

Improved Full state Hybrid Projective Synchronization of Chaotic Systems with the Different Order

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Abstract

Chaos synchronization has received a signification attention in nonlinear science. Over the past two decades, many types of synchronization have been announced. This paper introduces another novel type of chaos synchronization-improved full state hybrid projective synchronization (IFSHPS), which includes FSHPS, complete synchronization, anti-synchronization and projective synchronization as its special item. The study first proposed the synchronization that response system also has the constant diagonal matrix. Based on the Lyapunov stability theorem and the adaptive control techniques, the IFSHPS scheme is given and illustrated with generalized Lorenz system and Lü system as example. With the effective scheme parameters identification and IFSHPS of chaotic system can be realized simultaneously. Numerical simulations are presented to demonstrate the effectiveness of the proposed synchronization scheme.

Keywords: IFSHPS, active control, lyapunov stability theory.

1. Introduction

During the last two decades, synchronization in chaotic dynamic systems has received a great deal of interest among scientists from various research fields [1-7] such as neural networks, secure communications and time series analysis of chaotic systems. The idea of synchronization is to use the output of the master system to control the slave system so that the output of response system follows the output of the master system asymptotically. In the literature, there exist many types of synchronization such as complete synchronization, anti-synchronization, projective synchronization [8-9]. In most cases of drive-response synchronization, all the states of response system

synchronize to the corresponding states of drive system in term of the same synchronization regime. For example, when we say that two systems are complete-synchronized (or phase-synchronized, or something else) with each other, it means that each pair of the states between the interactive systems is complete-synchronized (or phase-synchronized, or something else).

Recently, Hu etc [10, 11, 12] investigated a new type of chaotic synchronization-full state hybrid projective synchronization (FSHPS). The feature of this synchronization phenomenon is that the scaling factors of the synchronization can be arbitrarily designed to different state variables by means of control. Do the phenomenons that drive system and response system have scaling factors at the same time? There is no double that it is an interesting problem. But to the best of our knowledge, there is no investigation results reported in the literature. Inspired by above works, in this paper, we show that the phenomenon, which called improved full state hybrid projective synchronization (IFSHPS), does exist. Especially, response system also has the constant diagonal matrix.

For two dynamical systems

$$\dot{x}(t) = F(x) \leftarrow \text{drive system}, \quad (a)$$

$$\dot{y}(t) = G(x, y) \leftarrow \text{response system}, \quad (b)$$

where $x = (x_1, x_2, \dots, x_n)^T$, $y = (y_1, y_2, \dots, y_n)^T \in \mathbb{R}^n$ are state variables of the drive system (a) and the response system (b).

Definition 1 (IFSHPS). For the drive system (a) and response system (b), if there exists nonzero constant matrix $\alpha = \text{diag}(\alpha_1, \alpha_2, \dots, \alpha_n)$ $\beta = \text{diag}(\beta_1, \beta_2, \dots, \beta_n) \in \mathbb{R}^{n \times n}$ such that $\lim_{t \rightarrow \infty} \|\alpha y - \beta x\| = 0$. We are said to be IFSHPS (an acronym for improved full-state hybrid projective synchronization), where α and β are called scaling matrix, $\alpha_1, \alpha_2, \dots, \alpha_n$ and $\beta_1, \beta_2, \dots, \beta_n$ are called scaling factors.

At present, most of theoretical results about synchronization of chaos focus on the systems whose models are identical, similar or with mismatched parameters. However, in many practical world such as laser array and biological systems, it is hardly the case that every component can be assumed to be identical. More and more applications of chaos synchronization in secure communications make it much more important to synchronize two different and different order chaotic systems in recent years. So, the study of synchronization for strictly different dynamical systems and different order dynamical systems is both very important from the perspective of control theory and very necessary from perspective of practical application.

The aim of this paper is to study the IFSHPS of drive and response systems with fully unknown parameters. The rest of this Letter is organized as follows. In section 2, the adaptive IFSHPS scheme of chaotic systems with different order presented. Generalized Lorenz and Lü chaotic systems are applied to investigate IFSHPS, and numerical simulations are used to verify the effectiveness of the proposed scheme in Section 3. Finally, a conclusion ends the paper.

2. Adaptive IFSHPS of chaotic systems with different order

In this section we will choose generalized Lorenz system, Lü system to illustrate the adaptive IFSHPS with different order. Our purpose is to achieve the IFSHPS of Lü system and the former three states of generalized Lorenz system. Therefore, we divide the drive system (Generalized Lorenz system [13]) into two parts.

$$\begin{cases} \dot{x}_1(t) = a(x_2 - x_1) + dx_4 \\ \dot{x}_2(t) = bx_1 - x_1x_3 - x_2 \\ \dot{x}_3(t) = x_1x_2 - cx_3 \end{cases} \quad (1)$$

and

$$\dot{x}_4(t) = -x_1 - ax_4 \quad (2)$$

which has a chaotic attractor as shown in Fig.1 when $a=1$, $b=26$, $c=0.7$, $d=1.5$ with the initial values $x_1(0)=1$, $x_2(0)=1$, $x_3(0)=1$, $x_4(0)=1$.

The response system (Lü system [14]) is

$$\begin{cases} \dot{y}_1(t) = a_1(y_2 - y_1) + u_1 \\ \dot{y}_2(t) = -y_1y_3 + c_1y_2 + u_2 \\ \dot{y}_3(t) = y_1y_2 - b_1y_3 + u_3 \end{cases} \quad (3)$$

which has a chaotic attractor as shown in Fig.2 when $a_1=36$, $b_1=3$ and $c_1=14$ with the initial values $y_1(0)=2$, $y_2(0)=3$, $y_3(0)=4$.

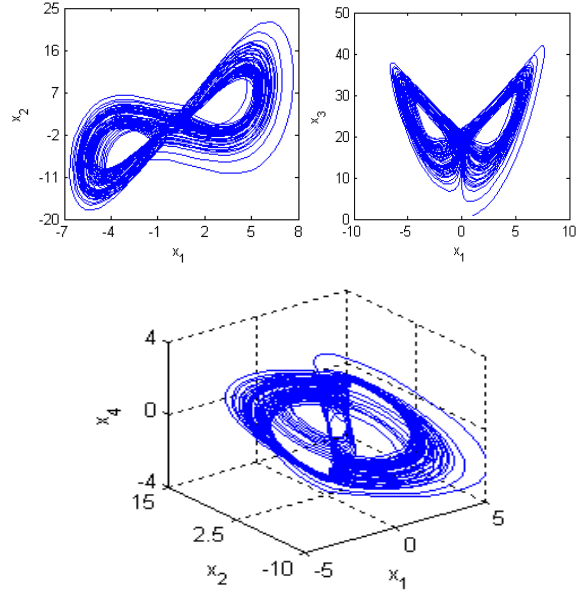


Figure 1. Phase map of Generalized Lorenz system

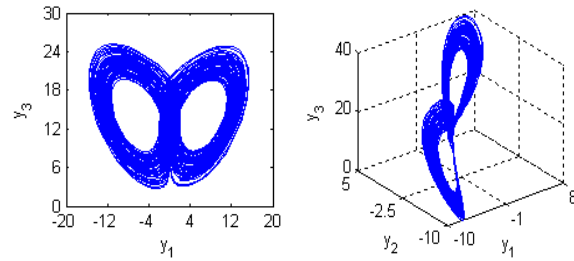


Figure 2. Phase map of Lü system

Our purpose is to achieve the IFSHPS between drive system and response system with unknown parameters. In the study, a , b , c , d and a_1 , b_1 , c_1 are unknown system parameters, $(u_1, u_2, u_3)^T$ is the controller to be determined.

Let us define the state errors between the drive system (1) and the response system (2) as

$$e_1 = \alpha_1 y_1 - \beta_1 x_1, e_2 = \alpha_2 y_2 - \beta_2 x_2, e_3 = \alpha_3 y_3 - \beta_3 x_3$$

The following is the error dynamics:

$$\begin{cases} \dot{e}_1 = \alpha_1(a_1(y_2 - y_1) + u_1) - \beta_1(a(x_2 - x_1) + dx_4) \\ \dot{e}_2 = \alpha_2(-y_1y_3 + c_1y_2 + u_2) - \beta_2(bx_1 - x_1x_3 - x_2) \\ \dot{e}_3 = \alpha_3(y_1y_2 - b_1y_3 + u_3) - \beta_3(x_1x_2 - cx_3) \end{cases} \quad (4)$$

Theorem: If the controller is choose as follow:

$$\begin{cases} u_1 = 1/\alpha_1(-\alpha_1(y_2 - y_1) + \beta_1 x_4 \hat{d} \\ \quad + \beta_1(x_2 - x_1)\hat{a} - e_1), \\ u_2 = 1/\alpha_2(\alpha_2 y_1 y_3 + \beta_2(-x_2 - x_1 x_3) \\ \quad - \alpha_2 y_2 \hat{c}_1 + \beta_2 x_1 \hat{b} - e_2), \\ u_3 = 1/\alpha_3(-\alpha_3 y_1 y_2 + \beta_3 x_1 x_2 + \alpha_3 y_3 \hat{b}_1 \\ \quad - \beta_3 x_3 \hat{c} - e_3) \end{cases} \quad (5)$$

and the estimates $\hat{a}$, $\hat{b}$, $\hat{c}$, $\hat{d}$, $\hat{a}_1$, $\hat{b}_1$ and $\hat{c}_1$ obey the following update laws:

$$\begin{cases} \dot{\hat{a}} = -\beta_1 e_1(x_2 - x_1) \\ \dot{\hat{b}} = -\beta_2 e_2 x_1 \\ \dot{\hat{c}} = \beta_3 e_3 x_3 \\ \dot{\hat{d}} = -\beta_1 e_1 x_4 \end{cases} \quad (6)$$

and

$$\begin{cases} \dot{\hat{a}}_1 = \alpha_1 e_1(y_2 - y_1) \\ \dot{\hat{b}}_1 = -\alpha_3 e_3 y_3 \\ \dot{\hat{c}}_1 = \alpha_2 e_2 y_2 \end{cases} \quad (7)$$

Then the error system is globally stable. Hence, the response system (3) associated the proposed control law (5) and the drive system (1) can globally asymptotically achieve the IFSHPS, and the unknown parameters can be estimated.

Proof: Choose the following Lyapunov function V as follows:

$$V = \frac{1}{2}(e_1^T e_1 + e_2^T e_2 + e_3^T e_3 + \hat{a}^T \hat{a} + \hat{b}^T \hat{b} + \hat{c}^T \hat{c} + \hat{d}^T \hat{d} \\ + \hat{a}_1^T \hat{a}_1 + \hat{b}_1^T \hat{b}_1 + \hat{c}_1^T \hat{c}_1)$$

where

$\hat{a} = (\tilde{a} - a)$, $\hat{b} = (\tilde{b} - b)$, $\hat{c} = (\tilde{c} - c)$, $\hat{d} = (\tilde{d} - d)$,
 $\hat{a}_1 = (\tilde{a}_1 - a_1)$, $\hat{b}_1 = (\tilde{b}_1 - b_1)$, $\hat{c}_1 = (\tilde{c}_1 - c_1)$. With the choice of the controller (5) and updating laws (6) and (7), the derivative with respect to time is

$$\dot{V} = -e^T e < 0$$

that is, $\dot{V} < 0$ is negative. Then according to the Lyapunov stability theorem, the IFSHPS of chaotic systems (1) and (3) is achieved.

3. Simulations results

In this section numerical simulations are given to verify the effectiveness of adaptive method for achieving synchronization of two different order systems.

3.1. Complete synchronization simulation results

Firstly, we give the special cases of projective synchronization where $\alpha_i = 1$ and $\beta_i = 1$, $i = 1, 2, 3$ is complete synchronization. The system “unknown” parameters are chosen to be $a = 1$, $b = 26$, $c = 0.7$, $d = 1.5$, $a_1 = 36$, $b_1 = 3$ and $c_1 = 14$. So that generalized Lorenz and Lü systems have chaotic attractor. The initial states of the drive system and response system are $X = (1, 1, 1, 1)^T$, $Y = (2, 3, 4)^T$, respectively. The simulations results are shown from Fig.3-6.

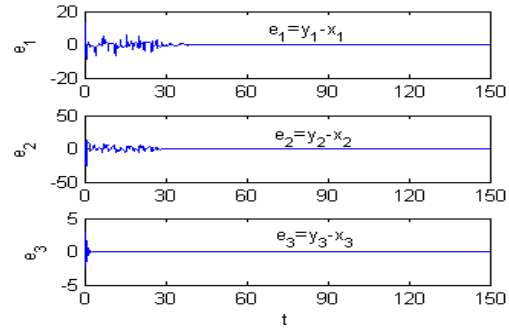


Figure 3. The time evolution of the errors with the scaling factors $\alpha_i = 1$ and $\beta_i = 1$, $i = 1, 2, 3$

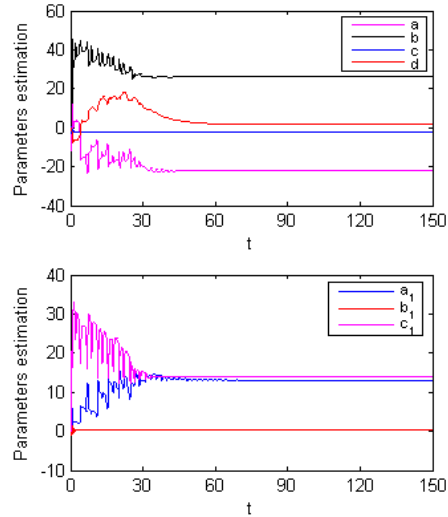


Figure 4. Parameters a, b, c, d and a_1, b_1, c_1 , estimation of chaotic systems

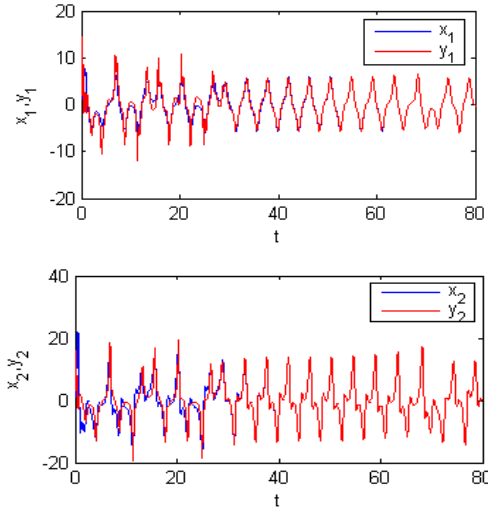


Figure 5. Time series of generalized Lorenz and Lü chaotic systems

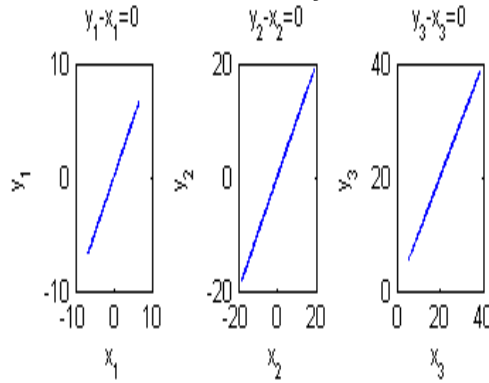


Figure 6. Complete synchronization of chaotic systems with the different order: signal x_i and y_i , $i=1,2,3$

Fig.3 shows the behavior of complete synchronization error states via adaptive control method. Fig.4 shows the Parameters estimation. The time series of x_1 and y_1 ; x_2 and y_2 are shown in Fig.5. Red line depicts the response system; blue line depicts the drive system with the unknown parameters and different initial values. The plots of x_1 versus y_1 , x_2 versus y_2 and x_3 versus y_3 after transient states are shown in Fig.6.

3.2. Anti-synchronization simulation results

Secondly, where $\alpha_i = 1$ and $\beta_i = -1$, $i = 1, 2, 3$ is anti-synchronization. The same simulations results as complete synchronization are shown from Fig.7-10. The following maps give the anti-synchronization error, parameters estimation, time series and the attractors in R^2 .

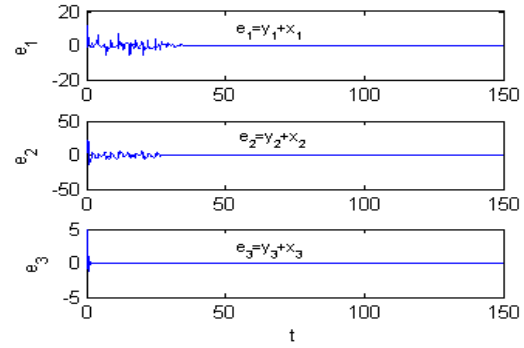


Figure 7. The time evolution of the errors with the scaling factors $\alpha_i = 1$ and $\beta_i = 1$, $i=1,2,3$

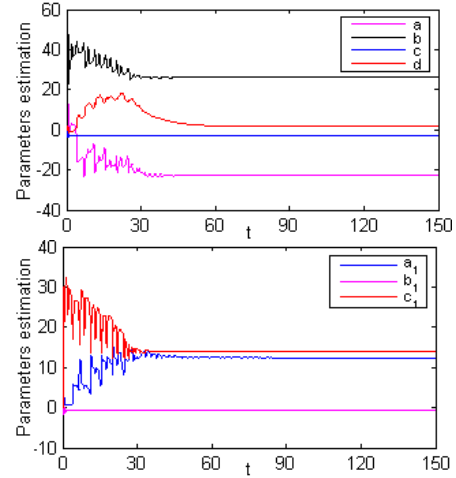


Figure 8. Parameters a, b, c, d and a_1, b_1, c_1 , estimation of chaotic systems

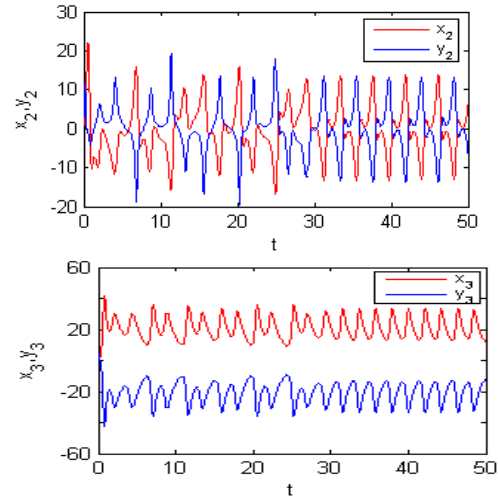


Figure 9. Time series of anti-synchronization

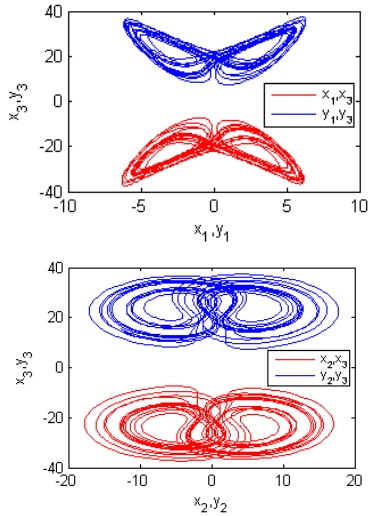


Figure 10. The attractors in R^2

3.3. IFSHPS simulation results

In this section, we give the IFSHPS simulation results. Without loss of generality, we set the scaling factors $\alpha_1 = 2$, $\alpha_2 = 0.5$, $\alpha_3 = 1$, $\beta_1 = -0.5$, $\beta_2 = 1.5$, $\beta_3 = 2$.

The simulation results are shown in Fig.11, 12, 13, and 14. From Fig.11 we can see the error vector e converge to zero as time t goes to infinity. Fig.12 shows the results of parameters identification. In Fig.13 blue line depicts the response system; red line depicts the drive system with the unknown parameters and different initial values. The plots of x_1 versus y_1 , x_2 versus y_2 and x_3 versus y_3 after transient states are shown in Fig.14. The above numerical simulations show that the IFSHPS of two different structure chaotic systems with unknown parameters is realized.

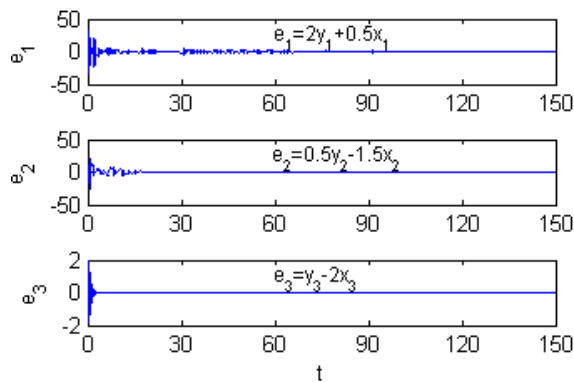


Figure 11. IFSHPS errors of generalized Lorenz and Lü chaotic systems with fully unknown parameters

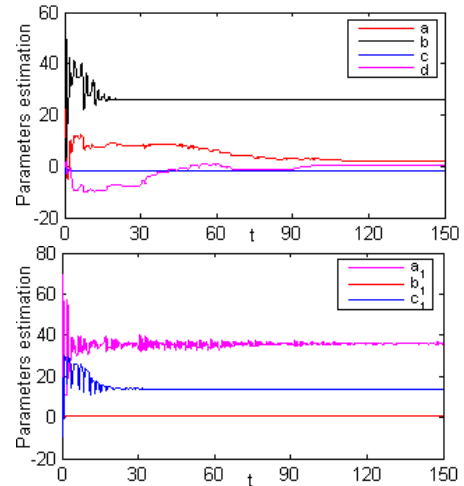


Figure 12. Parameters a, b, c, d and a_1, b_1, c_1 , estimation of chaotic systems

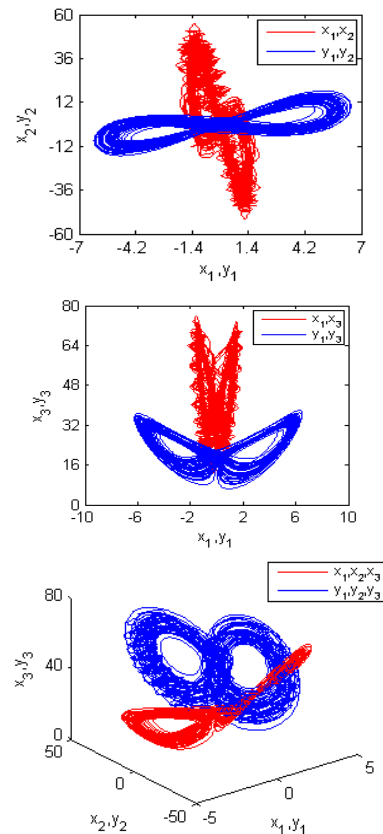


Figure 13. The attractors in R^2 and R^3

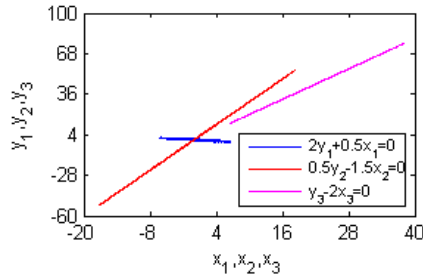


Figure 14. Improved FSHPS of chaotic systems with the different order: signal x_i and y_i , $i=1,2,3$

4. Summary

The paper first proposed the synchronization that response system also has scaling factors α . In this letter, we used the term “IFSHPS” (an acronym for improved full-state hybrid projective synchronization.) distinguish the “FSHPS”. The scaling factors α and β of this synchronization can be arbitrarily designed to different state variables by means of control.

In this paper the adaptive IFSHPS scheme has been proposed for chaotic systems with fully unknown parameters. A unified controller and a parameters update law are designed to achieve the chaotic systems with different order, based on the Lyapunov stability theorem. In the simulation, the special cases of the IFSHPS are also given. Many groups of numerical examples are presented to illustrate the effectiveness of the proposed synchronization.

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