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Source: *The Mathematical Gazette*, Vol. 43, No. 346 (Dec., 1959), pp. 256-268
Published by: Mathematical Association
Stable URL: http://www.jstor.org/stable/3610652
Accessed: 28-11-2015 20:10 UTC

MATHEMATICS IN WARSHIP DESIGN

By S. J. Palmer

Members of the Royal Corps of Naval Constructors are responsible for the design and construction of H.M. Ships, and one of the main features of their training is the study of mathematics. The aim of this article is to give the VIth form mathematics teacher an acquaintance with the kind of mathematics used in ship design; he may then be able to encourage some of his pupils to consider the prospect of a career in the Royal Corps. Promising mathematicians with a strong practical bent can enter from school under a scheme advertised in this number of the Gazette.

Now it is probably well known that a large warship is one of the most complex engineering projects to which man puts his hand; it must be able to propel itself through oceans at high speed, to withstand the stresses and motions imposed by the seas, to carry and operate the latest weapons and detection devices, to withstand attack from the enemy, and to house and feed its ship's company in reasonable comfort in all weather conditions between the Arctic and the Tropics. The design of such a ship poses a number of problems for the Constructor and his colleagues in the Royal Naval Scientific Service, problems as diverse as calculating the trunk sizes of complex ventilation systems or the thickness of shielding around nuclear reactors. In common with general practice in industry many of these questions can be solved satisfactorily by engineering research and experience, but the problems which are peculiar to ships and, in particular, to warships, usually cannot be dealt with in this way. The reason for this is that warships are big and expensive and it is not practicable to build and test prototypes; nevertheless no risks can be taken with their speed, stability, or structural strength and, since they must be at least as efficient as similar warships produced in other countries, there is no room for large factors of safety. Under these conditions the design can only be tackled by a logical, mathematical, approach.

At the present time, however, mathematical theories still fall short of solving completely some of the more complex problems in ship hydrodynamics and ship structures, and the designer is compelled to check and adjust the results of his calculations by tests on scale models. The examples which follow illustrate some of these problems and the way in which experiments with small models either check or fill the gaps in theories which are still not completely satisfactory.

The first four examples are concerned with ship hydrodynamics. The model tests are mainly carried out at the Admiralty Experiment Works near Gosport and the equipment is impressive: there are long tanks (the largest 900′ long × 40′ wide) in which ship models are propelled or towed at speeds up to 40 feet per second, water tunnels in which model propellers can be examined under working conditions in stroboscopic lighting, and a large manoeuvring basin in which models can be remotely controlled in complex wave systems. As with all scale model work great accuracy in workmanship and measurement is essential.

1. *Ship Resistance*

The resistance (R) of a ship depends on its dimensions (L) and speed (V), upon gravity, and upon the viscosity (μ) and density (ρ) of the fluid in which it moves. From dimensional analysis

$$\frac{R}{(\rho/g)V^2L^2}=f\left(\frac{VL}{\mu/\rho},\frac{V^2}{gL}\right)$$

That is, the non-dimensional resistance coefficient is a function of two variables, the first depending on viscosity (called skin friction), and the second depending on gravity (called the wavemaking resistance).

When designing a ship's hull it is necessary to find the shape which will have the least resistance and to make an accurate estimate of this resistance. At the present time this is done by experiments with scale models and it is not possible to define the shape by a convenient mathematical expression. It is usually assumed (the Froude hypothesis) that skin friction and wavemaking resistance are separate components and that the total resistance is the sum of the two. Much of the theoretical work which has been done on skin friction has been published and need not concern us here, but the parallel work on wavemaking is not so well known.

Careful observation of a ship or model moving at uniform speed in calm water reveals that abaft the bow, and enclosed within lines angled at about 30° to the centreline of the ship, there are two wave systems, one with its crests at about 60° to the centreline and the other at right angles to the centreline. Similar waves are generated at the stern, the bow and stern systems having a phase difference which depends on the length of the ship and its speed. The interference of the two systems to produce the waves left behind the ship leads to a periodic variation of resistance with speed; if crests of the bow waves pitch into troughs of the stern waves then the resulting waves abaft the ship are small and the resistance is small and, conversely, if the crests coincide the resistance is high. If the speed of the ship is steadily increased the two wave systems will alternately fall into and out of phase and, since this affects the pressure distribution around the ship, the resistance fluctuates.

Progress on the development of an adequate theory has been continuous, if slow, since Michell published the fundamental theory in 1898, and reasonably accurate estimates of the wavemaking resistance of certain geometrical forms can now be made. These generally have been obtained by selecting complex variables, or distributions of sources and sinks, which will generate ship-like forms. For example, the source distribution for a very fine form with parabolic sections, extending over the central plane ($y = 0$) between limits $-a \leqslant x \leqslant a$, and $0 \leqslant z \leqslant d$, is given by

$$\sigma = \frac{bcx}{\pi a^2}\left(1 - \frac{z^2}{d^2}\right)$$

where

$$\begin{aligned} \sigma &= \text{sources per unit area,} \\ c &= \text{speed of body along } 0x \\ 2a &= \text{length of body,} \\ 2b &= \text{maximum breadth of body,} \\ d &= \text{draught} \end{aligned}$$

It is true that a great deal more must be done before we can calculate the wavemaking resistance of normal ship forms, but the solutions which have already been obtained have made a valuable contribution to understanding what happens and they have stimulated and clarified model research.

2. *Ship Propulsion*

All ships have to have a means of propulsion and the usual way is to use a screw propeller to convert the power of an engine into a driving force. This force is developed by setting water in motion backwards and, of course, the kinetic energy of this water is lost. If m is the mass of the water flowing through the propeller in unit time and v is the increase in the velocity of the water due to the propeller action, then the thrust is proportional to mv and the kinetic energy lost is proportional to mv^2. It follows that to reduce the loss of energy v should be as small as possible, which, for a given thrust, can only be done by making m as large as possible, that is, by making the propeller as large as possible. This simple fact is often not appreciated by people who build model boats and then fit them with tiny, and therefore inefficient, propellers.

The problem of representing the action of a screw propeller has claimed the attention of mathematicians since the beginning of this century and today rather complex mathematical models are used. The propeller blades are represented by line vortices having varying circulation along their length. This variation leads to the shedding of free vortex lines in a direction coinciding with the resultant motion and the problem is to calculate the velocities induced by these vortex lines around the sections of the propeller blades. When this is done the lift and drag can be evaluated at each section and these can be integrated to give the thrust on the ship and the torque required from the engine.

Even this fairly complex analysis is found to be inadequate for the broad bladed propellers used to propel ships, but the theory provides an indispensable guide to the effect of varying parameters such as the pitch and shape of the blades.

Until a quantitatively more accurate theory is available the ship designer must continue to base his propeller designs on the results of model experiments, particularly as in these experiments he is also able to allow for the interaction between the hull and the screw.

One might ask why, if experimental methods are satisfactory, should we continue to search for a mathematical solution? The answer is clear, the experimenter can measure the thrust, torque and efficiency of his models and compare one with another, but he is almost completely in the dark about why these results are obtained, and will remain so until the mathematician can produce an adequate theory.

3. *Ship Motions*

The prediction of the motion of a ship in rough seas and the effect of the shape of the ship on this motion are questions which are currently attracting a good deal of attention among ship designers of all nations. A warship should be able to maintain high speed and fire its missiles accurately in spite of bad weather, and a passenger ship should not be too uncomfortable even in the worst seas it is likely to meet. The first step is to formulate the equations of motion for a ship moving at steady speed through regular waves, that is, waves of sinusoidal form whose crests are straight and extend indefinitely in either direction. For example, the simple equations for pitch and heave may be written in the form

$$a\ddot{z} + b\dot{z} + cz = Fe^{i\omega t}$$

$$A\ddot{\theta} + B\dot{\theta} + C\theta = Me^{i\omega t}$$

where z is the vertical displacement, or heave,
θ is the angle of pitch
F is the heaving force imposed on the ship by the waves
and M is the pitching moment imposed on the ship by the waves.

In these equations only the real part of the right-hand side is to be taken.

In fact cross-coupling occurs between pitch and heave and the equations become more generally

$$a\ddot{z} + b\dot{z} + cz + d\ddot{\theta} + e\dot{\theta} + f\theta = Fe^{i\omega t}$$

$$A\ddot{\theta} + B\dot{\theta} + C\theta + E\ddot{z} + F\dot{z} + Gz = Me^{i\omega t}$$

The coefficients on the left-hand side of these equations can be regarded as constants and they can be calculated from the lines plan of the ship although, in practice, it is easier to evaluate them by model experiments. When they are known the motion of the ship in regular waves can be predicted with reasonable accuracy.

It might appear that this prediction of motion in regular waves is still far removed from forecasting what happens to a ship in the violent and confused seas which are encountered in storms, and so it would be were it not for the oceanographers and mathematicians who have taken records of such seas and then, by statistical analysis, resolved them into a very large number of very small sine waves. This neat handling by mathematicians of one of Nature's most fearsome spectacles is a story in itself, but it is sufficient for our present purpose that the ship designer can be given a distribution of wave energy against frequency for the worst seas likely to be encountered in any area. From this he can calculate the motions of a ship in regular components of the confused sea and then obtain the total motion by adding together the individual responses.

It would be wrong, however, to give the impression that this problem has been solved. What we can say is that we know how to approach it and, in time, when certain details of the theoretical work have been improved, we expect to be able to forecast from the lines plan of a new ship what its probable motion will be in typical rough weather conditions and, most important of all, how to modify the shape of the ship to reduce this motion.

4. *Submarine Control*

With the advent of nuclear propulsion and of guided missiles which can be launched from a submerged body, increasing attention is being given to the design of large, high speed submarines. One of the many problems is to predict the behaviour of such a submarine, weighing perhaps several thousand tons, when manouvering at high speed deeply submerged.

The submarine moves in three dimensions under the action of weight, buoyancy, resistance, propeller thrust and the hydrodynamic lifts and moments on the control surfaces (the rudders and hydroplanes). With certain assumptions, such as neglecting the compressibility of the hull, the equations of motion can be written down and expressions can be obtained for the stability of the motion in the vertical and horizontal planes. Thus the linear equations for the dynamic stability of the disturbed motion in the vertical plane of a submarine moving on a level path can be expressed as:

$$a\dot{\theta} + b\dot{\beta} + c\beta = 0$$

$$e\ddot{\theta} + f\dot{\theta} + g\theta + h\beta = 0$$

where: θ = the angle of the submarine's axis to the horizontal
β = the angle of incidence of the submarine, i.e., the angle of the axis to the instantaneous direction of advance of its C.G.

The coefficients represent either mass and inertia or hydrodynamic derivatives determined by the way in which the hydrodynamic force and moment on the submarine vary with incidence β and angular velocity $\dot{\theta}$.

The equations of motion are thus simultaneous linear differential equations for β and θ, which can be solved in the usual way by the substitutions

$$\beta = \beta_c e^{\lambda t}, \qquad \theta = \theta_c e^{\lambda t}$$

The elimination of θ_c and β_c between the two equations thus obtained leads to a cubic equation in λ known as the stability cubic. If any of the roots of the stability cubic are real and positive, or complex with positive real parts, the disturbed motion will increase with time and so be unstable. By this means a criterion for dynamic stability is obtained and a method provided whereby the sizes of the control surfaces can be determined.

So far it has not been possible to calculate the coefficients in these expressions but they can be measured by experiments on models rotating in a circular path.

Examples

The following examples are concerned with the design of warships' structures. Here again the Admiralty have impressive apparatus for testing scale models, but this time the models are of steel and aluminium structures and the laboratories are at the Naval Construction Research Establishment, near Rosyth. Of the many pieces of equipment perhaps the most imposing (it is the largest, of its kind, in the world) is a testing frame enclosing a volume 69 ft. × 39 ft. × 33 ft. in which large scale models can be subjected to forces up to 2,000 tons along the axis with additional forces up to 500 tons in any direction. While the models are being tested strain gauges record the stresses at a large number of critical points. As always, an adequate theory is essential if the designer is to understand why the stress at any point is high and if he is to extrapolate the model results to the full scale with confidence.

5. *Longitudinal Strength Calculation*

The structure of a warship must be adequate to withstand, without excessive stress or deflection, the forces imposed on it during service. When a ship moves among waves the distribution of buoyancy along the length is continuously varying, and this causes the ship to bend, either "hogging" or "sagging" according to its position relative to the wave crest. The purpose of the longitudinal strength calculation is to find the largest stresses which a ship is likely to experience as a result of these bending actions. If, in a proposed design, the calculated stresses are unacceptably high, then the scantlings of the structure must be increased and the calculation repeated until the required standard of strength is obtained.

It has been shown by tests on ships at sea that when the hull bends the resulting distribution of stress is in accordance with the linear bending theory; that is, the longitudinal bending stress at any point in the cross-section of a ship is proportional to the distance of that point from the neutral axis of bending. It follows that the longitudinal bending stress f at a point distant y from the neutral axis can be calculated from the equation

$$f = \frac{My}{I} \tag{1}$$

in which M is the external bending moment acting on the ship, and I is the second moment of area about the neutral axis of the hull cross-section at the longitudinal position considered. The position of the neutral axis must satisfy the condition that

$$\Sigma\, y \,.\, \delta A = 0$$

where y is the distance of any element δA from the neutral axis, the summation including all longitudinally continuous material in the cross-section. Having established the position of the neutral axis, the second moment of area I is then found from the summation:

$$I = \Sigma y^2 \,.\, \delta A.$$

To calculate the bending stress from equation (1) it remains to find the bending moment M resulting from the distribution of weight and buoyancy along the length of the ship. For this the ship is assumed to be at rest with either the crest or trough of a wave amidships, the former giving rise to the "hogging" condition and the latter to the "sagging" condition. For the purpose of this standard calculation the wave length L is taken equal to the ship length, and the wave height h as 1/20th of the length. The profile of the wave surface is assumed to be a trochoid whose horizontal and vertical co-ordinates x and y are related by the equations

$$x = \frac{L\theta}{2\pi} + \frac{h}{2}\sin\theta$$

$$y = \frac{h}{2}(1 - \cos\theta)$$

A profile of this "standard" wave is then placed over a profile of the ship, and its position adjusted by trial and error until the buoyancy is equal to the weight of the ship and the centre of buoyancy is vertically below the centre of gravity of the ship. This meets the initial assumption that the ship is poised, in equilibrium, on the wave. The evaluation of the buoyancy and centre of buoyancy of the part of the ship beneath the surface of the wave is usually made by a mechanical integrator, the instrument being moved by hand around the drawing of the immersed sections of the ship.

In this way a curve of buoyancy per unit length can be plotted. The difference between this and the curve of weight per unit length gives the net loading at each section of the ship. If the net load per foot run at any distance x from amidships is denoted by q, then the shearing force F and bending moment M acting on the ship at that point are

$$F = \int_0^x q \, . \, dx + C_1$$

$$M = \int_0^x \int_0^x q \, . \, dx \, . \, dx + C_1 x + C_2$$

in which the constants of integration are such that the shear force and bending moment are zero at the ends of the ship. Integration of the load curve to obtain the shear force, and double integration to obtain the bending moment, may either be done numerically, using Simpson's or similar rules, or mechanically, using an integraph which automatically plots the integral of a curve when the pointer is moved along the curve.

The stress distribution at any section of the ship can then be calculated from the shear force and bending moment at that section and the thickness of the structural members can be adjusted to bring these stresses to values which experience has shown to be suitable.

6. *Longitudinal Strength of Superstructures*

The longitudinal strength calculation, using the linear bending theory as described above, strictly applies only to the main hull girder of a ship up to the uppermost continuous deck. Wherever the longitudinal structure of the ship contains breaks or discontinuities the assumptions of the linear theory are no longer valid and a different approach is required to find the stresses in the discontinuous structure. The ship's superstructure is an important case in point. This will extend over only a part of the length of the ship and may thus be regarded as a short box structure rigidly attached to a longer beam which is itself subjected to known bending and shearing forces obtained from the weight and buoyancy curves as described above. The base of the superstructure is therefore forced, by its attachment to the hull, to bend and stretch; and the problem is to find the stresses in the superstructure resulting from these actions.

It can be shown that when a flat panel of plating is subjected to forces applied in its own plane, the resulting stress distribution in the plate is governed by the equation

$$\frac{\partial^4\varphi}{\partial x^4} + \frac{2\partial^4\varphi}{\partial x^2\,\partial y^2} + \frac{\partial^4\varphi}{\partial y^4} = 0 \tag{1}$$

in which x and y are orthogonal co-ordinate axes, and φ is the Airy Stress Function. The direct stresses f_x and f_y and the shear stress fxy are related to the stress function φ by the equations

$$\left.\begin{aligned} f_x &= \frac{\partial^2\varphi}{\partial x^2} \\ f_y &= \frac{\partial^2\varphi}{\partial y^2} \\ f_{xy} &= -\frac{\partial^2\varphi}{\partial x\,.\,\partial y} \end{aligned}\right\} \tag{2}$$

Thus if, for a single deck superstructure, φ_1 denotes a stress function governing stress in the sides of the superstructure and φ_2 a stress function in the top of the superstructure, then both φ_1 and φ_2 must satisfy equation (1) and also the appropriate conditions of stress, strain or curvature at the edges of the panels.

For this type of problem it is convenient to express the stress function in the form of an infinite series

$$\varphi = \Sigma(A\cosh nky + B\sinh nky + Cy\cosh nky + Dy\sinh nky)\cos nkx \tag{3}$$

in which n takes integral values and the summation includes as many terms as are required to express accurately in series form the external bending moment applied to the ship. Equation (3) satisfies equation (1), and hence it only remains to find the integration constants A, B, C and D in equation (3), to make φ satisfy the necessary edge conditions for the separate plate panels forming the sides and top of the superstructure. Thus the stress function for the vertical sides of the superstructure must be such that at its lower edge the strains and curvatures conform to those in the hull; in this way the interaction of hull and superstructure is taken into account. There are sufficient edge conditions to enable all the constants of integration to be determined, and from these the stress functions φ are obtained. The stresses at any point in the superstructure are then found from equations (2).

The mathematical procedure described above, in which a solution to the governing differing equation is found which satisfies the relevant boundary conditions, is typical of the technique used in solving many ships structural problems. A similar application of this method is described in the next section.

7. *Strength of Flat Grillages under Lateral Loading*

The structure of a warship is largely composed of panels of plating reinforced by stiffeners to withstand either axial or lateral loading. A plate panel reinforced by an orthogonal system of stiffeners is generally referred to as a plated grillage, and the calculation of the strength and stiffness of grillages is thus of basic importance in warship structural design. A typical case occurs, for example, in the flight deck of an Aircraft Carrier. The deck has to be designed to withstand, without yielding or undue deflection, the loads which may occur due to aircraft taking-off, or landing. This particular design problem has stimulated considerable research into the strength of flat grillages under point loads, and techniques of elastic analysis have been developed for this problem which are equally applicable to grillages under uniform lateral pressure.

It has been found that the lateral deflection w at any point (x, y) on a flat grillage under lateral pressure of intensity p is governed by the equation

$$A\frac{\partial^4 w}{\partial x^4} + B\frac{\partial^4 w}{\partial x^2\, \partial y^2} + C\frac{\partial^4 w}{\partial y^4} = p$$

in which A, B, and C are coefficients which depend on the flexural and torsional stiffnesses of the grillage and can be calculated from the dimensions and elastic properties of the plating and its stiffeners. The solution of the above equation generally follows similar lines to those outlined above for the superstructure calculation. A solution of the differential equation is found, and contains constants of integration which are then chosen to satisfy the conditions at the boundaries of the grillage. Thus if the stiffeners are effectively encastré at the edges of the grillage, that is, they do not deflect laterally or rotate at that edge, then the boundary conditions are

$$w = 0$$

$$\frac{\partial w}{\partial x} = 0$$

at the edge considered. Such boundary conditions can always be written in terms of the deflection w or its derivatives, and these conditions enable constants of integration to be found. From the general expression for deflection so obtained, bending moments and hence stresses in the structure can be calculated, since these also can be written in terms of deflection derivatives and the flexural properties of the grillage.

This differential equation solution of grillage problems is only useful where there are a large number of stiffening beams in both directions and where both structure and loading are regular. In many practical cases, however, the stiffening beams have, for other reasons, to be of differing sizes and are often irregularly spaced, and the solution by this method becomes prohibitively complex. As an alternative in such cases use is often made of the Energy Method which embodies the principle that if a system is in a position of stable equilibrium then its total energy is a minimum. Now the total energy of a laterally loaded grillage consists of two parts, the strain energy (V) of bending of the grillage, and the potential energy (W) of the load applied to the panel. The bending energy (V) can be expressed in terms of the known flexural properties of the grillage and the deflection w or its derivatives; and the potential energy W involves only the loading p and the deflection w. Thus, for example, the deflected surface of the grillage might be assumed in the form

$$w = a_1F_1 + a_2F_2 + a_3F_3$$

in which F_1, F_2, and F_3 are functions of the co-ordinates x and y, and a_1, a_2, and a_3 are constants yet to be determined. This assumed form of deflection should satisfy the boundary conditions for the grillage. Then the total energy

$$U = V + W$$

can be expressed in terms of the known flexural properties, the loading, and the constants a_1, a_2, and a_3 in the deflection expression. Since the total energy must be a minimum for equilibrium then

$$\frac{\partial U}{\partial a_1} = \frac{\partial U}{\partial a_2} = \frac{\partial U}{\partial a_3} = 0$$

and these three equations enable a_1, a_2, and a_3 to be found, and hence the full expression for deflection determined. Bending moments and stresses are then calculated in the usual way from the deflection expression. It will be seen that the success of this method depends on choosing an assumed deflection form which closely approximates actual conditions. If this is done the results will be sufficiently accurate for design purposes. In this connection experimental evidence from models or full-scale structures is of great value in selecting deflection expressions for use with the energy method.

In the past both methods of analysis described above have been widely used in the design of ship structural elements. The scope of both methods has hitherto been limited mainly by the amount of time and labour available for carrying out the necessary algebraic or arithmetical analysis. However, with the recent development of electronic computing, the scope of theoretical structural analysis will be greatly increased. Calculation of stresses in complex structures under irregular loading now becomes possible, and already computing programmes have been prepared to deal with a number of standard structural problems in warships.

8. *Submarine Pressure Hull*

The hull of a submarine must be able to withstand the pressure due to several hundred feet of water and this must be done with the minimum weight of structure so that as much weight as possible can be allowed for the armament, stores and machinery. Fortunately there are two reasons why the design of a submarine hull lends itself to precise mathematical treatment; one is that the loading at the maximum depth is known accurately and the other is that the section is circular. Basically the hull is a cylindrical shell strengthened by ring stiffeners and divided into compartments by bulkheads. The designer's job is to calculate the optimum size and spacing of the stiffeners and the thickness of the cylindrical plating.

The structure may fail in several ways and for each possible method of collapse differential equations can be set up for the radial displacement of the cylinder. A typical equation is

$$\frac{d^4w}{dx^4} + \frac{hw}{Ia^2} = \frac{p}{IE}\left(1 - \frac{\mu}{2}\right)$$

where w, h and a are the radial displacement, thickness and radius of the cylinder, I is the M.I. of the frame, x is distance measured along the axis, p is the pressure, E is Young's modulus, and μ is Poisson's ratio.

The solution to this equation has four arbitrary constants which can be determined from the boundary conditions of slope, displacement, shear force, and curvature. The equation for w given by this solution can be successively differentiated to give bending moments and shearing forces, and from these the stresses can be calculated.

The above, it must be admitted, is somewhat less sophisticated than the treatment which is found to be necessary for an accurate assessment of the stresses in the hull. This generally involves solutions to differential equations in the form of series and some experiment work to determine the effect of the interaction between the different modes of failure. By such means it is possible to design a cylindrical hull which will withstand a given pressure and have the minimum weight.

This, of course, is only the first stage of the investigation; many areas of the cylinder have to be pierced for hatches, pipes and valves, or joined to decks or machinery seats, and these drastically upset the stress pattern. The stress at each discontinuity has to be analysed by theory and experiment and this requires a concentrated effort by mathematicians and constructors which would, no doubt, surprise those who were not familiar with this subject.

S. J. P.

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DIRICHLET

By H. Davenport

This year has brought the centenary of Dirichlet's death (5 May 1859), and it is fitting that we should recall some of his achievements and his part in the development of mathematics.

Gustav Peter Lejeune Dirichlet was born on 13 February 1805, and was the son of the postmaster at Düren, near Cologne. Among the schools he attended was a Gymnasium at Cologne, where one of his teachers was the physicist Ohm. When the time came for University study in 1822, Dirichlet persuaded his parents to let him go to Paris, which was then the world centre of mathematics. Laplace and Legendre were still alive and active, Fourier was at the height of his career, with a group of brilliant young men round him, and Cauchy had already begun his massive development of the theory of functions.

Dirichlet spent three years in Paris and profited greatly from them. Besides attending lectures, he devoted himself to reading and re-reading Gauss's *Disquisitiones Arithmeticae*. By long continued efforts he mastered it, and he was probably the first to do so, even though it had appeared more than 20 years earlier. The two great influences of the Paris period: his regular studies and his mastery of the *Disquisitiones*, show themselves throughout all his work.

The first paper he published (on the equation $x^5 + y^5 = z^5$) sufficed to establish Dirichlet's reputation, and brought him the friendship of Fourier and of Alexander von Humboldt, who exerted himself to further his career. Soon after he returned to Germany, Dirichlet was appointed Professor at Berlin, and stayed there until 1855 when he accepted the invitation to succeed Gauss at Göttingen. (His own successor in 1859 was Riemann.) Thus most of Dirichlet's work belongs to the Berlin period. Jacobi was also