A Resolver-to-Digital Conversion Method for Fast Tracking

Choong-Hyuk Yim, In-Joong Ha, *Member, IEEE,* and Myoung-Sam Ko, *Senior Member, IEEE*

***Abstract*—In this paper, we propose a new resolver-to-digital (R/D) conversion method in which a bang-bang type phase comparator is employed for fast tracking. We eliminate from the R/D conversion loop the low-pass filter that is needed to reject carrier signal and noise. Instead, we employ two prefilters outside the R/D conversion loop that take the role of the low-pass filter. We can thereby construct a fast and accurate tracking R/D converter. Some simulation and experimental results as well as mathematical performance analysis are presented to demonstrate the superior tracking performance of our R/D converter over conventional tracking R/D converters.**

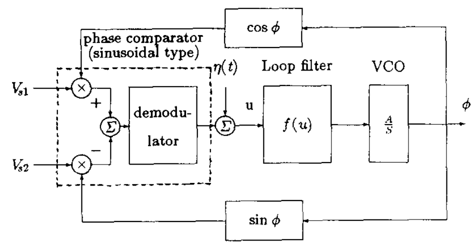


Fig. 1. Block diagram of a conventional tracking R/D converter, where $u \triangleq K\sin(\theta - \phi) - \eta(t)$.

Nomenclature

θ	= resolver shaft angle
ϕ	= digital angular position output
e	= tracking error $(\theta - \phi)$
V	= amplitude of resolver format signal
A	= integrator gain
B	= (bang-bang type) phase comparator gain
ω	= frequency of carrier signal
a_i	= coefficients of bandstop filter
b_i	= coefficients of 3rd-order low-pass filter
c_i	= coefficients of phase lead/lag circuit
d_i	= coefficients of 2nd-order low-pass filter
a, b, c	= coefficients of loop filter in RDC 19200-304
V_{s1}, V_{s2}	= resolver format signals
η	= internal noise
ζ	= additive noise to input
K	= loop gain

I. Introduction

RESOLVERS are transducers that are used to sense the angular position of rotational machines. They resemble small motors and have magnetically coupled rotor and stator windings. Operation of tracking resolver-to-digital (R/D) converters is based on the resolver format signals induced on the stator windings of the resolvers. As is depicted in Fig. 1, a widely used conventional tracking R/D converter contains three basic components: a phase comparator, a loop filter, and a voltage-controlled oscillator (VCO). The phase comparator consists of a sine multiplier, a cosine multiplier, a subtractor, and a demodulator. The VCO basically functions as a digital integrator. Further details of conventional R/D converters are given in Section II.

Manuscript received November 15, 1991; revised April 11, 1992.
The authors are with the Department of Control and Instrumentation Engineering, Automation and Systems Research Institute, Seoul National University, Seoul 151-742, Korea.
IEEE Log Number 9202502.

The results presented in [6]–[8] treat the problems associated with the nonideal characteristics of resolvers. Hung and Hung show in [8] that the accuracy of a given resolver can be enhanced by a digital compensation technique utilizing the difference between the measurement of the true shaft position and the output of the resolver. Recently, Hanselman [6], [7] investigated the effects of the nonideal characteristics of resolvers on position accuracy and proposed some signal processing techniques that reduce or eliminate the position error due to these nonideal characteristics.

It has been known, aside from nonlinearity-induced inaccuracies, that tracking R/D conversion is a most robust method for obtaining high-resolution information from the resolver format signals. The tracking performance of tracking R/D converters depends mainly on the dynamic, not static, characteristics of resolvers and R/D conversion loops. In this paper, we assume that the nonideal characteristics of resolvers have been properly compensated and we attempt to improve the dynamic characteristics of R/D conversion loops.

The prior works closely related to ours can be found in [1], [2], [11]. These papers concern phase-locked loops (PLL) but not R/D converters. However, the operation of tracking R/D converters is exactly the same as that of PLL's in synchronous mode. Ahmed and Wong [2] show that the optimum nonlinear device used in the feedback path of the PLL, which attains synchronization in the shortest possible time, is the on-off device. In [2], however, the dynamics of the low-pass filter located in the PLL are assumed to be fast enough to be neglected. Ahmed and Aboud [1] consider the practical situation where the dynamics of the low-pass filter are significant

0278-0046/92$03.00

and show that the optimal solution is to place a polynomial type of static nonlinearity with finite power between the low-pass filter and the VCO. Williamson [11] proposed the nonlinear loop filter that can enhance acquisition performance without significant degradation due to the effects of noise.

The results on PLL in [1], [2], [11] can provide fast tracking performance but cannot guarantee zero steady-state tracking error, and thus cannot be directly used for tracking R/D conversion. In the method proposed in [2], the time lag due to the dynamics of the low-pass filter causes limit cycles in the output signal even when the low-pass filter is a brickwall ideal low-pass filter. The schemes proposed in [1], [11] are basically the Type I systems and produce constant tracking error for a ramp input.

Our tracking R/D converter adopts the on-off device considered first in [2] as the phase comparator. However, we eliminate from the R/D conversion loop the low-pass filter which was needed [1], [2], [11] to reject carrier signal and noise. Instead, we employ two prefilters outside the R/D conversion loop that take the role of the low-pass filter, thereby, eliminating the source of limit cycles in the output and constructing a fast and accurate tracking R/D converter.

Since our tracking R/D converter attains the ideal situation considered in [2], it can provide fast tracking. We prove, via Lyapunov's direct method, that the tracking error converges to zero if the input rate does not exceed the converter maximum tracking rate specified by the physical limitation such as the saturation voltage of op amps. Ahmed and Wong [2] proved only that the on-off device minimizes the first time to reach the zero tracking error. We propose two approaches to the design of the two prefilters and a method of reducing the effect of noise on tracking accuracy. In order to demonstrate the practical use of our conversion method, we also present some experimental results that show the superior tracking performance over the conventional tracking R/D converters.

II. New Resolver-to-Digital Conversion Scheme

We first describe the tracking operation of a widely used conventional tracking R/D converter. Suppose that the current angular position of the resolver is at angle θ. Then the resolver format signals induced on the secondary windings of an ideal resolver will be

$$V_{s1} = V \sin\theta \cos\omega t \qquad V_{s2} = V\cos\theta\cos\omega t \tag{1}$$

if the primary winding is excited with an ac reference signal $V\cos\omega t$. Because the tracking R/D converters function by translating the relative magnitudes of V_{s1} and V_{s2}, they are relatively immune to voltage drop in cables.

As we can see from Fig. 1, these resolver format signals V_{s1} and V_{s2} are multiplied by cosine and sine of the current state of the up/down counter, respectively. Subtraction of the resulting signals gives the modulated error signal $V\sin(\theta - \phi)\cos\omega t$. This signal is then demodulated by the phase-sensitive detector that utilizes the ac reference signal $V\cos\omega t$. An analog dc error signal is therefore produced, which is proportional to $\sin(\theta - \phi)$. This dc error signal is delivered through a loop filter to the VCO. The VCO internally consists of an analog integrator and an up/down counter. The analog integrator outputs a pulse to the up/down counter when the integrated value reaches a specified quantity and then the integrator is reset. It is clear from the above discussion that the integrator in the VCO outputs pulses to the up/down counter until the final state of the up/down counter equals to the resolver shaft angle, i.e., $\phi = \theta$ [3], [9].

In the situation when the tracking error is small, the dc error signal is positively proportional to the tracking error since $\sin e \cong (2/\pi)e$ if $|e| < (\pi/2)$. However, in the case of large tracking error, the dc error signal becomes negatively proportional to the tracking error since $\sin e \cong -(2/\pi)(e - \pi\,\mathrm{sgn}(e))$ if $(\pi/2) < |e| \le \pi$. Hence, the pulse rate to the up/down counter rather decreases as $|e|$ exceeds $(\pi/2)$. Therefore, the nonlinear characteristics of the sinusoidal phase comparator result in poor tracking performance in the case of a large initial tracking error.

In the PLL schemes proposed in [1], [11], the loop filter in Fig. 1 consists of a low-pass filter and a polynomial type of static nonlinearity with finite power. As a result, these schemes have the dynamic characteristics of a Type I system and cannot exactly track even a ramp input signal. In most of conventional tracking R/D converters (for instance, see [3], [9]), the loop filter in Fig. 1 has the form:

$$\text{Linear loop filter: } \mathscr{L}[f(u)] = \frac{c(bs+1)}{s(as+1)}.$$

Therefore, the conventional tracking R/D converters are the Type II systems and hence can track a ramp input exactly but with poor transient performance due to the integral action of the linear loop filter.

To overcome such drawbacks of the conventional tracking R/D converters and the PLL schemes, we eliminate the demodulator and the loop filter from the R/D conversion loop and instead adopt a bang-bang type phase comparator as is shown in Fig. 2. The rejection of the carrier signal and the noise, usually of high frequency, is performed by two prefilters, each of which consists of a demodulator and a linear filter. Such a R/D conversion method has two important advantages over the previous methods.

First, this approach allows for digital implementation of the conversion loop. That is, the VCO can be implemented by digital circuits since the output of the bang-bang type phase comparator has only two states. The resulting digital VCO has excellent performance in linearity and, furthermore, makes the conversion loop more insensitive to the environmental disturbances such as humidity, temperature, and offsets of op amps.

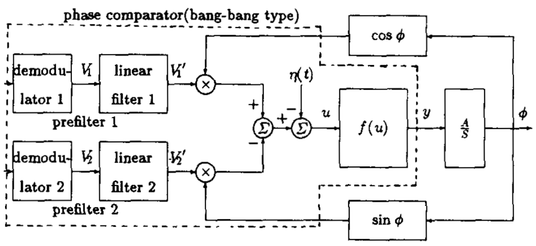


Fig. 2. Block diagram representation of the new tracking R/D converter, where $f(u) \triangleq \mathrm{B}\,\mathrm{sgn}\,(u)$ and $u \triangleq K \sin(\theta - \phi) - \eta(t)$.

Second, this approach provides good tracking performance. As we will soon show, we can prove under some reasonable assumptions that the new tracking R/D converter always drives the tracking error to zero if the input rate does not exceed its maximum tracking rate (AB in Fig. 2). Furthermore, we can show by following exactly the arguments used in [2] that replacement of the bang-bang type phase comparator in Fig. 2 by any other memoryless device with finite maximum power B will only prolong the acquisition time, that is, the first time to reach $\phi = \theta$.

Now, we present the mathematical performance analysis of our converter. For the simplicity of performance analysis, we make two ideal assumptions:

A1) Two prefilters are identical and produce negligible time lags.

A2) There exists no noise, i.e., $\eta = 0$.

Under these ideal assumptions, the R/D converter in Fig. 2 can be simplified to the one in Fig. 3 and can be described by the following simple equation.

$$\dot{\phi} = Af(u) \qquad u = K \sin(\theta - \phi) \tag{2}$$

where

$$f(u) \triangleq B\,\mathrm{sgn}\,(u). \tag{3}$$

The signum function in (3) is a discontinuous device and can be approximated by a continuous device

$$f(u) \triangleq \begin{cases} \dfrac{Bu}{\sin\delta} & \text{if } |u| \le \sin\delta \\ \mathrm{B}\,\mathrm{sgn}\,(u) & \text{if } |u| \ge \sin\delta \end{cases} \tag{4}$$

which is well known as the saturation function. Our performance analysis is performed only on the continuous system given by (2) and (4) since the saturation function in (4) approaches the signum function in (2) and (3) as δ tends to zero.

Finally, we assume that

A3) $\mu \triangleq AB - \sup_{t \ge 0} |\dot{\theta}(t)| > 0$.

This condition simply requires that the input rate must not exceed the product of the phase comparator gain and the integrator gain. This is quite a natural requirement since $\sup_{t \ge 0} |\dot{\phi}| = AB$. Now, we are in order to state Theorem1.

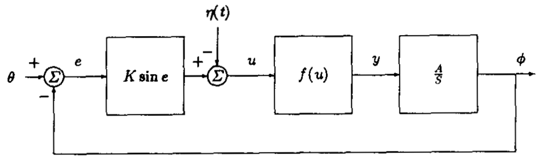


Fig. 3. Equivalent representation of the new tracking R/D converter with ideal prefilters, where $f(u) \triangleq \mathrm{B}\,\mathrm{sgn}\,(u)$.

Theorem 1 (Zero Tracking Error Theorem): Suppose that A1)–A3) hold for the new tracking R/D converter shown in Fig. 2. Then, the tracking error is "ultimately bounded" so that

$$|e(t)| \le \epsilon + \sin^{-1}\left(\frac{\sin\delta}{K}\right) \qquad t \ge T(\epsilon, e(t_0)) + t_0 \tag{5}$$

for any ϵ such that

$$0 < \epsilon < \pi - 2\sin^{-1}\left(\frac{\sin\delta}{K}\right) \tag{6}$$

where

$$T(\epsilon, \delta, e(t_0)) \triangleq \begin{cases} 0 \quad \text{if } |e(t_0)| \le \epsilon + \sin^{-1}\left(\dfrac{\sin\delta}{K}\right) \\ \left(\dfrac{2}{\mu}\right) \ln\left[\dfrac{1 - \cos\left(\dfrac{e(t_0)}{2}\right)}{1 - \cos\dfrac{\epsilon + \sin^{-1}\left(\dfrac{\sin\delta}{K}\right)}{2}}\right] \\ \quad \text{if } \epsilon + \sin^{-1}\left(\dfrac{\sin\delta}{K}\right) < |e(t_0)| < \pi - \sin^{-1}\left(\dfrac{\sin\delta}{K}\right). \end{cases} \quad \square$$

Theorem 1 states that if A3) is satisfied, the tracking error is kept smaller than "the ultimate bound" $\epsilon + \sin^{-1}(\sin\delta/K)$ after a finite time T, depending on the positive constants ϵ, δ and the initial tracking error $e(t_0)$. Note that the choice of ϵ and δ is completely free. Therefore, we can make the ultimate bound of the tracking error as small as possible by reducing the linear region of the saturation function in (4).

Theorem 1 has other important implications. From the proof of Theorem 1 given in the Appendix, observe that the assertion in Theorem 1 still holds even if the saturation function in (4) is replaced by any continuous device as follows.

$$f(u) \triangleq \begin{cases} g(u) & \text{if } |u| \le \sin\delta \\ \mathrm{B}\,\mathrm{sgn}\,(u) & \text{if } |u| \ge \sin\delta \end{cases} \tag{4$'$}$$

where g is a continuous function of u such that $g(\sin\delta) = \mathrm{B}\,\mathrm{sgn}\,(\sin\delta)$. For the case of the bang-bang type phase comparator, i.e. (3), the assertion in Theorem 1 holds with $\delta = 0$. Therefore, if A3) is satisfied, then our tracking R/D converter in Figs. 2 or 3 ensures that

B1) $\text{Lim}_{t \to \infty} |e(t)| = 0$ if $|e(t_0)| < \pi$.
B2) For any ϵ such that $0 \le \epsilon < \pi$, $|e(t)| \le \epsilon$, and $t \ge t_0$ if $|e(t_0)| \le \epsilon$.

This implies that the maximum tracking rate of the new tracking R/D converter is AB and is limited only by the physical limitations faced in hardware implementation. As is shown in Section IV, this is not necessarily true in the case of conventional tracking R/D converters.

Finally, we discuss at some length the prior works closely related to ours. As mentioned earlier, the bang-bang type phase comparator and its continuous approximation were first considered by Ahmed and Wong [2]. They placed these nonlinear devices between the brickwall ideal low-pass filter and the VCO. Under the assumption that the dynamics of the brickwall ideal low-pass filter are fast enough to be neglected, they showed that these nonlinear devices minimize exactly or approximately the first time to acquire lock-in.

However, the dynamics of the low-pass filter cannot, in practice, be neglected since the filter has to reject the carrier signal and the noise. In fact, the responses of their PLL scheme can involve limit cycles or steady state error due to the dynamics of the low-pass filter. Recall that, in our new R/D conversion method, the low-pass filter is removed from the R/D conversion loop and two prefilters replace its role. As a result, our R/D converter does not have such a drawback.

Since our R/D converter can operate under the ideal situations considered in the prior work, the analysis for the case of ideal brickwall low-pass filter in [2] can be directly applied to our tracking R/D converter and supports that our method is the best way of achieving fast tracking convergence. Furthermore, our Theorem 1 assures that the tracking error always converges to zero if the input rate satisfies A3). Neither Theorem 1 in this section nor experimental results in Section IV have been performed in the prior works [1], [2], [11].

Thus, our result can be viewed to improve the prior works so as to achieve fast and precise tracking performance. Of course, two prefilters must not produce significant distortion and time lag in the input signals. We present the compensation method to resolve this problem in Section III.

III. Compensation for Time Lag Due to Simple Prefilters

In order for the new tracking R/D converter proposed in Section II to have the desired performance stated in Theorem1, the two prefilters must not cause significant time lag in the input signal but have to reject effectively the carrier signal and the noise. Furthermore, the gains of two prefilters are required to be identical. To meet such requirements for prefilters, we propose the linear filter, which consists of three bandstop filters, one low-pass filter, and one phase lead/lag compensator.

Bandstop filter:

$$G_1(s) = \frac{s^3 + a_1 s^2 + a_2 s + a_3}{s^3 + a_4 s^2 + a_5 s + a_3}$$

Low-pass filter:

$$G_2(s) = \frac{1}{b_1 s^3 + b_2 s^2 + b_3 s + 1}$$

Phase lead/lag compensator:

$$G_3(s) = \frac{c_1 s^2 + c_2 s + 1}{c_3 s^2 + c_4 s + 1}.$$

In order to explain the roles of the above filters and compensator, we consider the Fourier series of the output of the demodulator 1 shown in Fig. 2.

$$\begin{aligned} V_1 &= V \sin\theta |\cos\omega t| + \zeta \\ &= \frac{2V}{\pi} \sin\theta \left(1 + \frac{2}{3}\cos 2\omega t - \frac{2}{15}\cos 4\omega t + \cdots\right) + \zeta \end{aligned}$$

where ζ is the input noise. The three bandstop filters are used to eliminate the second term in the parentheses. The bandstop frequency of each bandstop filter is chosen to be twice that of the carrier signal. Therefore, we need three filters in order to reduce the effect of the second term less than 1 LSB. The low-pass filter is used to reject the noise as well as the third and higher terms in the bracket. Finally, the phase lead/lag compensator is required to compensate for the time lag of the input signal caused by the bandstop filters and the low-pass filter.

Unless the bandstop filters are employed, we have to design a low-pass filter that can eliminate all harmonic terms in the bracket. However, such a low-pass filter usually causes significant time lag in the input signal, which is difficult to be compensated for. As will be seen from the simulation and experimental results in Section IV, the prefilters designed as above meet all requirements for prefilters and hence ensure that our R/D converter has the properties stated in Theorem 1. However, these "asymptotically ideal prefilters" need careful and time-consuming labor, not only in the design but also in the hardware implementation. In practice, it is possible but not easy to construct analog circuits so that two linear filters of 14th order are nearly identical and have the desired dynamic characteristics.

For this reason, we propose an alternative approach to the design of prefilters. We adopt simpler prefilters for the rejection of noise and carrier signal and add a time lag compensator to reduce the significant time lag caused by these simpler prefilters. The block diagram representation of this alternative approach is depicted in Fig. 4, where H_k, $k = 1, 2, 3, 4$ are linear filters. Apparently, this alternative approach looks more complex in the respect of overall structure than the original one. However, it should

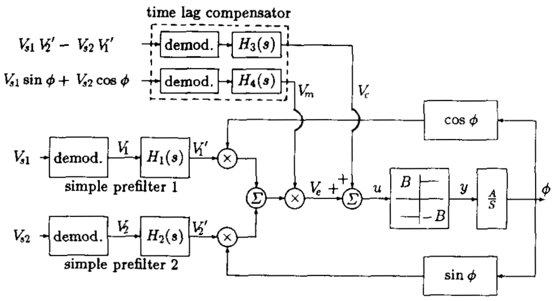


Fig. 4. Block diagram representation of the proposed compensation method.

be noticed that the linear filters H_k, $k = 1, 2, 3, 4$ required in the alternative approach can be chosen to be simple low-pass filters of the form

$$H_k(s) = \frac{1}{d_{1k}s^2 + d_{2k}s + 1}.$$

The rigorous mathematical analysis on the performance of the time-lag compensator for the general input is very difficult. Instead, we present a mathematical justification on the role of the time-lag compensator under some special assumptions. We assume that

(C1) The dynamic characteristics of two simple prefilters are identical.

(C2) The outputs of two simple prefilters can be represented as

$$V_1' = V\sin(\theta - \alpha(\dot{\theta})), \quad V_2' = V\cos(\theta - \alpha(\dot{\theta})) \quad (7)$$

where α denotes the phase delay of the input signal due to the dynamics of prefilters. We also assume that

(C3) e and $\alpha(\dot{\theta})$ are slow-varying.

By (C2), we can write

$$\begin{aligned} V_{s1}V_2' - V_{s2}V_1' &= \left[V^2 \sin\theta \cos(\theta - \alpha(\dot{\theta})) \right. \\ &\quad \left. - V^2 \cos\theta \sin(\theta - \alpha(\dot{\theta}))\right] \cos\omega t \\ &= V^2 \sin(\alpha(\dot{\theta})) \cos\omega t. \end{aligned} \quad (8)$$

By (C3), we have

$$V_m = V\cos e. \quad (9)$$

By (7) and (9), we can write V_e as

$$\begin{aligned} V_e &= V_m\left[V\sin(\theta - \alpha(\dot{\theta}))\cos\phi \right. \\ &\quad \left. - V\cos(\theta - \alpha(\dot{\theta}))\sin\phi\right] \\ &= V^2 \cos e \sin(e - \alpha(\dot{\theta})). \end{aligned} \quad (10)$$

On the other hand, by (C3), we can write

$$V_c = V^2 \sin(\alpha(\dot{\theta})). \quad (11)$$

From (10) and (11), we finally obtain the expression of u as follows.

$$u = V_e + V_c = V^2 \cos(e - \alpha(\dot{\theta}))\sin e. \quad (12)$$

Recall that the operation of the bang-bang type phase comparator depends only on the polarity of its input u. Therefore, if e and $\alpha(\dot{\theta})$ are sufficiently small (precisely speaking, if $|e - \alpha(\dot{\theta})| < (\pi/2)$), we have

$$y = \text{sgn}(u) = \text{sgn}(\sin e). \quad (13)$$

This shows that the proposed compensation method enables the R/D converter with simple prefilters to recover the ideal form indicated in Fig. 3 and hence to preserve the properties (B1) and (B2) stated in Section II. Consequently, without the tracking performance being degraded significantly, the structure of prefilters can be made simple and the bandwidth of prefilters can be chosen narrow so as to be more effective for the rejection of carrier signal and noise.

The assumptions (C1)–(C3) made for the justification of our compensation method seem to be reasonable for many practical situations. For instance, when the input signal is a ramp function, these assumptions are all valid except for the initial short interval of transition period. The simulation results presented in Section IV will prove that our compensation method is still useful even for the case of general situation.

IV. Simulation and Experimentation

In this section, we investigate through simulation and experimentation the practical use of the methods developed in the previous sections. In simulation and experimentation, we compare the tracking performance of our new converter with that of a conventional R/D converter popularly used in industries. The data used for simulation and experimentation are listed in Table I. The numerical values of A, a, b, and c represent the data of the typical conventional tracking R/D converter RDC 19200-304 manufactured by the DDC Co.

In the simulation results shown in Fig. 5, the input signal was chosen as a triangular wave with the period 7.04 ms and the amplitude 0.943 rad. Note that this input signal satisfies the condition (A3) in Section II since $\sup_{t \geq 0} |\dot{\theta}(t)| = 535.8$ rad/s.

When asymptotically ideal prefilters are used, the output response of our R/D converter tracks the input signal exactly and hence is overlapped with the input signal. When the simple prefilters with $d_{1k} = d_1$, $d_{2k} = d_2$, $k = 1, 2, 3, 4$ are used, we see that the output response of our R/D converter severely deviates from the input signal. When the time lag compensator is inserted into our R/D converter with simple prefilters, its tracking performance

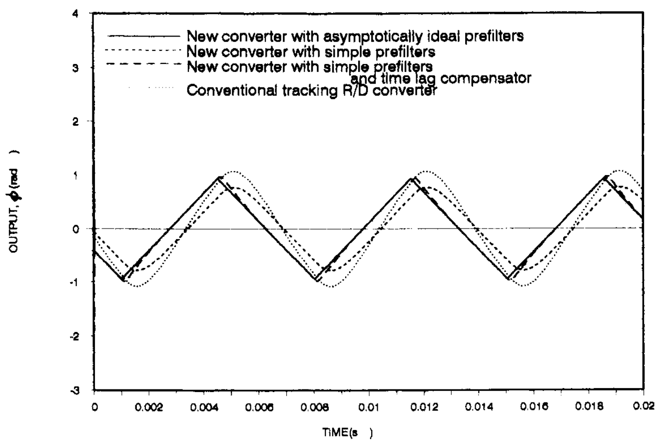


Fig. 5. Simulation results for the triangular input.

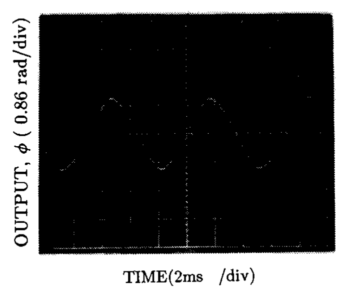
TABLE I
DATA USED IN SIMULATION AND EXPERIMENTATION

Coefficients of bandstop filter	a_1	44642.96
	a_2	9.96492E8
	a_3	8.93E13
	a_4	223214.0
	a_5	9.9649E9
Coefficients of low-pass filter	b_1	1.794E − 13
	b_2	3.18E − 9
	b_3	1.392E − 4
Coefficients of phase lead/lag circuit	c_1	8.454E − 8
	c_2	4.95E − 4
	c_3	1.0292E − 9
	c_4	5.56E − 5
Coefficients of 2nd-order low-pass filter	d_1	1.40E − 7
	d_2	6.49E − 4
Integrator gain	A	125.8
Phase comparator gain	B	9.99
Frequency of carrier signal	ω	18850
Coefficients of loop filter in RDC 19200-304	a	1.6667E − 4
	b	1.6667E − 3
	c	17.5

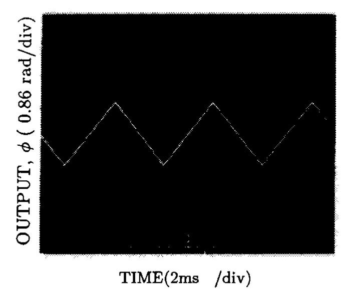
greatly improves and is better than that of the conventional R/D converter. However, some tracking error occurs near the corner points of the input signal. This is because the assumptions (C2) and (C3) are not necessarily valid near these points.

For our experimental study, we have constructed a prototype of our new converter with asymptotically ideal prefilters. For a fair comparison, it was designed to have a resolution of 12 b/r and a maximum tracking rate of 200 r/s (which corresponds to $AB = 1256.64$ rad/s, here), just as the RDC 19200-304.

In the first experiment, the input signal was chosen to be the same triangular wave as was used in the simulation. The output responses of our new R/D converter with the asymptotically ideal prefilters and the conventional R/D converter RDC 19200-304 are drawn in Fig. 6(a) and (b), respectively. The experimental result in Fig. 6 is a good



(a)



(b)

Fig. 6. Experimental results for the triangular input. (a) Conventional converter. (b) New converter with asymptotically ideal prefilters.

match with the simulation result in Fig. 5 and exhibits the superior tracking performance of our converter over the conventional converter. In the second experiment, the input signal was chosen as the step input. The output responses of the conventional converter and our converter with asymptotically ideal prefilters for small and large step inputs are shown in Figs. 7 and 8, respectively. From Figs. 7 and 8, we observe that the settling time of our new converter is a factor of 6 shorter than that of the conventional converter. In both experiments, we checked through use of a logic analyzer that the tracking error of our converter was kept within 1 LSB.

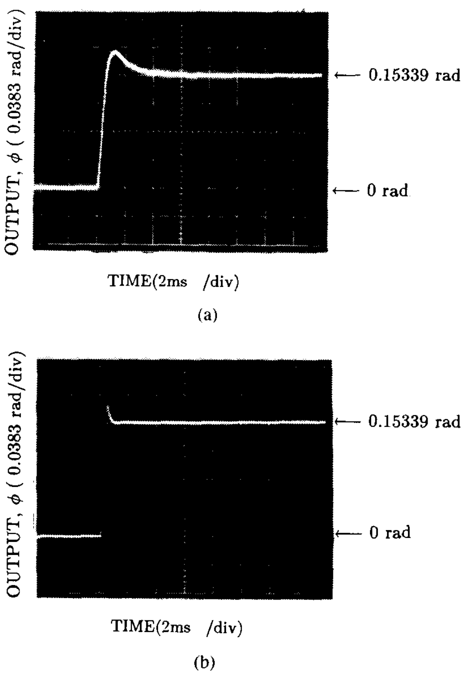


Fig. 7. Experimental results for the 0.15339-rad (100 LSB) step input. (a) Conventional Converter. (b) New converter with asymptotically ideal prefilters.

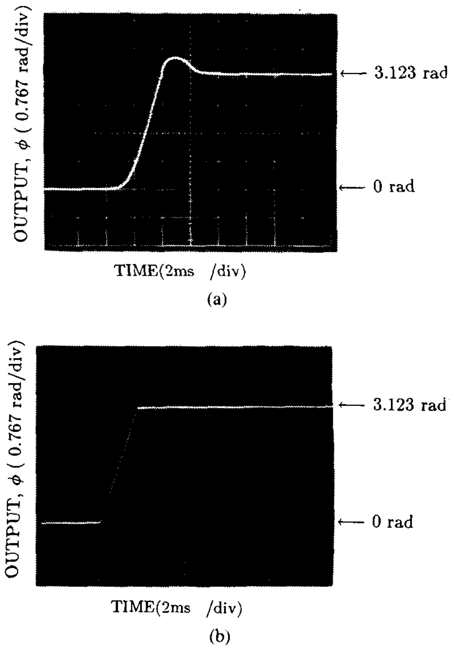


Fig. 8. Experimental results for the 3.123-rad (2036 LSB) step input. (a) Conventional converter. (b) New converter with asymptotically ideal prefilters.

Finally, we discuss the effects of noise on tracking performance of our new converter. The noise, which can affect the tracking performance, can be divided into two kinds. One is the high-frequency noise that accompanies the input signal and the other is the internal noise that is independent of the input signal. The high-frequency noise accompanying the input signal can be effectively eliminated by the prefilters proposed in the preceding sections.

The internal noise that is independent of the input, however, is added directly to the error signal and cannot be prefiltered. This noise is denoted by η as is shown in Figs. 2 and 3. To investigate the effect of this internal noise on the output, we choose a rectangular wave input signal with period 6.5 ms and amplitude 0.0153 rad (10 LSB). From the experimental result in Fig. 9(a), we see that when $K = 150$ the tracking error exceeds 1 LSB due to the internal noise. However, the experimental result in Fig. 9(b) shows that, by simply increasing the loop gain K sufficiently, we can reduce the effect of the internal noise on the output to less than 1LSB. This can be explained as follows.

If we include the noise term in (2) and (3), we have

$$\dot{\phi} = AB\,\mathrm{sgn}\,[K \sin(\theta - \phi) - \eta(t)]. \tag{14}$$

Let $\bar{\eta}$ be the function of time such that

$$\sin(e - \bar{\eta}) = \sin e - \frac{\eta}{K}. \tag{15}$$

If K is large and e is small, (15) uniquely determines $\bar{\eta}$ for each e and η and furthermore $\bar{\theta} = \theta - \bar{\eta}$ satisfies the assumption (A3). By (15), we can write (14) as

$$\dot{\phi} = AB\,\mathrm{sgn}\,[K \sin(\bar{\theta} - \phi)]. \tag{14$'$}$$

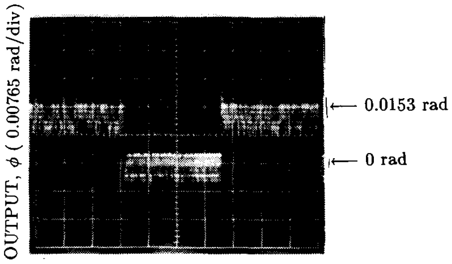
By (14)′ and the property (B1), we have $|\phi - \bar{\theta}| \to 0$ as $t \to \infty$. This, with (15), ensures that

$$\left|\sin e - \frac{\eta}{K}\right| \to 0 \quad \text{as} \quad t \to \infty. \tag{16}$$

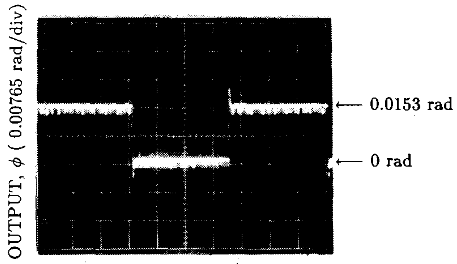
Consequently, if $|\eta(t)| < N$, $t \geq 0$ and K is large, there will be a finite time $t_0 > 0$ such that

$$|e(t)| \leq \sin^{-1}\left(\frac{N}{K}\right), \qquad t \geq t_0. \tag{17}$$

Note from Theorem 1 that, in the case of our converter, excessive increase in the loop gain K does not degrade tracking performance. Therefore, loop gain K can be increased freely for the reduction of the effects of noise. However, in other PLL or R/D converters that use linear loop filters, very large loop gain may reduce the effects of noise significantly but may result in oscillatory or unstable output responses.



(a)



(b)

Fig. 9. Output responses of the new R/D converter for the rectangular input when (a) $K = 150$ and (b) $K = 780$.

V. Conclusions

Through mathematical performance analysis, simulation, and experimentation, we have shown that the proposed R/D converter can track any arbitrary input quickly and precisely as long as the input rate does not exceed its maximum tracking rate AB and has superior tracking performance over conventional converters. The analysis on the effect of noise shows that the position error due to noise can be reduced to less than 1 LSB by increasing the loop gain K sufficiently large without deteriorating tracking performance. For the industrial use of our new converter, however, we have to add some circuits that can automatically control the offsets and drifts of op amps and multipliers. We also need to build a hysteresis type threshold into the conversion loop in order to eliminate the jitter around the null (zero-error) point, due primarily to quantizing noise.

Appendix

Proof of Theorem 1

We prove our assertion via Lyapunov's direct method. This method was proposed originally for the stability analysis on the equilibrium states of nonlinear systems (see Chap. 5 of [10]) but has recently been generalized to regulation [4] and tracking [5] of uncertain nonlinear systems. Nonetheless, a systematic approach to the construction of Lyapunov functions is still unknown. Here, we have found a Lyapunov function for the system (2), (4) by physical insight.

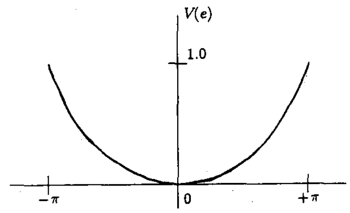


Fig. 10. Graphic representation of the Lyapunov function V.

Choose a Lyapunov function V (Fig. 10) as

$$V = 1 - \cos\left(\frac{e}{2}\right). \tag{18}$$

Then, the total derivative of V along the solution of (2) and (4) is

$$\begin{aligned}\dot{V} &= \left(\frac{1}{2}\right)\dot{e}\sin\left(\frac{e}{2}\right)\\ &= \left(\frac{1}{2}\right)(\dot{\theta} - Af(K\sin e))\sin\left(\frac{e}{2}\right)\\ &\le -\left(\frac{1}{2}\right)Af(K\sin e)\sin\left(\frac{e}{2}\right)\\ &\quad + \left(\frac{1}{2}\right)\left|\sin\left(\frac{e}{2}\right)\right|\sup_{t\ge 0}|\dot{\theta}(t)|.\end{aligned} \tag{19}$$

First, we prove by contradiction that

$$|e(t)| < \epsilon + \sin^{-1}\left(\frac{\sin\delta}{K}\right), \quad t \ge t_0$$

if

$$|e(t_0)| < \epsilon + \sin^{-1}\left(\frac{\sin\delta}{K}\right). \tag{20}$$

If our assertion is false, the continuity of e assures that there exists $t_1, t_2 > t_0$ such that

$$\sin^{-1}\left(\frac{\sin\delta}{K}\right) \le |e(t)| < \epsilon + \sin^{-1}\left(\frac{\sin\delta}{K}\right), \quad t_1 \le t < t_2 \tag{21}$$

but

$$|e(t_2)| = \epsilon + \sin^{-1}\left(\frac{\sin\delta}{K}\right). \tag{22}$$

By (6), (21), and (22),

$$|\sin e| \ge \frac{\sin\delta}{K} \qquad t_1 \le t \le t_2. \tag{23}$$

By this with (4), (19), and (21),

$$\dot{V} \le -\left(\frac{1}{2}\right) AB \sin\left(\frac{e}{2}\right) \frac{\sin e}{|\sin e|} + \left(\frac{1}{2}\right) \left|\sin\left(\frac{e}{2}\right)\right| \sup_{t\ge 0} |\dot{\theta}(t)| = -\left(\frac{1}{2}\right)\left|\sin\left(\frac{e}{2}\right)\right| \left(AB - \sup_{t\ge 0} |\dot{\theta}(t)|\right), \quad t_1 \le t \le t_2. \tag{24}$$

This with (A3) and (21)–(23) implies that

$$\dot{V}(t) < 0 \qquad t_1 \le t \le t_2. \tag{25}$$

Therefore, $V(t_2) < V(t_1)$ and hence $|e(t_2)| < |e(t_1)|$. However, this with (21) and (22) leads to the contradiction:

$$\epsilon + \sin^{-1}\left(\frac{\sin\delta}{K}\right) < \epsilon + \sin^{-1}\left(\frac{\sin\delta}{K}\right). \tag{26}$$

Thus, (20) is true. On the other hand, by (6) and (24),

$$\dot{V}(t_0) < 0 \qquad \text{if} \qquad |e(t_0)| = \epsilon + \sin^{-1}\left(\frac{\sin\delta}{K}\right). \tag{27}$$

Since V is continuous in t, this, with (20), implies that

$$|e(t)| \le \epsilon + \sin^{-1}\left(\frac{\sin\delta}{K}\right), \qquad t \ge t_0$$

if

$$|e(t_0)| \le \epsilon + \sin^{-1}\left(\frac{\sin\delta}{K}\right). \tag{28}$$

Next, we prove (5) for the case of

$$\epsilon + \sin^{-1}\left(\frac{\sin\delta}{K}\right) < |e(t_0)| < \pi - \sin^{-1}\left(\frac{\sin\delta}{K}\right). \tag{29}$$

It should be clear that if (29) holds, the statement in (20) with (6) implies that

$$|e(t)| < \pi - \sin^{-1}\left(\frac{\sin\delta}{K}\right). \tag{30}$$

Now, suppose that

$$\epsilon + \sin^{-1}\left(\frac{\sin\delta}{K}\right) \le |e(t)| < \pi - \sin^{-1}\left(\frac{\sin\delta}{K}\right), \quad t_0 \le t \le t_0 + T. \tag{31}$$

Then, it follows from (6) and (31) that

$$|\sin e| \ge \frac{\sin\delta}{K}, \qquad t_0 \le t \le t_0 + T. \tag{31$'$}$$

By (4), (19), (30), and (31)′, we have

$$\dot{V} \le -\left(\frac{1}{2}\right)\left(AB - \sup_{t\ge 0} |\dot{\theta}(t)|\right)\left|\sin\left(\frac{e}{2}\right)\right| \le -\left(\frac{1}{2}\right)\mu V, \qquad t_0 \le t \le t_0 + T \tag{32}$$

since $1 - \cos(e/2) < |\sin(e/2)|$, if $|e| < \pi$. This implies that

$$V(t) \le V(t_0)\exp\left[-\frac{\mu(t - t_0)}{2}\right], \qquad t_0 \le t \le t_0 + T. \tag{33}$$

In particular,

$$V(t_0 + T) \le V(t_0)\exp\left(-\frac{\mu T}{2}\right). \tag{34}$$

This with the definitions of T and V implies that

$$1 - \cos\left(\frac{e(t_0 + T)}{2}\right) \le 1 - \cos\left(\frac{\epsilon + \sin^{-1}\left(\frac{\sin\delta}{K}\right)}{2}\right). \tag{35}$$

Thus, we have shown that

$$|e(t_0 + T)| \le \epsilon + \sin^{-1}\left(\frac{\sin\delta}{K}\right) \tag{36}$$

if (31) holds. Now, it is not difficult to see that (28) and (36) lead to (5) for the case of (29). □

REFERENCES

[1] N. U. Ahmed and M. Aboud, "Optimum design of a feedback controller for a phase-locked loop," *IEEE Trans. Commun.*, pp. 1521–1524, Dec. 1975.

[2] N. U. Ahmed and S. H. Wong, "On optimum phase-locked loop," *Int. J. Electron.*, vol. 36, no. 2, pp. 253–259, 1974.

[3] G. Boyes, Ed., *Synchro and Resolver Conversion.* Analog Devices Inc., 1980.

[4] M. J. Corless and G. Leitmann, "Continuous state feedback guaranteeing uniform ultimate boundedness for uncertain dynamic systems," *IEEE Trans. Automat. Contr.*, vol. AC-26, pp. 1139–1144, 1981.

[5] I. J. Ha and E. G. Gilbert, "Robust tracking in nonlinear systems," *IEEE Trans. Automat. Contr.*, vol. AC-32, pp. 763–771, 1987.

[6] D. C. Hanselman, "Resolver signal requirements for high accuracy resolver-to-digital conversion," *IEEE Trans. Ind. Electron.*, vol. 37, no. 6, pp. 556–561, Dec. 1990.

[7] ——, "Signal processing techniques for improved resolver-to-digital conversion accuracy," *IECON'90*, pp. 6–10, 1990.

[8] S. T. Hung and J. C. Hung, "Digitally compensated resolvers," *IECON'84*, pp. 625–627, 1984.

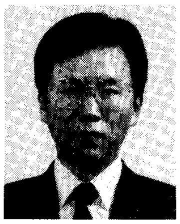
[9] *Synchro Conversion Handbook.* ILC Data Device Co., 1982.

[10] M. Vidyasagar, *Nonlinear Systems Analysis.* Englewood Cliffs, NJ, Prentice-Hall, 1978.

[11] D. Williamson, "Improved phase-locked loop performance via nonlinear loop filters," *IEEE Trans. Commun.*, vol. COM-27, no. 3, pp. 542–556, Mar. 1979.

Choong-Hyuk Yim was born on Dec. 10, 1965. He received the B.S. and M.S. degrees from Seoul National University, Seoul, Korea, in 1987 and 1989, respectively. He is presently working toward the Ph.D. degree in control and instrumentation engineering at Seoul National University.

His principal research interests are nonlinear control theory and its application to electric machines.



In-Joong Ha (M'87) received the B.S. and M.S. degrees in electronics engineering from Seoul National University, Seoul, Korea, in 1973 and 1980, respectively, and the Ph.D. degree in computer, information, and control engineering (CICE) from the University of Michigan, Ann Arbor, in 1985.

He is presently an Associate Professor in the Department of Control and Instrumentation Engineering, Seoul National University, Seoul, Korea. From 1973 to 1981, he worked in the area of missile guidance and control at the Agency for Defence Development in Daejeon, Korea. From 1982 to 1985, he was a Research Assistant at the Center for Research on Integrated Manufacturing, University of Michigan, Ann Arbor. From 1985 to 1986, he worked as a Senior Research Engineer at General Motors Research Laboratories, Warren, MI. His fields of interest are nonlinear control theory and its application to robotics, electric machines, and missiles.

Dr. Ha received the 1985 Outstanding Achievement Award in the CICE program.

Myoung-Sam Ko (SM'87) was born in Hamhung, Korea, on Jan. 1, 1930. He received the B.S., M.S., and Ph.D. degrees in electrical engineering from Seoul National University, Seoul, Korea, in 1955, 1960, and 1972, respectively.

He is presently a Professor in the Department of Control and Instrumentation Engineering, Seoul National University, Seoul, Korea, where he is also the Director of Automation and Systems Research Institute. He was a Visiting Scholar at the University of New Mexico and the University of Tokyo, in 1968 and 1979, respectively. In 1978, he served as the Chairman of the IEEE Korean Section. From 1987 to 1988, he was the President of the Korean Institute of Electrical Engineers (KIEE). His recent research interests are digital control and microprocessor applications to motors, multijoint robots, and textile manufacturing process.

Dr. Ko received a KIEE Paper Award, an IEEE Centennial Medal, and a Korean National Camellia Award, in 1974, 1984, and 1989, respectively, for his extraordinary activities in the field of control and instrumentation engineering.