convenient to represent cost and duration of a task in unit quantities of work. If d_i is the time required to complete a unit quantity of work of task i and W_{ij} is the total quantity of work required in segment j of task i, then the actual duration, D_{ij}, can be expressed as

$$D_{ij} = \mu_{ij} d_i W_{ij} \tag{1}$$

where μ_{ij} = job condition factor reflecting effects of variable conditions such as weather, soil conditions, terrain, site congestion, learning effects, and so on.

FORMULATION OF SCHEDULING OPTIMIZATION PROBLEM

A general mathematical formulation of the scheduling problem is presented in this section. The advantage of such a general formulation is that it can be specialized and reduced for the solution of specific and, perhaps, less complicated scheduling problems. Furthermore, it can be effectively integrated with the general neural dynamics model for solution of optimization problems developed by Adeli and Park (1995a).

Both nonrepetitive and repetitive tasks are considered in the formulation. The nonrepetitive tasks correspond to the activities of the traditional network methods such as CPM. A nonrepetitive task involves no internal logic because it is performed only once. A repetitive task, on the other hand, may have an elaborate internal logic that connects the segments assigned to various crews. By specifying appropriate constraints, work continuity considerations and multiple-crew strategies can be modeled. A crew rarely performs at ideal productivity throughout a job; its performance is affected by the varying job conditions. This is included in our scheduling model by means of a factor, μ_{ij}, that modifies the ideal productivity of a crew to reflect the effect of the job conditions. The external logic of each task is specified by means of a full set of precedence relationships and/or stage (distance) and time buffers.

Development of the general scheduling formulation for a construction project such as highway construction involves the following steps divided into three main categories (headings) as follows:

Break Down Work into Tasks, Crews, and Segments

Step 1

Break down the project into N_T tasks. Identify nonrepetitive and repetitive tasks. Let N_{NT} and N_{RT} be the number of nonrepetitive and repetitive tasks, respectively. If $N_{RT} = 0$, skip steps 2–5 and go to step 6.

Step 2

For each repetitive task i, choose the number of crews to be used (N_{Ci}). Nonrepetitive tasks have only one crew that performs over one segment only.

Step 3

Assign N_{Si}^k segments of the highway to crew k of repetitive task i. The segments are chosen considering the job conditions and quantity of work required—factors affecting the production rate. In addition, predetermined breaks in the work of a crew may influence the choice of segments. The segments are not required to have equal lengths or to be constructed in sequence. Each segment is identified by Z_{ij}^k and $Z_{ij}^{k'}$, the beginning and ending distances at which repetitive task i is performed by crew k over segment j. Note that each crew of a task is assigned a unique set of segments; two crews cannot perform the same task over the same portion of the highway.

Specify Internal Logic of Repetitive Tasks

For each crew of a repetitive task, do the following:

Step 4

Specify the work continuity relationship between segments j and $j + 1$, in the following form:

$$T_{ij}^k + D_{ij}^k + S_{ij}^k \le T_{i(j+1)}^k \quad (2)$$

where T_{ij}^k = time at which crew k of task i starts work on segment j; D_{ij}^k = duration of work for crew k of task i on segment j; and S_{ij}^k = idle or slack time of crew k of task i between segments j and $j + 1$. For continuity of work S_{ij}^k must be equal to 0. If a task has only one crew, skip step 5 and go to step 6.

Step 5

Define the start of a crew with respect to previous crew(s). The following precedence relationships of start-to-start, finish-to-finish, and start-to-finish are used:

Start-to-start (SS)

$$T_{i1}^k + L_{SSi}^{kl} \le T_{i1}^l \quad (3)$$

Finish-to-finish (FF)

$$T_{iN_{Si}^k}^k + D_{iN_{Si}^k}^k + L_{FFi}^{kl} \le T_{iN_{Si}^l}^l + D_{iN_{Si}^l}^l \quad (4)$$

Start-to-finish (SF)

$$T_{i1}^k + L_{SFi}^{kl} \le T_{iN_{Si}^l}^l + D_{iN_{Si}^l}^l \quad (5)$$

where superscripts l and k = current and previous crews, respectively; and L_{SSi}^{kl}, L_{FFi}^{kl}, and L_{SFi}^{kl} are start-to-start, finish-to-finish, and start-to-finish time lags between crews k and l, respectively. These time lags may be given as a function of quantity of work and/or time. If more than one relationship is specified for a particular crew, only one will govern in the final minimum cost schedule obtained from the optimization algorithm. This particular relationship usually is not known in advance, and all possible relationships have to be specified in the optimization model.

Specify External Logic of Repetitive and Nonrepetitive Tasks

Step 6

Describe the sequencing of the tasks in the project. Each task can be linked with any number of previous tasks by specifying one or more of the following precedence relationships:

Start-to-start (SS)

$$T_{i1}^1 + L_{SSij} \le T_{j1}^1 \quad (6)$$

Finish-to-start (FS)

$$T_{iN_{Si}^k}^k + D_{iN_{Si}^k}^k + L_{FSij} \le T_{j1}^1 \quad k = 1, \ldots, N_{Ci} \quad (7)$$

Start-to-finish (SF) (specified when the task has only one crew)

$$T_{i1}^1 + L_{SFij} \le T_{jN_{Sj}^l}^l + D_{jN_{Sj}^l}^l \quad l = 1 \quad (8)$$

Finish-to-finish (FF) (specified when both tasks have one crew only)

$$T_{iN_{Si}^k}^k + D_{iN_{Si}^k}^k + L_{FFij} \le T_{jN_{Sj}^l}^l + D_{jN_{Sj}^l}^l \quad k = l = 1 \quad (9)$$

The quantities L_{SSij}, L_{FSij}, L_{SFij}, and L_{FFij} are the respective time lags between task j and a previous task i. The FS relationship can be used to ensure continuity from one task to another by specifying $L_{FSij} = 0$. The relationships represented by (6)–(9) can also be written for any given crew or segment of a task rather than the whole task. For example, consider the case where crew B of task Y is the same as crew A of a previous task X. Crew B can start work only after crew A has finished. Therefore, an FS relationship has to be specified between crew A of task X and crew B of task Y

Step 7

Define the space and/or time buffer between tasks. These constraints are essential if interference of crews on different tasks is to be prevented. If task i precedes task j by a distance buffer B_{Sij}, the following constraints have to be satisfied:

$$Z_j(T_{in}^k) + B_{Sij} \le Z_{in}^k \quad k = 1, \ldots, N_{Ci}, n = 1, \ldots, N_{Si}^k \quad (10)$$

$$Z_j(T_{in}^k + D_{in}^k) + B_{Sij} \le Z_{in}^{k'} \quad k = 1, \ldots, N_{Ci}, n = 1, \ldots, N_{Si}^k \quad (11)$$

$$Z_{jn}^k + B_{Sij} \le Z_i(T_{jn}^k) \quad k = 1, \ldots, N_{Cj}, n = 1, \ldots, N_{Sj}^k \quad (12)$$

$$Z_{jn}^k + B_{Sij} \le Z_i(T_{jn}^k + D_{jn}^k)' \quad k = 1, \ldots, N_{Cj}, n = 1, \ldots, N_{Sj}^k \quad (13)$$

The term $Z_i(T_{jn}^k)$ denotes the location of task i at the time T_{jn}^k. For tasks with a constant production rate during a segment of work, $Z_i(T_{jn}^k)$ is found by a linear interpolation between the values at the start (Z_{im}^l) and the finish ($Z_{im}^{l'}$) of segment m performed by crew l of task i.

$$Z_i(T_{jn}^k) = Z_{im}^l + \frac{(T_{jn}^k - T_{im}^l)(Z_{im}^{l'} - Z_{im}^l)}{(T_{im}^l + D_{im}^l) - T_{im}^l} \quad (14)$$

Similarly, if task i precedes task j by the time buffer B_{Tij}, then we have the following constraints:

$$T_{in}^k + B_{Tij} \le T_j(Z_{in}^k) \quad k = 1, \ldots, N_{Ci}, n = 1, \ldots, N_{Si}^k \quad (15)$$

$$(T_{in}^k + D_{in}^k) + B_{Tij} \le T_j(Z_{in}^{k'}) \quad k = 1, \ldots, N_{Ci}, n = 1, \ldots, N_{Si}^k \quad (16)$$

$$T_i(Z_{jn}^k) + B_{Tij} \le T_{jn}^k \quad k = 1, \ldots, N_{Cj}, n = 1, \ldots, N_{Sj}^k \quad (17)$$

$$T_i(Z_{jn}^{k'}) + B_{Tij} \le (T_{jn}^k + D_{jn}^k) \quad k = 1, \ldots, N_{Cj}, n = 1, \ldots, N_{Sj}^k \quad (18)$$

Likewise, $T_i(Z_{jn}^k)$ is found by a linear interpolation between the starting time (T_{im}^l) and the stopping time ($T_{im}^l + D_{im}^l$) for segment m performed by crew l of task i.

$$T_i(Z_{jn}^k) = T_{im}^l + \frac{(Z_{jn}^k - Z_{im}^l)\{(T_{im}^l + D_{im}^l) - T_{im}^l\}}{(Z_{im}^{l'} - Z_{im}^l)} \quad (19)$$

The optimization problem can now be formulated as the minimization of direct cost

$$C_D = \sum_{i=1}^{N_{NT}} W_i C_i(d_i) + \sum_{i=1}^{N_{RT}} \sum_{k=1}^{N_{Ci}} \sum_{j=1}^{N_{Si}^k} W_{ij}^k C_i(d_i^k) \quad (20)$$

subject to the scheduling constraints [(2)–(13) and (15)–(18)], plus initial constraint

$$T_{11}^1 = \text{const} \quad (21)$$

project duration constraints

J. Constr. Eng. Manage., 1997, 123(4): 450-458

$$T_{ij}^k + D_{ij}^k \le D^{\max} \quad i = 1, \ldots, N_T,\ k = 1, \ldots, N_{Ci},\ j = 1, \ldots, N_{Si}^k \tag{22}$$

task duration constraints

$$(d_i^k)^{\min} \le d_i^k \le (d_i^k)^{\max} \quad i = 1, \ldots, N_T,\ k = 1, \ldots, N_{Ci} \tag{23}$$

and nonnegativity constraints

$$T_{ij}^k,\ d_i^k \ge 0 \quad i = 1, \ldots, N_T,\ k = 1, \ldots, N_{Ci},\ j = 1, \ldots, N_{Si}^k \tag{24}$$

where C_i = direct cost per unit quantity of work for task i; d_i^k = time required by crew k of task i to complete a unit quantity of work based on resource allocation only; $(d_i^k)^{\min}$, $(d_i^k)^{\max}$ = minimum and maximum possible values of d_i^k, respectively; and $D^{\max}$ = maximum acceptable project duration. Note that in this formulation, (1) can be written for each crew k of task i as

$$D_{ij}^k = \mu_{ij}^k d_i^k W_{ij}^k \tag{25}$$

ARTIFICIAL NEURAL NETWORKS AND SCHEDULING

Artificial neural networks (ANN) are a functional abstraction of the biological neural structures of the central nervous system. Their computing abilities have been proven in the fields of prediction and estimation, pattern recognition, and optimization (Adeli and Yeh 1989; Adeli and Zhang 1993; Adeli and Hung 1995; Adeli and Park 1995a–c; Adeli and Park 1996).

An ANN model for the complete scheduling of construction projects has not been presented in the literature. Alsugair and Chang (1994) used a back-propagation learning network to capture human knowledge of allocating construction resources. The ANN determines the size and number of equipment units required for earthmoving processes. Mohammad et al. (1995) formulated the problem of optimally allocating available yearly budget to bridge rehabilitation and replacements projects among a number of alternatives as an optimization problem using the Hopfield network.

NEURAL DYNAMICS COST OPTIMIZATION MODEL FOR CONSTRUCTION PROJECTS

Formulation

Defining $\mathbf{X} = \{T_{ij}^k, d_i^k | i = 1, N_T, k = 1, N_{Ci}, j = 1, N_{Si}^k\}$ as the vector of decision variables, the optimization problem can be written as

Minimize

$$C_D = f(\mathbf{X}) \tag{26}$$

subject to inequality constraints

$$g_j(\mathbf{X}) \le 0 \quad j = 1, \ldots, J \tag{27}$$

and equality constraints

$$h_k(\mathbf{X}) = 0 \quad k = 1, \ldots, K \tag{28}$$

where $g_j(\mathbf{X})$ = jth inequality constraint function; $h_k(\mathbf{X})$ = kth equality constraint function; J = total number of inequality constraints; and K = total number of equality constraints. Using the exterior penalty function method, a pseudo-objective function is defined as

$$P(\mathbf{X}, r_n) = f(\mathbf{X}) + \frac{r_n}{2}\left\{\sum_{j=1}^{J} [g_j^+(\mathbf{X})]^2 + \sum_{k=1}^{K} [h_k(\mathbf{X})]^2\right\} \tag{29}$$

where $g_j^+(\mathbf{X}) = \max\{0, g_j(\mathbf{X})\}$; and r_n = a penalty parameter that magnifies constraint violations.

A dynamic system is defined as

$$\frac{d\mathbf{X}}{dt} = \dot{\mathbf{X}} = F(\mathbf{X}) \tag{30}$$

where $\mathbf{X} = \{X_1(t), X_2(t), \ldots, X_N(t)\}^T$ = state vector tracing a trajectory in N-dimensional space where the superscript T indicates the transpose of a vector and $N = \sum_{i=1}^{N_T} \sum_{k=1}^{N_{Ci}} N_{Si}^k + \sum_{i=1}^{N_T} N_{Ci}$. The dynamic system evolves until it reaches an equilibrium point. The stability of such an equilibrium point is ensured by satisfying the Lyapunov stability theorem, which states that a solution $\hat{\mathbf{X}}$ to the system of differential equations $\dot{\mathbf{X}} = 0$ is stable if

$$\frac{dV}{dt} \le 0 \quad \text{for all nonzero } \mathbf{X} \tag{31}$$

where $V(\mathbf{X})$ = Lyapunov functional defined as an analytic function of state variables such that $V(\mathbf{0}) = 0$ and $V(\mathbf{X}) > 0$ for all $|\mathbf{X}| > 0$ (Kolk and Lerman 1992). The objective (direct cost) function and the constraint functions in our construction cost optimization model individually satisfy the conditions for a Lyapunov functional. Therefore, the pseudo-objective function P defined by (29) is also a valid Lyapunov functional, V.

Following Adeli and Park (1995a), by defining

$$\frac{d\mathbf{X}}{dt} = \dot{\mathbf{X}} = -\nabla f(\mathbf{X}) - r_n\left\{\sum_{j=1}^{J} g_j^+ \nabla g_j(\mathbf{X}) + \sum_{k=1}^{K} h_k \nabla h_k(\mathbf{X})\right\} \tag{32}$$

where $\nabla f(\mathbf{X})$, $\nabla g_j(\mathbf{X})$, and $\nabla h_k(\mathbf{X})$ = gradients of objective function, jth inequality constraint, and kth equality constraint, respectively, the Lyapunov stability theorem for the dynamic system is satisfied.

$$\frac{dV}{dt} = \left(\frac{dV}{d\mathbf{X}}\right)\left(\frac{d\mathbf{X}}{dt}\right) = -\left[\nabla f(\mathbf{X}) + r_n\left\{\sum_{j=1}^{J} g_j^+ \nabla g_j(\mathbf{X}) + \sum_{k=1}^{K} h_k \nabla h_k(\mathbf{X})\right\}\right]^2 \le 0 \tag{33}$$

This also shows that the dynamic system evolves such that the value of the pseudo-objective function always decreases. Eq. (32) is in fact the learning rule of the neural dynamics model.

For an equilibrium point $\mathbf{X}$ to be local optimum solution, we also need to satisfy the Kuhn-Tucker optimality conditions

$$\frac{\partial L}{\partial X_i} = \frac{\partial f(\mathbf{X})}{\partial X_i} + \sum_{j=1}^{J} u_j \frac{\partial g_j(\mathbf{X})}{\partial X_i} + \sum_{k=1}^{K} v_k \frac{\partial h_k(\mathbf{X})}{\partial X_i} = 0; \quad i = 1, \ldots, N \tag{34}$$

$$g_j(\mathbf{X}) + s_j^2 = 0; \quad j = 1, \ldots, J \tag{35}$$

$$h_k(\mathbf{X}) = 0; \quad k = 1, \ldots, K \tag{36}$$

$$u_j s_j = 0; \quad j = 1, \ldots, J \tag{37}$$

$$u_j \ge 0; \quad j = 1, \ldots, J \tag{38}$$

$$v_k = \text{unrestricted in sign} \tag{39}$$

where L = Lagrangian function defined as a linear combination of objective and constraint functions

$$L(\mathbf{X}, \mathbf{u}, \mathbf{v}, \mathbf{s}) = f(\mathbf{X}) + \sum_{j=1}^{J} u_j[g_j(\mathbf{X}) + s_j^2] + \sum_{k=1}^{K} v_k h_k(\mathbf{X}) \tag{40}$$

where s_j = slack term for jth inequality constraint; and u_j and v_k = Lagrangian multipliers corresponding to jth inequality and kth equality constraint, respectively.

Finally, the optimum solution to the direct cost optimization problem can be found by the integration

$$\mathbf{X} = \int \dot{\mathbf{X}}\, dt \tag{41}$$

J. Constr. Eng. Manage., 1997, 123(4): 450-458

This integration can be performed by the Euler or Runge-Kutta method.

Topological Characteristics

The neural network topology for the neural dynamics construction cost optimization model is shown in Fig. 3. The nodes in the network represent the variables and constraints of the problem. The variable layer has N nodes corresponding to the total number of decision variables. The constraint nodes are divided into N_{NT} layers corresponding to nonrepetitive tasks, N_{RT} layers corresponding to repetitive tasks, and an initial constraint node. Nodes are grouped within each layer into the constraint categories described in a previous section. Variable and constraint nodes are fully interconnected (interlayer connections). In addition, recurrent and intralayer connections are also used and are described subsequently.

Associated with each connection is a weight whose magnitude and sign affect the impulse the connected node will receive. Both excitatory (positive connection weights) and inhibitory (negative connection weights) connections are used in our model. The coefficients of the constraint functions are assigned to the excitatory connections from the variable layer to the constraint nodes. The gradients of the constraint functions are assigned to the inhibitory connections from the constraint nodes to the variable layer. The gradients of the objective function are assigned to the recurrent inhibitory connections of the variable layer. A weight of one is assigned to the intralayer connections. This allows the outputs of nodes in competition to be compared.

The output of the variable layer is the current state vector **X**. Because the coefficients of the constraint functions are encoded in the excitatory connections from the variable layer to the constraint nodes, the input to a constraint node is the magnitude of the constraint at any given state, that is, $g_j(\mathbf{X})$ for an inequality constraint j, and $h_k(\mathbf{X})$ for an equality constraint k. The output of a constraint node will depend on the type of the constraint it represents. For an inequality constraint j, the output is

$$O_{cj} = \begin{cases} 0 & \text{when } g_j(\mathbf{X}) \le 0 \\ r_n g_j(\mathbf{X}) & \text{when } g_j(\mathbf{X}) > 0 \end{cases} \tag{42}$$

and for an equality constraint k the output is

$$O_{ck} = \begin{cases} 0 & \text{when } h_k(\mathbf{X}) = 0 \\ r_n h_k(\mathbf{X}) & \text{when } h_k(\mathbf{X}) \ne 0 \end{cases} \tag{43}$$

Eqs. (42) and (43) represent the activation functions. They are chosen such that the output of a constraint node is the penalized constraint violation. When more than one equation is specified for a particular category of constraint, such as external logic constraint, a competition is created between the outputs of the nodes in that group. For a group of n nodes with outputs $O_{c1}, O_{c2}, \dots, O_{cj}, \dots, O_{cn}$ such that

$$O_{cj} = \max\{O_{c1}, O_{c2}, \dots, O_{cj}, \dots, O_{cn}\} \tag{44}$$

The outputs after competition are as follows:

$$O_{cj} = O_{cj} \quad \text{and} \quad O_{c1}, O_{c2}, \dots, O_{cn} = 0 \tag{45, 46}$$

Let w_{ji} and w_{ki} be the connection weight from the jth and kth inequality and equality constraint node, respectively, to the ith variable node, and Y_i be the weight of the recurrent connection to a node i in the variable layer. Then, the input to the ith variable node is given by

$$I_{vi} = Y_i + \sum_{j=1}^{J} w_{ji} O_{cj} + \sum_{k=1}^{K} w_{ki} O_{ck} \tag{47}$$

The new value of the ith decision variable is obtained by the integration

$$X_i^{\text{new}} = \int I_{vi}\, dt \tag{48}$$

This integration is done by the Euler or the Runge-Kutta methods. In the construction cost optimization problem, we found the simple Euler method to yield accurate results.

The network operates until no change in the decision variables occurs within a given tolerance, that is, when $\dot{\mathbf{X}} = 0$. **X** is the solution to the minimum direct cost construction scheduling problem.

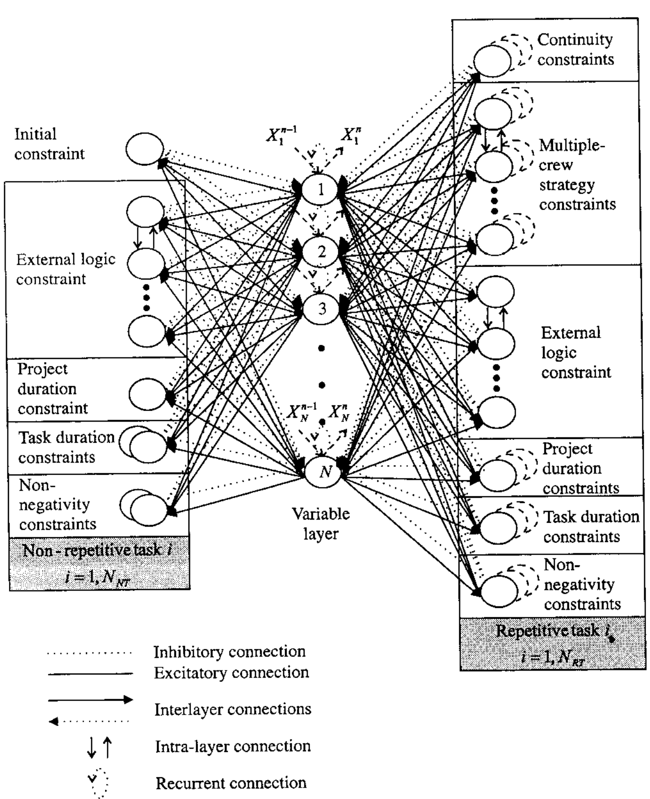

FIG. 3. Neural Network Topology for Neural Dynamics Cost Optimization Model

ILLUSTRATIVE EXAMPLE

A 5-km-long, two-lane highway construction project is used to illustrate the capabilities of the computational model pre-

TABLE 1. Description and Type of Tasks in Illustrative Example

Task number (1)	Description (2)	Type (3)
1	Clear and grub site for temporary offices plus right-of-way	Nonrepetitive
2	Grade site for temporary offices	Nonrepetitive
3	Erect temporary offices	Nonrepetitive
4	Construct temporary roads	Nonrepetitive
5	Move in	Nonrepetitive
6	Grade asphalt concrete plant site	Nonrepetitive
7	Erect asphalt concrete plant	Nonrepetitive
8	Construct culverts	Repetitive
9	Clear and grub right-of-way	Repetitive
10	Earthwork	Repetitive
11	Lay subbase	Repetitive
12	Lay base	Repetitive
13	Pave	Repetitive
14	Finish shoulders	Repetitive

J. Constr. Eng. Manage., 1997, 123(4): 450-458

sented in this paper. The work required is divided into 14 repetitive and nonrepetitive tasks summarized in Table 1. Tasks 1–5 represent the establishment of a temporary site office at the beginning of the 5-km-long stretch. The erection of an asphalt concrete plant at a distance of 2.5 km from the beginning of the roadway (at the center of the project) is represented by tasks 6 and 7 (together with a portion of task 9).

Cost-Duration Relationship

The relationship between direct cost and duration for unit quantity of work for each task is given in Table 2. An initial cost of $5,000 and, thereafter, a daily cost of $500 is used as the indirect cost for this example.

TABLE 2. Direct Cost-Duration Relationship for Each Task

Task number (1)	Direct cost-duration relationship (2)	Range (d) (3)
1	$C = -300d + 1{,}050$	$1.0 \le d \le 1.5$
2	$C = -280d + 960$	$0.5 \le d \le 2.0$
3	$C = -200d + 250$	$0.25 \le d \le 0.50$
4	$C = -200d + 550$	$0.50 \le d \le 1.25$
5	$C = -150d + 550$	$1.0 \le d \le 2.0$
6	$C = -280d + 960$	$0.5 \le d \le 2.0$
7	$C = -400d + 5{,}700$	$5 \le d \le 8$
8	$C = 1{,}600/d$	$2 \le d \le .3$
9	$C = -300d + 1{,}050$	$1.0 \le d \le 1.5$
10	$C = (1{,}600 + 500d)/d$	$1.0 \le d \le 2.0$
11	$C = -200d + 850$	$0.75 \le d \le 1.25$
12	$C = -200d + 950$	$0.75 \le d \le 1.25$
13	$C = -200d + 900$	$0.75 \le d \le 1.25$
14	$C = -100d + 800$	$2 \le d \le 4$

Scheduling Logic

The way in which each task is performed and the logic in which the tasks are carried out for a given project is not always well defined. Different schedulers may have different ideas for breaking down and sequencing each task. Often schedulers are constrained by the scheduling model available, forcing them to make simplifying assumptions. The flexible computational model presented in this paper, however, allows schedulers greater control over the progress of work and enables them to complete the job more efficiently.

Details of the breakdown of repetitive tasks into crews and segments, the start and finish distances, the quantities of work required, and the job condition factors for segments of work are given in Table 3. A constant number (1,000 m) is used as the start distance of the project to avoid division by 0 in the computation.

How the variation in the quantities of work and the job condition factors affect the breakdown of tasks can be explained by the clear and grub operations represented by tasks 1 and 9. Fig. 4 shows the areas that have to be cleared and grubbed and the type of vegetation involved. Task 1 operates on the first 200 m of the roadway but also includes the area for the site office. Task 9 covers the remaining length of the highway including the site for the asphalt concrete plant. A new segment of work is required whenever there is a change in the quantity of work required per unit length of the highway and/or a change in the job condition factor. Each change will affect the production rate. To reflect such a change, a separate segment of work is defined. Whenever there are no such changes, such as for task 14, there is no need to break down the work into smaller segments.

Base laying and paving operations (tasks 12 and 13) require material from the asphalt concrete plant. Therefore, as the op-

TABLE 3. Task Details for Illustrative Example

Task number (1)	Crew number (2)	Segment number (3)	Distances (m) Start (4)	Distances (m) Finish (5)	Quantity of work (6)	Unit (7)	Job condition factor (8)
1	1	—	1,000	1,200	3	hectares	1.0
2	1	—	1,000	1,200	3	hectares	1.0
3	1	—	1,000	1,200	5	units	1.0
4	1	—	1,000	1,200	3	100 m	1.0
5	1	—	1,000	1,200	100	percent	1.0
6	1	—	3,500	3,650	1.5	hectares	1.0
7	1	—	3,500	3,650	100	percent	1.0
8	1	1	1,300	1,305	1	culvert	1.0
		2	2,750	2,755	1	culvert	1.0
		3	5,500	5,505	1	culvert	1.0
9	1	1	1,200	3,000	4.5	hectares	1.0
		2	3,000	3,500	1.25	hectares	1.1
		3	3,500	3,650	1.875	hectares	1.1
		4	3,650	6,000	5.875	hectares	1.15
10	1	1	1,000	3,000	10	1,000 m^3	1.0
		2	3,000	3,500	6	1,000 m^3	1.15
	2	1	3,500	5,000	8	1,000 m^3	1.05
		2	5,000	6,000	5	1,000 m^3	1.0
11	1	1	1,000	2,000	4.25	1,000 m^3	1.0
		2	2,000	4,000	8.5	1,000 m^3	1.05
		3	4,000	6,000	8.5	1,000 m^3	1.0
12	1	1	3,500	2,250	2.6	1,000 m^3	1.0
		2	2,250	1,000	2.6	1,000 m^3	1.05
	2	1	3,500	4,750	2.6	1,000 m^3	1.0
		2	4,750	6,000	2.6	1,000 m^3	1.05
13	1	1	3,500	2,250	1.25	1,000 m^3	1.0
		2	3,250	1,000	1.25	1,000 m^3	1.05
	2	1	3,500	4,750	1.25	1,000 m^3	1.0
		2	4,750	6,000	1.25	1,000 m^3	1.05
14	1	1	1,000	6,000	3	hectares	1.0

J. Constr. Eng. Manage., 1997, 123(4): 450-458

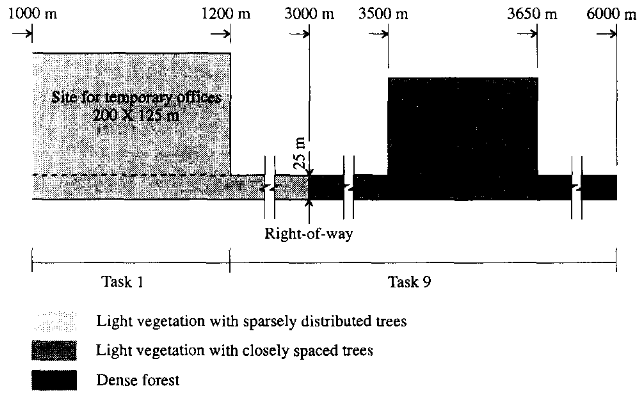

FIG. 4. Areas and Types of Vegetation That Have to Be Cleared by Tasks 1 and 9

eration moves away from the plant, more time will be taken to do the same amount of work. In the writers' example, the time required for operations beyond 1,250 m from the plant was increased by 5%, indicated by the job condition factor of 1.05. Instead of a step function, a continuous linear or nonlinear function may be used for the job condition factor, which will reflect the impact of increasing haul distances on the rate of operation.

The internal logic of repetitive tasks is given in Table 4. Work continuity relationships between segments of work and multiple-crew strategies are specified. Usually no slack time ($S_{ij}^k = 0$) is allowed between segments of the work of a crew. However, the slack term can be any function of the decision variables. The authors use a nonlinear slack term to model the continuity of work constraint of task 8 in the form

$$S = 1 - \beta \leq 1.0 \tag{49}$$

where $\beta < 1.0$ = fractional portion of finishing time (starting time plus duration) of previous segment of work. For example, for a finishing time of 10 d and 2 h (10.25 d assuming 8 h per day), $\beta = 0.25$. This ensures that the work on the next segment will start on the following day. As a result adequate time is provided for the crew to move from one location to the next. Multiple-crew strategies become important when more than two crews are used.

TABLE 4. Internal Logic of Repetitive Tasks

Task number (1)	Crew number (2)	Segment number (3)	Continuity relationship (4)	Multiple-Crew Strategy: Predecessor crew (5)	Multiple-Crew Strategy: Relationship (6)
8	1	1	—	—	—
		2	$S = 1 - \beta$[a]		
		3	$S = 1 - \beta$[a]		
9	1	1	—	—	—
		2	$S = 0$		
		3	$S = 0$		
		4	$S = 0$	—	—
10	1	1	—		
		2	$S = 0$	1	$SS, L = 0$
	2	1	—		
		2	$S = 0$		
11	1	1	—	—	—
		2	$S = 0$		
		3	$S = 0$		
12	1	1	—	—	—
		2	$S = 0$		
	2	1	—	1	$FF, L = 0$
		2	$S = 0$		
13	1	1	—	—	—
		2	$S = 0$		
	2	1	—	1	$SS, L = 0$
		2	$S = 0$		

[a]β is the fractional portion of the finishing time of the previous segment of work.

TABLE 5. External Logic of Tasks

Task (1)	Predecessor (2)	Relationship (3)
Task 1		
Task 2	Task 1	$FS, L = 0$
Task 3	Task 2	$SS, L = 0$
	Task 4	$FS, L = 0$
Task 4	Task 2	$FS, L = 0.25D$
Task 5	Task 3	$FS, L = 0$
Task 6	Segment 3, Crew 1, Task 9	$FS, L = 0$
Task 7	Task 6	$FS, L = 0$
Task 8		
Crew 1, Segment 1	Task 9 at 1,300 m	$FS, L = 0$
Crew 1, Segment 2	Task 9 at 2,750 m	$FS, L = 0$
Crew 1, Segment 3	Task 9 at 5,500 m	$FS, L = 0$
Task 9	Task 1	$FS, L = 0$
Task 10	Task 9	Space buffer, B = 150 m
Task 11	Task 10	Space buffer, B = 150 m
Task 12	Task 7	$FS, L = 0$
	Task 11	Time buffer, B = 2 d
Task 13	Task 12	Time buffer, B = 2 d
Task 14	Task 13	Time buffer, B = 2 d

Table 5 gives the external logic of tasks for the illustrative example. The external logic of the first five tasks, which are nonrepetitive, can also be shown by an activity-on-node diagram (Fig. 5). Standard precedence relationships [(6)–(9)] are used to link the tasks. The time lag term, however, may be any function of the decision variables.

The construction of a culvert cannot start unless the area has been cleared and grubbed. Therefore, the external logic of task 8 requires that work on any culvert be delayed until the crews of repetitive task 9 have worked through the corresponding location. A space buffer of 150 m is provided around earthmoving operations (task 10) to make sure adequate space is available for the equipment. Tasks 12 and 13 cannot start before the completion of the asphalt concrete plant. A minimum time buffer of 2 d is provided between tasks 11, 12, 13, and 14.

Solution of Problem

The direct cost optimization problem is solved for project durations of 60, 65, 70, 80, 90, and 100 d. The penalty parameter, r_n, is taken (Adeli and Park 1995a) as

$$r_n = r_0 + (n/\alpha) \tag{50}$$

where r_0 = initial penalty; n = iteration number; and α = a

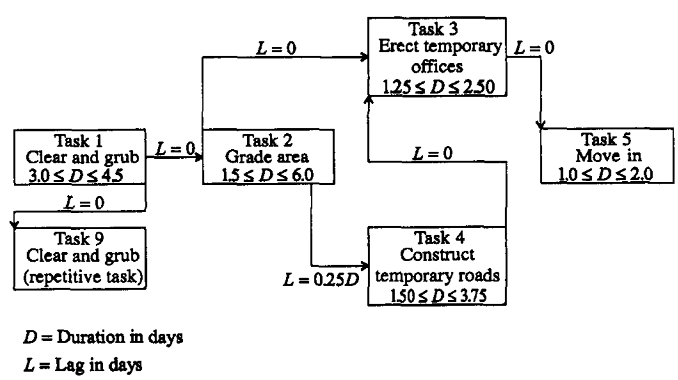

FIG. 5. Activity-On-Node Diagram for First Five Tasks of Illustrative Example

J. Constr. Eng. Manage., 1997, 123(4): 450-458

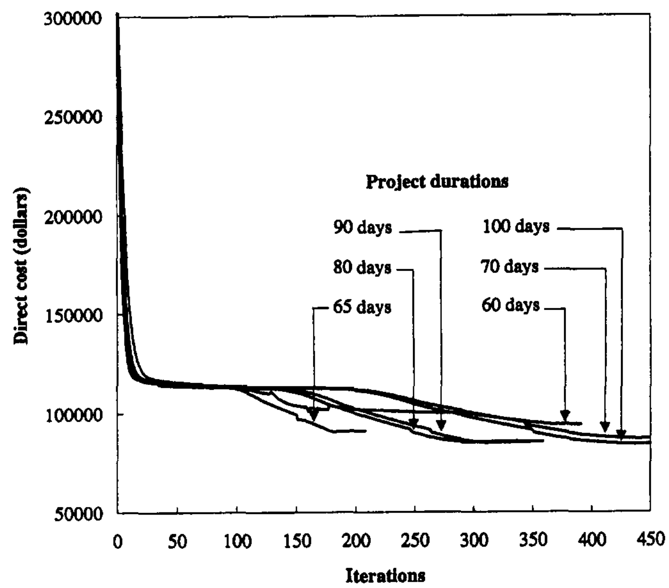

FIG. 6. Direct Cost Convergence Curves for Illustrative Example

TABLE 6. Direct, Indirect, and Total Costs Variation for Illustrative Example

Duration (d) (1)	Direct cost (dollars) (2)	Indirect cost (dollars) (3)	Total cost (dollars) (4)
60	94,118	30,000	124,118
65	91,215	32,500	123,715
70	87,314	35,000	122,414
80	85,742	40,000	125,742
90	85,438	45,000	130,438
100	84,411	50,000	134,400

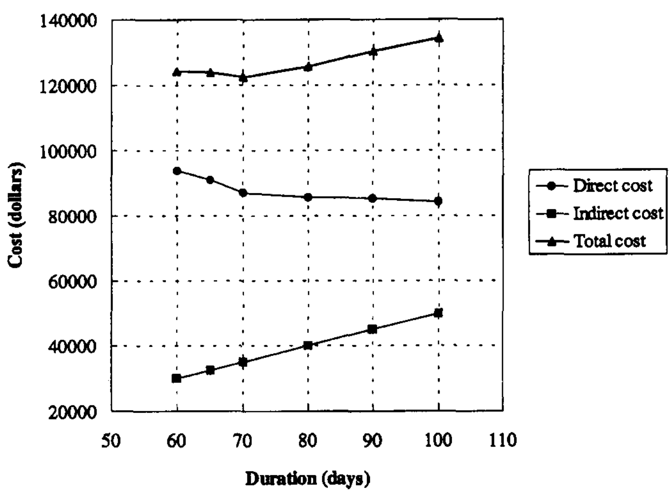

FIG. 7. Time-Cost Trade-Off Curve for Illustrative Example

positive number. Through this relationship, the penalty is increased gradually in each iteration to avoid the possibility of numerical ill-conditioning. As stopping criteria, a change of less than $1 in the original objective (direct cost) function and a maximum of 450 iterations are chosen. The convergence curves for the solutions are given in Fig. 6. Table 6 and Fig. 7 show the variation of direct, indirect, and total costs for different values of project duration. From Fig. 7 and Table 6 a project duration of 70 d leads to the minimum total cost. The final global optimum schedule is shown as a linear planning chart in Fig. 8.

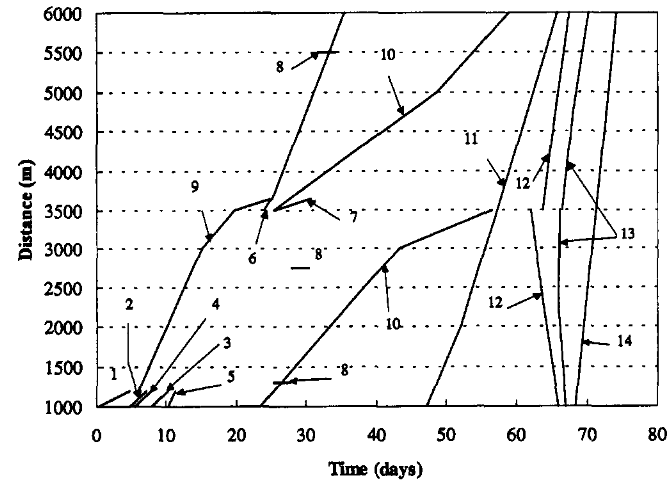

FIG. 8. Linear Planning Chart for Minimum Total Cost Schedule

CONCLUSION

A general formulation was presented for the scheduling of construction projects. Both repetitive and nonrepetitive tasks are considered in the formulation. By specifying appropriate constraints, work continuity considerations and multiple-crew strategies can be modeled. The effects of varying job conditions on the performance of a crew are taken into account by introducing a job conditions factor that modifies the task duration computed on the basis of resource allocation only. This factor can be a constant, a linear, or a nonlinear function, depending on the complexity of the situation. An optimization formulation is presented for the construction project scheduling problem with the goal of minimizing the direct construction cost. Any linear or nonlinear function can be used for task direct cost-duration relationships. The nonlinear optimization problem is then solved by the neural dynamics model developed recently by Adeli and Park (1995a). For any given construction duration, the model yields the optimum construction schedule for the minimum construction direct cost automatically. By varying this construction duration, one can solve the cost-duration trade-off problem and obtain the global optimum schedule and the corresponding minimum construction cost.

The new construction scheduling model provides the capabilities of both CPM and LSM approaches. In addition, it provides features desirable for repetitive tasks, such as highway construction projects, and allows schedulers greater flexibility in modeling construction projects more accurately. In particular, the model is suitable for studying the effects of change order on the construction cost. The new scheduling model can be specialized for the solution of specific and, perhaps, less complicated scheduling problems.

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APPENDIX II. NOTATION

The following symbols are used in this paper:

B_{Sij} = space (distance) buffer between tasks i and j;
B_{Tij} = time buffer between tasks i and j;
C_i = direct cost of executing unit quantity of work of task i;
c_D = total project direct cost;
D_{ij}^k = the actual duration of executing segment j by crew k of task i;
D^{max} = maximum acceptable project duration;
d_i^k = duration per unit quantity of work of task i performed by crew k based on resource allocation only;
$(d_i^k)^{max}$ = maximum possible value of d_i^k;
$(d_i^k)^{min}$ = minimum possible value of d_i^k;
$f(\mathbf{X})$ = objective function;
$g_j(\mathbf{X})$ = jth inequality constraint;
$h_k(\mathbf{X})$ = kth equality constraint;
J = number of inequality constraints;
K = number of equality constraints;
$L(\mathbf{X}, \mathbf{u}, \mathbf{v}, \mathbf{s})$ = Lagrangian function;
L_{FFij} = finish-to-finish time lag between task j and preceding task i;
L_{FSij} = finish-to-start time lag between task j and preceding task i;
L_{SFij} = start-to-finish time lag between task j and preceding task i;
L_{SSij} = start-to-start time lag between task j and preceding task i;
L_{FFi}^{kl} = finish-to-finish time lag between new crew l and previous crew k of task i;
L_{SFi}^{kl} = start-to-finish time lag between new crew l and previous crew k of task i;
L_{SSi}^{kl} = start-to-start time lag between new crew l and previous crew k of task i;
N = number of decision variables;
N_{Ci} = number of crews used for task i;
N_{NT} = number of nonrepetitive tasks;
N_{RT} = number of repetitive tasks;
N_T = number of tasks in the project;
N_{Si}^k = number of segments over which crew k of task i performs;
$P(\mathbf{X}, r_n)$ = pseudo-objective function;
r_n = penalty parameter;
r_0 = initial value of penalty parameter;
S = slack time;
S_{ij}^k = idle or slack time of crew k after segment j of task i;
s_j = slack for the jth inequality constraint;
T_{ij}^k = time at which crew k of task i starts work on segment j;
$T_i(Z_{jn}^k)$ = time of task i defined by location Z_{jn}^k;
u_j = Lagrangian multiplier for jth inequality constraint;
$V(\mathbf{X})$ = Lyapunov function;
v_k = Lagrangian multiplier for kth equality constraint;
W_{ij} = quantity of work required in segment j of task i;
$\mathbf{X}$ = vector of decision variables;
Z_{ij}^k = distance at which crew k of task i starts work on segment j;
$Z_{ij}^{k'}$ = distance at which crew k of task i finishes work on segment j;
$Z_i(T_{jn}^k)$ = location of task i defined by time T_{jn}^k;
β = parameter used in slack term; and
μ_{ij}^k = job condition factor for segment j performed by crew k of task i.

J. Constr. Eng. Manage., 1997, 123(4): 450-458