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Reduced-parameter fractional-order modeling of large dynamical system: Application to Gas Turbine

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***Abstract*—Nowadays, gas turbines are popularly used for power generation. Widespread installations and the dynamic nature of this system has increased the necessity of its accurate modeling and control. It is a large and complex dynamical system and exact identification of its parameters like temperature and speed are the important issues. Noticeable research activities are going on so far in this field in order to understand and simplify the nonlinear behavior of these complex systems. However, the need for simple and compact models has been a strong motivation for researchers. Most of the advanced controllers are model based, if model is bulky or having large parameter then controller design become tedious and time consuming. Online implementation of such model driven strategy is also the main concern. Further it results in sluggish operation and also deteriorates the closed loop performance. Hence simplified and compact model is an important requirement. This paper proposes, application of fractional calculus theory to find compact model of gas turbine. A simple and compact fractional-order models are obtained using parameter reduction technique. A local optimization problem is posed to represent large parameter integer-order systems by few parameters fractional-order models. The dynamics of original system are retained in compact fractional-order models because of its memory type nature. Results shows that the proposed models completely represent the dynamic of large parameter integer-order system.**

***Index Terms*—Reduced parameter modeling, Large parameter integer-order system, Compact fractional-order modeling, Optimization technique.**

I. Introduction

Worldwide installations of gas turbines are increased and is now one of the major part of modern industry. It acts as mechanical drivers for compressor and very important equipment for power generation. Obtaining the accurate model this complex device and develop simulation is the crucial requirement. Combined cycle power plant is the complex arrangement and involves many parameters. The gas turbine system for electricity generation includes combustion chamber, compressor, turbine, inlet guide vane, fuel system, positioner and control unit. In order to know the complete dynamic nature of gas turbine it is important to see the individual component behaviour [1], [2]. The simple and compact model of gas turbine which completely represent the dynamic nature is very important and which also help to design superior controller [3]. Some well known and popular gas turbine model available in the literature are Rowen's model [4], IEEE model, Frequency dependent model, GGOV1 model, CIGRE model and Detailed model [5]. All these model requires intensive computer simulations and design of advanced controller adds computational complexities. In the literature, the transfer function based model is used to represent dynamic nature of gas turbine. In mathematical modeling the output speed and exhaust temperature are controlled variable. Inlet fuel flow and guide vane signal are manipulated variable [6]. Gas turbine is large parameter and complex dynamical systems for which exact identification of the parameters is very important and design of temperature and speed controller are the crucial task. Noticeable research activities are going in this field in order to understand and simplify the nonlinear behavior of these systems. However, the need for simple, compact models for performance optimisation has been a strong motivation for researchers.

Designs methodology for many advanced controller are model based, if model is bulky or having large parameter then controller design become tedious and time consuming. Online implementation of such model driven controller is also the main concern which results in sluggish operation and also further deteriorates closed loop performance. Hence, simplified and compact model is an important requirement.

Presently, lots of research activities are found in the area of fractional-order (FO) modeling and control. It is the application of fractional calculus(FC), a branch of mathematics dealing with derivatives and integrals of non-integer order (real or complex) [7] [8]. FO models are compact and gives more realistic representation of real world and man-made systems. FO controllers are very popular and proved their efficacy over the conventional integer-order (IO) controllers. FO controller provides more flexibility in the design and ensures a better robust performance.

Fractional calculus has potential to represents many real world and man-made systems in to a compact form [9]. This paper proposes application of fractional calculus theory for modeling of gas turbine. It discusses the simple and compact parameter reduction technique used to represent the large parameter integer-order gas turbine model by the compact fractional-order model. Here we formulate the optimization problem which minimizes the error between the two model and gives the simple and compact fractional-order models.

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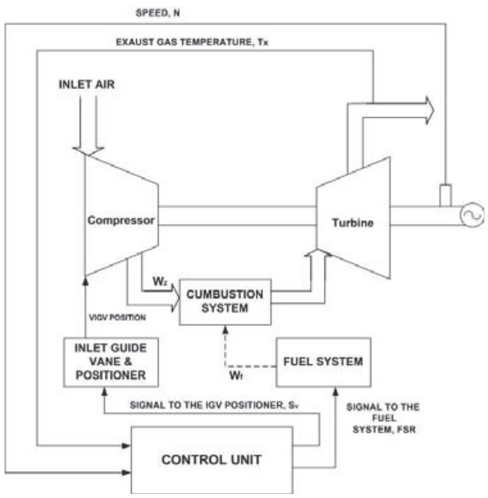

Fig. 1. A general arrangement of combined cycle power plant [6].

Rest of the paper is organized as follows. In Section 2, the basic arrangement of gas turbine and the transfer function model between the output turbine speed and inlet fuel flow rate is explained. The introduction to fractional calculus and fractional-order system is given in Section 3. Fractional-order modeling technique for parameter reduction is presented in in Section 4. Section 5, gives the step by step procedure to pose the optimization problem and solution using $MATLAB$ and the results obtained are presented. Finally, conclusion is drawn in Section 6.

II. Modelling of Gas Turbine

In power plant applications, the gas turbine which operates on natural gas, biomass gas and diesel is commonly used and preferred [6]. Fig. 1 shows the important blocks of power plant in combined cycle operation. The pressure of gas is first raised in compressor. The combustion chamber used to heat the pressured gas. Then high pressure and temperature gas is allow to pass in the turbine. The heat energy of the gas is converted in to mechanical energy. Accurate mathematical modeling and design of robust controller are the most important requirements of gas operated power plant. The purpose of controller is keep the output frequency and voltages constant in spite of load variations. In the literature, for gas turbines different thermodynamic models have been reported which include simple and applied models such as the Rowen model [4]. Aerothermodynamics, computational and the thermodynamic models are available for a power plant in combined cycled operation [5]. The parameters which can be measured are T_{amb} (Ambient temperature), IGV (inlet guide vane position), F (fuel flow), T_x (exhaust gas temperature), N (turbine speed) and P (produced power). These parameters are sampled at 1 s intervals while commenced loading up to the nominal load. The linear integer-order model is given in [6] between the turbine speed and fuel flow. The transfer functions $G_1(s)$ to $G_7(s)$ which gives relation between the control parameters of a gas turbine are given in [6] which were derived using ARX procedure. The parameters F and IGV are inputs which are calculated by the controller. P_d (demand power) and T_{amb} are treated as disturbances. The complete model for 4 inputs and 3 outputs is given below

$$\begin{bmatrix} N \\ P \\ T_x \end{bmatrix} = T. \begin{bmatrix} P_d \\ T_{amb} \\ F \\ IGV \end{bmatrix}, \tag{1}$$

where T is the transfer function matrix of system. The linear model of speed loop which is between the speed(N) and the input fuel flow(F) [3] is,

$$N = T_{13} = \left[\frac{G_1 G_3}{1 - G_2 G_3}\right].F. \tag{2}$$

After substituting the value of $G_1(s)$, $G_2(s)$ and $G_3(s)$ in Eqn.(2) and after further simplification it is derived as,

$$G_{IO}(s) \equiv \frac{N(s)}{D(s)}. \tag{3}$$

where,

$$N(s) = 7.08s^5 + 25.95s^4 + 28.17s^3 + 9.639s^2 + 0.955s$$

$$D(s) = 342.3s^6 + 915s^5 + 757.8s^4 + 196.4s^4 + 16.82s^2 + 0.18889s$$

This speed loop model Eqn. (3) is of high-order and integer type. This paper proposes a technique to reduce and represents this high-order integer model by simple and compact fractional-order models. These simple and compact models have all the dynamics of original system and can be easily used to design advanced controller.

III. Fractional Calculus and Fractional Order Dynamical System

Fractional calculus(FC) can be defined as a more generalization of derivatives and integrals to non-integer orders. It is as old as conventional calculus and its application to control theory is described in [7]. Many real world system can be represented in compact and simple manner with the use of FC. Fractional-order (FO) systems have memory or hereditary properties, while the integer-order (IO) systems have limited memory. The fractional operator is ${}_aD_t^{\alpha}$, where a and t are the limits and α, $(\alpha \epsilon \Re)$ is the order of the operation. Commonly used definitions are the Grunwald−Letnikov (GL) definition , Riemann−Liouville (RL) definition and Caputo FD definition which are presented in [7] [8]. The Laplace transform technique can be easily extended to FD and used for the analysis of linear FO systems.

IV. Parameter Reduction Methods

Fractional-order(FO) dynamic model is the best characterization of real world system than its counterpart. Conventional calculus is based on integer order differentiation and integration. The concept of fractional calculus has tremendous potential to change the way we see, model [8]. By getting motivated by this idea it is attempt to simplify the large parameter integer-order(IO) gas turbine system model [9]. The basic methodology and the optimization procedure to find simple and compact model is discussed here.

*1) **Step data based Parameter Reduction**:* The compact and simple FO model having only three parameter is considered to be obtained from the step response data of large parameter IO system, which is,

$$\frac{Y(s)}{U(s)} = G_{FO}(s) = \frac{b}{s^{\alpha} + a}, \tag{4}$$

The laplace transform it is,

$$D^{\alpha}y(t) + ay(t) = bu(t). \tag{5}$$

The response of this FO model to unit step input is calculated by finding the analytical solution,

$$y_m(t) = \mathcal{L}^{-1}\left[\frac{b}{s(s^{\alpha}+a)}\right] = \left(\frac{b}{a}\right) t^{\alpha}\mathscr{E}_{\alpha,\alpha+1}(-at^{\alpha}), \tag{6}$$

$$= \left(\frac{b}{a}\right)(1 - \mathscr{E}_{\alpha}(-at^{\alpha}). \tag{7}$$

The step response this is evaluated using Mittag-Leffler function. The basic Mittag-Leffler function is expressed as,

$$\mathscr{E}_{\alpha,\beta}(z) = \sum_{k=0}^{\infty}\frac{z^k}{\Gamma(\alpha k + \beta)}, \tag{8}$$

where ($\alpha > 0$, $\beta > 0$).
The response data for the large parameter IO system is obtained using step command in $MATLAB$.
Let,
y_p = Sampled step response data of IO plan ,
y_m= Sampled data of analytical solution of FO model.
As we want compact FO model y_m from the higher order IO plant model y_p. The parameter reduction is achieved by formulating optimization problem. The least square error criterion for parameter reduction is used. Thus the objective function is minimization of error between y_p - y_m,

$$f(b, a, \alpha) = \sum_{k=0}^{N}(y_p(k) - y_m(k))^2, \tag{9}$$

$$f(b, a, \alpha) = \sum_{k=0}^{N}\left[y_p(k) - \left(\frac{b}{a}\right)(1 - E_{\alpha}(-at_k^{\alpha}))\right]^2, \tag{10}$$

where,

$$t_k = t(k). \tag{11}$$

The a, b and α are unknown, the search domain given is $[b, a, \alpha]$ = $[0.011, 0.011, 0.01]$, the lower bounds $[b, a, \alpha] = [0.01, 0.01, 0.01]$ and the upper bounds $[b, a, \alpha] = [10, 10, 3]$. The considered frequency band is 10^{-3} to 10^{1} rad/s and the time is $t = 0 : 0.1 : 400s$.
The optimization problem Eqn.(9) is solved using $fmincon$ function of $MATLAB$ and the high-order IO system is reduced into the simple FO model.

*2) **Impulse data based Parameter Reduction**:* Similar procedure is followed for impulse input. The impulse response data of IO system is used. FO model having three parameter as shown below is to be obtained from the impulse response data is,

$$G_{FO}(s) = \frac{b}{s^{\alpha} + a}, \tag{12}$$

We now compare the impulse response of two system and use function $fmincon$ to minimize the error,

$$f(b, a, \alpha) = \sum_{k=1}^{N}(y_p(k) - y_m(k))^2, \tag{13}$$

Where,
y_p = Sampled data of impulse response of IO Plant,
y_m= Sampled data of impulse output of FO model.
The sampled response data of the compact FO model for unit impulse input is obtained analytical as shown below,

$$Y_{FO}(s) = y_m(s) = \frac{b}{s^{\alpha}+a}.U(s) \tag{14}$$

$$u(t) = \delta(t), \Rightarrow U(s). \tag{15}$$

$$y_m(t) = \mathcal{L}^{-1}\left[\frac{b}{s^{\alpha}+a}\right]. \tag{16}$$

We can solve as given in [8],

$$\mathcal{L}^{-1}\left[\frac{b}{s^{\alpha}+a}\right] = t^{\alpha-1}E_{\alpha,\alpha}(-at^{\alpha}), \tag{17}$$

so,

$$\mathcal{L}^{-1}\left[\frac{b}{s^{\alpha}+a}\right] = b.t^{\alpha-1}E_{\alpha,\alpha}(-at^{\alpha}), \tag{18}$$

$$y_m(t) = b.t^{\alpha-1}E_{\alpha,\alpha}(-at^{\alpha}. \tag{19}$$

V. MATLAB Implementation Procedure and Results

The procedure to find compact FO model is discussed below:

1) Choose the structure FO model to be obtained.
2) Obtain IO model step/impulse response sampled data (y_p).
3) Find the analytical solution of FO model for the step/impulse input (y_m).
4) A least square optimization problem is posed where objective function is error ($y_p - y_m$) minimization.
5) This problem is solved using $MATLAB$ function $fmincon$.

*1) **Step Model**:* After posing the optimization problem and solving it the higher order integer type model is represented by compact FO model as given below,

$$G_{Fstep}(s) = \frac{0.0541}{s^{1.0367} + 0.0108}. \tag{20}$$

Fig.2 and Fig.3 shows the step response and bode plot of original large parameter IO system and compact FO model. Fig.4 and Fig.5 shows the impulse and ramp response of compact FO model and original large parameter IO model.

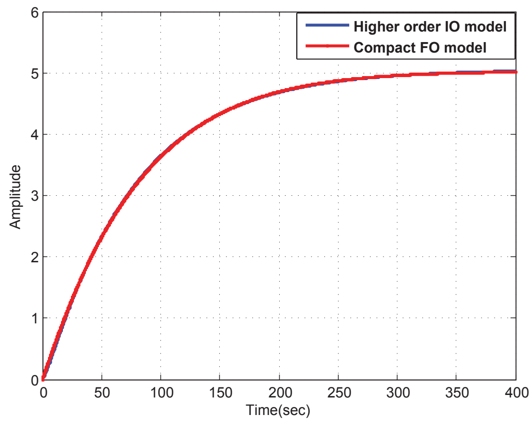

Fig. 2. Step response of original IO model and compact FO model.

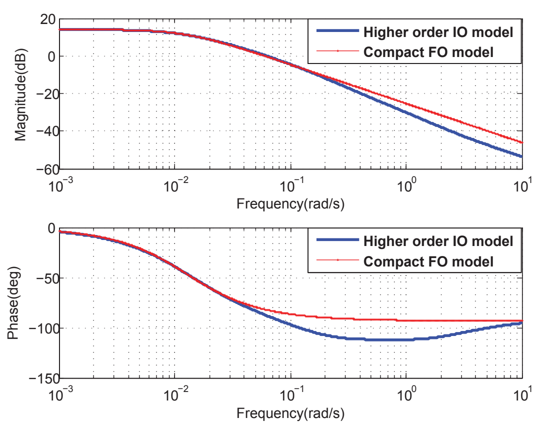

Fig. 3. Bode plot of original IO model and compact FO model.

*2) **Impulse Model**:* The optimization problem solved for impulse input gives the compact and simple FO model as,

$$G_{Fimpulse}(s) = \frac{0.0478}{s^{1.0737} + 0.01}. \tag{21}$$

Fig.6 and Fig.7 presents the impulse response and bode plot of original large parameter IO system and compact FO model. Fig.8 and Fig.9 shows the step and ramp response of compact FO model with original large parameter IO model. FO models obtained are simple and compact and retains the dynamic of IO model and hence it completely represent the large dynamic system.

*3) **Model Parameter Analysis**:* As given in Eqn.20, and Eqn.21 the parameters of the models are tabulated in Table.I. Two different FO model are obtained for the same integer-order model , one from step and other from impulse data. It is due to the function $fmincon$ which is local optimization algorithm. It is also observed that models are sensitive to initial conditions of parameters and the time span.

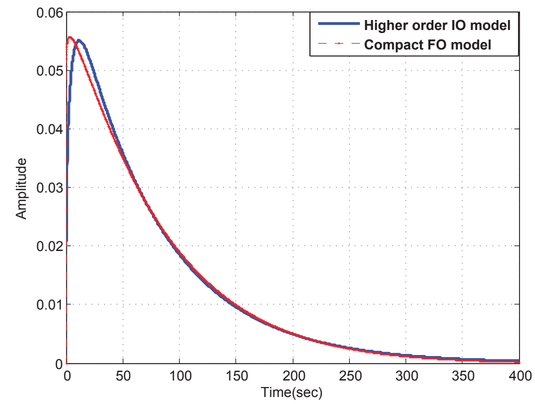

Fig. 4. Impulse response of original IO model and compact FO model.

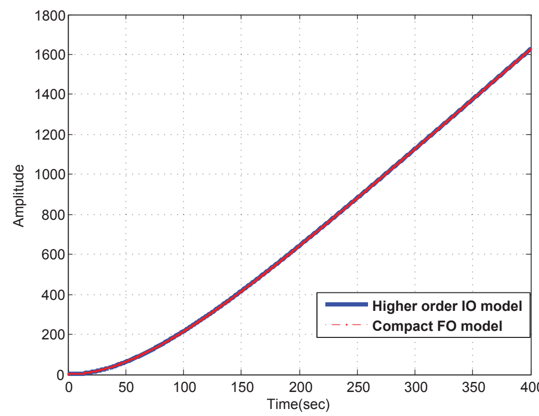

Fig. 5. Ramp response of original IO model and compact FO model.

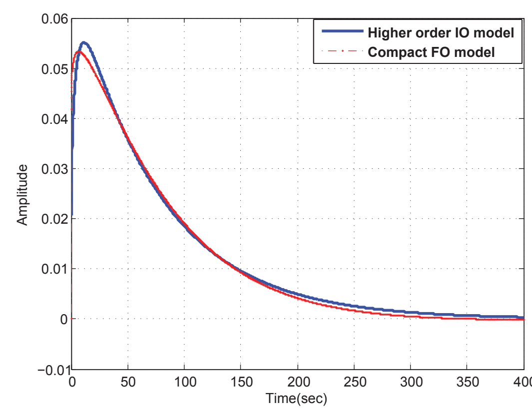

Fig. 6. Impulse response of original IO model and compact FO model.

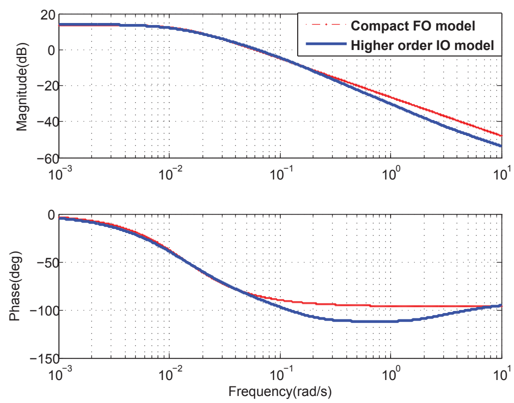

Fig. 7. Bode plot of original IO model and compact FO model.

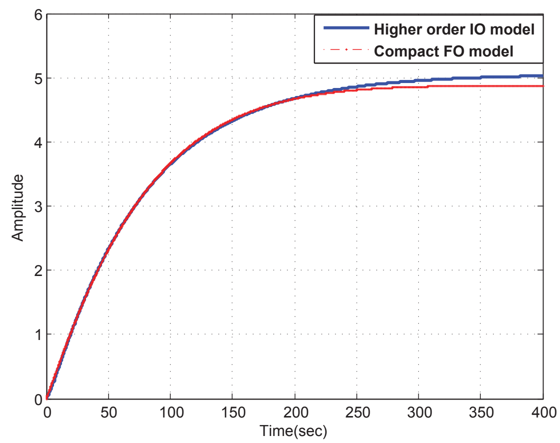

Fig. 8. Step response of original IO model and compact FO model.

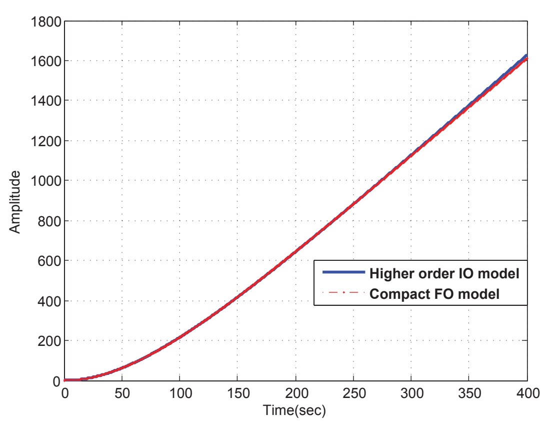

Fig. 9. Ramp response of original IO model and compact FO model.

VI. Conclusion

In this paper, a simple analytical and optimization based parameter reduction method is implemented successfully and the compact fractional-order models are obtained for high parameter integer-order gas turbine system. Resulting fractional-order model are having only three parameters compared with considered large parameter integer-order systems. Integer-order model output data for the step and impulse is used for parameter reduction. The optimization problem is formulated. Two different FO model are obtained for the same integer-order model and it is also observed that, these models are sensitive to initial conditions of parameters and the time span. The method presented is more general and attractive for the analysis and design of large dynamical systems. Resulting simple and compact models completely represents the dynamic of large system and can be used to design efficient controller.

TABLE I
FO Model Parameters

FO model	a	b	α	Error
Step data Model	0.0108	0.0541	1.0710	1.368
Impulse data Model	0.01	0.0478	1.0737	0.0091

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