

When Does A Supply Chain Member Benefit from Vendor-Managed Inventory?

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Enabled by the advances in information technology (IT), many supply chain partners have adopted the practice of vendor-managed inventory (VMI) to improve operations efficiency. While the IT investment for VMI implementation can be significant, the benefits of VMI to different supply chain members are not obvious based on anecdotal evidences and empirical studies. This paper studies the effects of VMI on a supply chain consisting of one manufacturer and one retailer to shed light on when an IT investment for VMI adoption can be justified. We show that whether the two supply chain members benefit from VMI depends on how the holding or shortage cost increment from the manufacturer to retailer compares with two corresponding critical values. We then develop comparative statics results on how these critical values change with respect to different parameters. Interestingly, the retailer is more likely to benefit from the adoption of VMI when its inventory holding cost is low, and the manufacturer is more likely to benefit from VMI adoption when its inventory holding cost is high, contradicting what our intuitions would suggest and what has been prescribed in the literature.

Key words: vendor-managed inventory; supply chain; log-concave; inventory holding cost

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1. Introduction

With the advances in information technology (IT), many supply chain partners have adopted vendor-managed inventory (VMI) to improve operations efficiency. Under VMI, a downstream firm shares demand information with its upstream vendor; in return, the vendor takes control of the downstream firm's inventory decisions rather than simply filling orders (Simchi-Levi et al. 2007). It was pioneered by the collaboration between Walmart and Procter & Gamble, and has since been adopted by many business partners to improve their supply chain performance (K. S. Associates 1993, Waller et al. 1999, Claassen et al. 2008). Data from Open Standards Benchmarking study in logistics by American Productivity and Quality Center (APQC) indicates that 52% of participating organizations have implemented VMI programs (Partida 2013).

Although VMI is expected to improve the performance of all supply chain members, industry reports have yielded somewhat mixed results. In the past, some vendors have resisted VMI implementation

(Hammond 1994). Spartan Stores, a Michigan grocery wholesaler, ended its VMI programs claiming increased store costs and planning inefficiencies resulting from VMI (Mathews 1995). Some downstream firms have also discontinued VMI programs due to lack of collaboration from their vendors (Sheffi 2002). VMI has also been criticized as harming the long run financial performance of vendors, and resulting in the loss of inventory management skills at downstream firms (Pohlen and Goldsby 2003). Many consumer packaged goods manufacturers have started to rethink the value of VMI (Cooke 2001). According to a recent study by APQC, VMI does not automatically lead to superior performance for vendors (Partida 2013).

On the other hand, many vendors in different industries have accrued the benefits brought by VMI. Kimberly-Clark, an American personal care corporation, was reported to have saved \$200 million over two years by adopting VMI with 44 of its customers (Nelson and Zimmerman 2000). According to the estimation of Datalliance, a VMI service provider, VMI can increase the sales for vendors by 5–25% while

reducing operating cost by 4–6% (Datalliance 2016). Some vendors are now offering VMI as a service to their customers. For example, Herndon Products, an aerospace and defense company, manages its customer's requirements based on demand, while maintaining preset inventory levels by part number and location (Herndon 2015). F. K. Machinery, a maintenance and repair supplies distributor not only manages inventory for its clients, but also tailors its VMI program to individual departments of its clients (FKMachinery 2016). Myriad Industries, a distributor of laboratory supplies, provides VMI services by managing customer inventory periodically (Myriad 2016).

In an empirical study, Clark and Hammond (1997) compare the performance of retailers who shared demand information and delegated the inventory control responsibilities to an upstream manufacturer (the Campbell Soup Company) with those that shared information but did not transfer the inventory decision rights. They find that the first group performed substantially better than the second group. That is, VMI is associated with significant improvements in retailer performance measures. Similarly, Lee et al. (1999) investigate the performance of 31 grocery retail chains who share information with Campbell for a VMI initiative, and provide empirical evidence that information sharing achieves dramatic performance improvements if it is used together with VMI. In contrast, Cachon and Fisher (1997) also study Campbell Soup's implementation of VMI but conclude that operating benefits could have been achieved even if the retailers had maintained control over inventory. Kulp et al. (2004) examine the effect of VMI on the profitability of manufacturers. Through an extensive survey in the food and consumer packaged goods industry, the authors find that VMI is directly and positively related to manufacturer margins. Corsten and Kumar (2005) empirically investigate whether the extent to which suppliers of a major retailer adopt VMI has a beneficial impact on their outcomes. The results demonstrate that whereas VMI adoption has a positive impact on supplier economic performance, it generates greater perceptions of negative inequity on the part of the supplier. In a recent study, Dong et al. (2014), using item-level dataset, show that VMI adds significant benefits to the downstream firm in terms of inventory and stockout reductions even without information sharing. These somewhat conflicting and inconclusive findings regarding the impacts of VMI on different supply chain members motivate us to analytically study when a supply chain member benefits from the adoption of VMI.

While it has been recognized that some failures of VMI implementation can be ascribed to imbalances in the net benefits realized by different supply chain

members (Ellegaard and Freytag 2010), analytical results regarding the impacts of VMI on the performance of individual supply chain members are rare. Even for the bare-bone model consisting of one manufacturer and one retailer, the literature offers little insight regarding when a firm would likely benefit from the implementation of VMI. For a model of a manufacturer selling to a newsvendor, Lee and Chu (2005) do present a sufficient condition for the retailer and a necessary condition for the manufacturer to benefit from VMI, but they do not examine how problem parameters (such as inventory holding cost) affect the value of VMI. Savaseneril and Erkip (2010) assess the motivation for a manufacturer to join VMI, and show that VMI provides increased flexibility in manufacturer's operations and may bring additional benefits. Mishra and Raghunathan (2004) and Kim (2008) have explored when a retailer might benefit from VMI in a setting with one retailer and two competing manufacturers. These two papers have the same model setup with only one difference: Mishra and Raghunathan (2004) consider a full backlog case, whereas Kim (2008) studies a lost sales (i.e., no backlog) case. Due to the difference in their assumptions, they reached different conclusions regarding the effect of VMI on retailer's performance. Specifically, Mishra and Raghunathan (2004) demonstrate that a retailer always benefits from VMI because VMI intensifies competition among manufacturers of competing brands and leads to higher inventory availability than retailer managed inventory (RMI). In contrast, Kim (2008) shows that VMI does not necessarily intensify brand competition and increase product availability so a retailer may be worse off by adopting VMI, depending on system parameters such as the inventory holding costs. Regarding the effect of inventory holding cost on the value of VMI to the retailer, Kim (2008, p. 625) asserts that "retailers may lose by adopting VMI when ... the holding costs are low." Such a claim, though intuitively appealing, is not necessarily true. In fact, a retailer is more likely to *benefit* from VMI when the inventory holding cost is *low*, as will be shown later in the paper. More broadly, the lack of analytical and accurate insights regarding the impacts of VMI on the performance of individual supply chain members inhibits fruitful discussions of this popular business practice with MBA students in classrooms and business executives in industry.

To examine the impacts of VMI on the performance of supply chain members, we first note that a key driver for VMI adoptions by retailers is savings on inventory cost. According to a recent study by Inventory Operations Consulting LLC, a consulting firm specializing in inventory management operations, the reality that downstream firms tend to adopt VMI is that VMI relieves them of the burden and

responsibility of managing the inventory (Piasecki 2016). Moreover, VMI is believed to be one of the most widely used techniques to reduce stockouts and increase fill rates (Gruen et al. 2002). As such, we use the classic infinite horizon periodic review inventory model to study how VMI affects the performance of supply chain members, because this inventory model captures the fundamental trade-off between holding inventory and incurring lost sales. Specifically, we consider a supply chain that consists of one manufacturer and one retailer with exogenous wholesale and retail prices, as the base model. The manufacturer makes to order, and the inventory replenishment at the retailer follows a base-stock policy to meet stochastic demand. The retailer's stocking levels are determined by the manufacturer under VMI and by the retailer under RMI. Furthermore, the inventory is owned by the party that makes the stocking level decision. We show that for such a setting, the bottom-line impact of VMI on both supply chain members depends on how the holding cost increment from the manufacturer to retailer compares with two critical values. Furthermore, the two critical values partition the holding cost increment continuum into three regions: small, medium, and big. When the holding cost increment is small, both firms lose from VMI; when it is medium, the retailer benefits while the manufacturer loses from VMI; when it is big, both firms benefit from VMI. Similarly, how the shortage cost increment from the manufacturer to retailer compares with two other critical values also determines which supply chain member is better off with VMI. When the shortage cost increment is small, both firms benefit from VMI; when it is medium, the retailer benefits while the manufacturer loses from VMI; when it is big, both firms lose from VMI.

Digging deeper, we then derive comparative statics results that shed light on how different problem parameters affect the value of VMI. In particular, our analysis reveals an unexpected role that inventory holding cost plays on the impacts of VMI. Under rather general assumptions on the demand distribution, we show that the retailer is more likely to benefit from VMI when its inventory holding cost is low, and the manufacturer is more likely to benefit from VMI when its inventory holding cost is high. Note that in our model (as in Mishra and Raghunathan 2004, Bernstein et al. 2006, Kim 2008 among many others), the firm that controls inventory also owns the inventory. Therefore, the direct effect of VMI on the retailer is relieving its burden of owning inventory. Naturally, one would expect that the retailer would be more likely to benefit from VMI when its inventory holding cost is high. However, such a line of argument fails to take into account the indirect effect of VMI that results from the changes in incentives for

choosing stocking levels. In our setting, for the retailer, the negative indirect effect of VMI due to the change in stocking levels may be more significant than the positive direct effect of VMI due to transferring inventory holding cost when its inventory holding cost is high. As a result, the retailer is less likely to benefit from VMI when its inventory holding cost is high. On the other hand, we find that, as expected, the retailer is more likely to benefit from VMI when its shortage penalty cost is high, whereas the manufacturer is more likely to benefit from VMI when its shortage penalty cost is low.

In order to explore the applicability and scope of our key insights, we then extend our analysis to a one-manufacturer-one-retailer supply chain with endogenous wholesale and retail prices. In such a supply chain, the manufacturer sets the wholesale price as a Stackelberg game leader, and the retailer decides on the retail price as a response. Once the wholesale and retail prices are determined, the manufacturer (retailer) chooses the stocking level under VMI (RMI). For the endogenous pricing case, we analytically prove that the insights developed for the base model (i.e., exogenous pricing case) continue to hold under the assumption that unsatisfied demand is fully backlogged. We further numerically show that our key insights remain qualitatively similar even when unsatisfied demand is partially backlogged.

In addition to developing new insights regarding when a supply chain member is likely to benefit from VMI, this work also makes some theoretical contributions to the inventory management literature. In particular, we extend the analysis of Lee and Chu (2005) for a single-period setting to an infinite-horizon setting. Furthermore, we explore the impacts of different problem parameters on when a supply chain member is likely to benefit from VMI. Toward this end, we need to assess how a change in a problem parameter affects the difference between the values of two endogenous profit functions; as a result, the standard comparative statics analysis approach cannot be applied directly. To overcome this technical difficulty, we identify a rather general class of demand distributions for which we can characterize the shapes of the two profit functions.

2. Related Literature

Overall, this paper contributes to the general realm of analytical study on the effects of two VMI components: demand information sharing and inventory control responsibility transferring. The study on the information-sharing component in supply chains with stochastic demands starts with Gavirneni et al. (1999), who analyze a two-echelon supply chain in which the downstream firm follows an (s, S) policy,

and find that information sharing benefits the upstream manufacturer most when demand variability is intermediate. Lee et al. (2000) assess the value of information sharing in a two-echelon supply chain when demands across different periods are correlated and show that the value of demand information sharing can be significant. Gavirneni (2002) studies a supply chain consisting of a capacitated manufacturer and a retailer facing demands in different periods that are independent, and shows that information sharing should be coupled with making adjustment on inventory ordering policies. Kurata and Yue (2008) study the value of information sharing in the form of scan-back trade in a supply chain with trade promotions, and demonstrate that supply chain members benefit from information sharing when it is combined with VMI in the form of buy-back trade. Based on observations from controlled experiments, Özer et al. (2011) argue that trust and trustworthiness are two important reasons for supply chain members to share information to improve supply chain efficiency. In a recent paper, Özer et al. (2014) investigate how supply chain members' countries of origin affect trust, trustworthiness, and strategic information sharing behavior in a cross-country supply chain.

The extant literature on the inventory-control-responsibility-transferring aspect of VMI focuses on its role in improving the overall supply chain efficiency. Cachon (2001) studies the inventory decisions in a two-echelon supply chain consisting of one manufacturer and multiple retailers. He shows that VMI can mitigate competitions between different supply chain members by centralizing stocking decisions in the hands of the manufacturer. Fry et al. (2001) consider the (z, Z)-type of VMI contract in a supply chain with one manufacturer and one retailer. They demonstrate that the optimal replenishment and production policies for the manufacturer are order-up-to policies, and use numerical analysis to compare the performance of a single supplier and a single retailer operating under VMI with the performance of those operating under RMI. Their study shows that VMI performs significantly better than RMI in many settings, but can perform worse in others. Kulp (2002) investigates the interrelation between the information environment and VMI. She finds that the overall supply chain's ability to benefit from VMI depends upon the extent to which the retailer reveals its internal accounting information to the manufacturer and the ability of the manufacturer to transmit and use this information in its decisions. If the information environment is not sufficiently precise or reliable, RMI may dominate VMI in terms of improving supply chain efficiency. Bernstein and Federgruen (2003) analyze the coordination of a two-echelon distribution system in which a supplier distributes a product to

multiple competing retailers, and construct a mechanism that can coordinate such a supply chain under VMI. Kraiselburd et al. (2004) analyze the role of VMI in improving the overall efficiency of a supply chain consisting of a manufacturer and a retailer. They show that VMI performs better (in terms of improving supply chain efficiency) when manufacturer effort is a substantial driver of consumer demand and when consumers are unlikely to substitute for another brand in case of a stockout. On the other hand, when substitution is significant, VMI can exacerbate, rather than mitigate, channel inefficiencies, and can perform worse than RMI. Bernstein et al. (2006) study a two-echelon supply chain with a single supplier servicing a network of retailers who compete with each other by selecting sales quantities. They show that the supply chain cannot be coordinated with a simple wholesale price scheme under RMI, but may be coordinated under VMI.

Our work is also related to the literature on the interplay between firms' operations decisions and information sharing. In the context of VMI, Raghunathan and Yeh (2001) analyze the impact of information sharing and continuous replenishment to study the factors that affect the value of VMI, and quantify the optimal number of retailers a manufacturer should partner with. Yao and Dresner (2008) extend the model of Raghunathan and Yeh (2001) and show that information systems, continuous replenishment planning, and VMI bring varying benefits in terms of inventory cost savings to firms, and that the benefits are not consistently distributed between retailers and manufacturers. Their findings also point to the managerial implications on how managers may decide the product sets for improved benefit realization under VMI. In the context of investing in information security, Liu et al. (2001) show that the nature (complementary or substitutable) of information assets possessed by different firms plays a crucial role in influencing their decisions on knowledge sharing and IT investment.

Different from the above streams of literature, our paper attempts to address a narrow but fundamental question for understanding the challenge faced by a supply chain member before investing new IT for VMI implementation: "What is in there for me?" Our results have significant implications on how supply chain members should implement VMI in order to leverage the value of IT by identifying the conditions under which both supply chain members benefit from the adoption of VMI. Based on our findings, it should be easy for supply chain partners to find common grounds and start implementing VMI with products for which the downstream firm's inventory holding cost is sufficiently large compared with the upstream firm's, or the downstream firm's shortage penalty cost

is small relative to the upstream firm's. Among these candidate products, the downstream firm should pursue implementing VMI for products with low inventory holding costs or high shortage penalty costs while the upstream firm should do so for products with high inventory holding costs or low shortage penalty costs.

Our work is tangentially related to the literature on value of IT investment, because a VMI implementation involves a huge investment in IT. It is often claimed that the actual benefits of IT are disappointing despite the huge investment in IT, and that IT expenditure has not been able to generate significant productivity gains (Stratopoulos and Dehning 2000). One possible explanation is that IT investments often involve major changes to business processes; a very large percentage of information systems fail to deliver benefits or to solve intended problems because the process of organizational changes is not properly addressed (Hong and Kim 2002). As a result, it is important for organizations to reengineer their business process (such as implementing VMI) in order to leverage the value of IT (Brynjolfsson and Hitt 1998). Our research aims to identify situations in which supply chain partners can better take advantage of the value of IT by studying when a supply chain member benefits from the adoption of VMI.

3. Base Model

Consider a supply chain that consists of one manufacturer and one retailer, with the retailer facing stochastic demand for one product. The manufacturer makes the product at a per-unit production cost c , and sells it to the retailer at an exogenous wholesale price w ; the retailer then sells the product to customers at an exogenous retail price p . Time is divided into periods of equal lengths. Customer demand in each period is random; the demands in different periods are assumed to be independent and identically distributed, characterized by distribution function F and density function f , with a finite mean μ and support $[0, B]$. Let $\bar{F}(x) = 1 - F(x)$.

The manufacturer makes to order, and inventory replenishment at the retailer's location follows a periodic-review base-stock policy to satisfy the stochastic demand. The inventory stocking decision (the stocking decision, hereafter) is made by either the manufacturer or the retailer, depending on the contract between supply chain members. Specifically, under the contract of retailer-managed inventory (RMI) the stocking decision is made by the retailer, while under the contract of vendor-managed inventory (VMI) it is made by the manufacturer. To focus on the main issues of our study and maintain consistency with the existing literature, we assume that the stocking

decision and inventory ownership are coupled (Mishra and Raghunathan 2004, Bernstein et al. 2006, Kim 2008), and all parameters and demand distributions are stationary. The wholesale and retail prices are both given exogenously; the only decision is to determine the base-stock level (stocking level, hereafter).

The sequence of events is as follows: First, the stocking level is determined. Under RMI, the retailer chooses the stocking level; under VMI, the manufacturer chooses the stocking level. The decision maker—the manufacturer under VMI and the retailer under RMI—chooses the stocking level to maximize its long-run average profit per period (average profit, for short hereafter). After the stocking level is determined, inventory replenishment follows a periodic review policy. At the beginning of a period, an order is placed to raise the inventory position at the retailer back to the predetermined stocking level; this order is instantaneously fulfilled by the manufacturer. Next, demand materializes, and is satisfied with on-hand inventory. In the case of stockout, a fixed fraction δ of the unsatisfied demand is backordered and to be fulfilled in the next period; the remaining $(1 - \delta)$ of the unsatisfied demand is lost. The backorder fraction δ reflects the customer loyalty toward either the retailer or the product (brand) in our model. The leftover inventory, if there is any, is carried over to the next period.

The per-unit holding cost for leftover inventory (inventory holding cost, hereafter) is h for the manufacturer and $\gamma_h h$ for the retailer. By assumption, the inventory holding costs for the manufacturer and retailer may be different. h represents that part of inventory holding cost that is common to both the manufacturer and retailer. The greater the value of h , the more inventory holding cost incurred by the manufacturer and retailer. In practice, the value of a product usually increases as the product moves to the downstream of the supply chain due to value-added activities of different supply chain members. As a result, the inventory holding cost to the retailer may be higher than to the manufacturer. To model this fact, we assume that $\gamma_h \geq 1$. $\gamma_h - 1$ then reflects the percentage increase in the inventory holding cost when the inventory owner changes from the manufacturer to the retailer. For ease of exposition, we refer γ_h as the holding cost increment (from the manufacturer to retailer).

As in Mishra and Raghunathan (2004) and Kim (2008), a shortage penalty cost is incurred by both the manufacturer and retailer whenever a stockout occurs. In particular, let the per-unit shortage penalty cost (shortage cost, hereafter) be s for the manufacturer and $\gamma_s s$ for the retailer. The higher the value of s , the more shortage cost that both the manufacturer and retailer have to afford for each occurrence of

a stockout. When a product is out of stock, an unsatisfied customer may choose to purchase an alternative brand, and the customer may continue to purchase that alternative brand if she or he likes it. Consequently, while the manufacturer does not sell the product directly to customers, it may still incur shortage costs for unfulfilled demand in addition to the lost revenue due to shortage. The shortage cost for the manufacturer may even be higher than that for the retailer if the customer switches brand but does not switch retailer. Hereafter, we refer γ_s , the ratio of the retailer's shortage cost to the manufacturer's, as the shortage cost increment (from the manufacturer to retailer).

Throughout this paper, we make the following assumption regarding the economic parameters to avoid trivial and uninteresting cases.

ASSUMPTION A1. $h \geq 0; s \geq 0; c \leq w \leq p$.

To develop analytical insights, we also make the following assumption regarding the density function $f(x)$ and the distribution function $F(x)$ of the demand random variable.

ASSUMPTION A2. (i) $f(x)$ is log-concave, that is, $\log[f(x)]$ is a concave function. (ii) $\bar{F}(x)F^2(x)/f(x)$ is unimodal.

Many probability distributions such as uniform and normal have log-concave density functions; see, for example, Bagnoli and Bergstrom (2005) for more distributions with log-concave density functions. Assumption A2.ii also holds for probability distributions that are commonly used in the operations management literature, such as uniform, normal, logistics, and Gamma distributions. Therefore, distributions such as uniform, normal, logistics, and Gamma with shape parameter greater than one satisfy both conditions of Assumption A2.

The effect of VMI on the performance of a supply chain member (or the overall supply chain) is assessed by comparing the average profit of the said supply chain member under VMI and RMI. Essentially, following the common practice in the literature (Mishra and Raghunathan 2004, Kim 2008), we treat RMI as the baseline case, and ignore the administration costs for both VMI and RMI. Table 1 summarizes the notation for ease of reference. In addition, we use superscripts R and V to index RMI or VMI, respectively.

4. Preliminary Analysis

Under RMI, the retailer chooses the stocking level to maximize its average profit. To develop the expression for the retailer's average profit for a given

Table 1 Summary of Notation

Notation	Definition
c, w, p	Unit production cost, wholesale price, and retail price
$h, s, \gamma_h h, \gamma_s s$	Inventory holding and shortage costs for the manufacturer and retailer
f, F	Density function and cumulative distribution function of period demand
δ	Fraction of unsatisfied customers who will choose to backorder
q	Stocking level
π_m, π_r	Manufacturer's expected profit, and retailer's expected profit

stocking level, consider a generic period t in steady state. Denote D_t as the demand in period t . At the beginning of the period, the inventory position is $(q - D_{t-1})^+ - \delta(D_{t-1} - q)^+$, where $(x)^+ = \max\{x, 0\}$. Then, an order of $q - [(q - D_{t-1})^+ - \delta(D_{t-1} - q)^+]$ is placed to raise the inventory position back to the stocking level q , and this order is delivered by the manufacturer instantaneously. Afterwards, the backordered demand from the previous period $\delta(D_{t-1} - q)^+$ is first satisfied using the on-hand quantity. Note that the backordered demand from one period is always satisfied in the next period. After the backordered demand is satisfied, the on-hand inventory level becomes q . Then, the on-hand inventory is used to satisfy the demand D_t in period t . So, at the end of period t , the leftover inventory is $(q - D_t)^+$, and the shortage is $(D_t - q)^+$. The sales in period t comes from two sources: the backordered demand $\delta(D_{t-1} - q)^+$ from the previous period, and the fulfilled demand $\min\{D_t, q\}$ of period t . Consequently, the sales in period t is $\min\{D_t, q\} + \delta(D_{t-1} - q)^+$. It follows that the retailer profit in period t is

$$-w(q - [(q - D_{t-1})^+ - \delta(D_{t-1} - q)^+]) + p(\delta(D_{t-1} - q)^+ + \min\{D_t, q\}) - \gamma_h h(q - D_t)^+ - \gamma_s s(D_t - q)^+,$$

where the first term is the purchasing cost, the second term is the sales revenue, and the last two terms are inventory holding and shortage costs, respectively. Collecting terms, we obtain the retailer profit in period t as

$$(p - w)\delta(D_{t-1} - q)^+ + p \min\{D_t, q\} - w \min\{D_{t-1}, q\} - \gamma_h h(q - D_t)^+ - \gamma_s s(D_t - q)^+.$$

Because the inventory system regenerates itself in one period, the retailer's average profit is (see, e.g., Ross 1966)

$$E[(p - w)\delta(D_{t-1} - q)^+ + p \min\{D_t, q\} - w \min\{D_{t-1}, q\} - \gamma_h h(q - D_t)^+ - \gamma_s s(D_t - q)^+],$$

where $E[\cdot]$ is the expectation operator. Using the fact that demands in different periods are identically distributed, we drop the subscript “ t ” and rewrite the retailer’s average profit for a given stocking level q as $\pi_r^R(q) = E\{(p-w)[\min\{D, q\} + \delta(D-q)^+] - \gamma_h h(q-D)^+ - \gamma_s s(D-q)^+\}$. Note that $\min\{D, q\} = D - (D-q)^+$ and $(q-D)^+ = q - D + (D-q)^+$. So,

$$\pi_r^R(q) = (p-w)\mu + [(1-\delta)(p-w) + \gamma_s s] [q - E(q-D)^+ - \mu] - \gamma_h h E(q-D)^+. \quad (1)$$

Similarly, the average profit (hereafter, profit for short) of the manufacturer under RMI is

$$\pi_m^R(q) = (w-c)\mu + [(1-\delta)(w-c) + s] [q - E(q-D)^+ - \mu]. \quad (2)$$

It is straightforward to show that $\pi_r^R(q)$ is a strictly concave function, and the retailer’s optimal stocking level q^R under RMI, the RMI stocking level for short hereafter, satisfies the following newsvendor-like formula:

$$F(q^R) = \frac{(1-\delta)(p-w) + \gamma_s s}{(1-\delta)(p-w) + \gamma_h h + \gamma_s s}. \quad (3)$$

For ease of exposition, define the *retailer critical ratio* as

$$\beta_r \equiv \frac{(1-\delta)(p-w) + \gamma_s s}{(1-\delta)(p-w) + \gamma_h h + \gamma_s s}. \quad (4)$$

Under VMI, the manufacturer bears the inventory holding cost and determines the stocking level to maximize its profit. Following the approach to obtain equation (1), we can derive the manufacturer profit for a given stocking level q under VMI:

$$\pi_m^V(q) = (w-c)\mu + [(1-\delta)(w-c) + s] [q - E(q-D)^+ - \mu] - h E(q-D)^+. \quad (5)$$

It can be shown that $\pi_m^V(q)$ is strictly concave in q . So, the manufacturer’s optimal stocking level q^V under VMI, the VMI stocking level for short hereafter, is uniquely determined by the first-order condition:

$$F(q^V) = \frac{(1-\delta)(w-c) + s}{(1-\delta)(w-c) + h + s}. \quad (6)$$

Again, for ease of exposition, define the *manufacturer critical ratio* as

$$\beta_m \equiv \frac{(1-\delta)(w-c) + s}{(1-\delta)(w-c) + h + s}. \quad (7)$$

Similar to equation (2), the retailer profit under VMI is

$$\pi_r^V(q) = (p-w)\mu + [(1-\delta)(p-w) + \gamma_s s] [q - E(q-D)^+ - \mu]. \quad (8)$$

For ease of exposition, define

$$w_o = \frac{(1-\delta)(p + \gamma_h c) - \gamma_h s + \gamma_s s}{(1-\delta)(1 + \gamma_h)}. \quad (9)$$

Lemma 1 compares the optimal stocking levels under the two inventory management regimes, and follows immediately from equations (3) and (6).

LEMMA 1. $q^V > q^R$ if and only if $w > w_o$, where w_o is defined in equation (9).

5. Impact of VMI

We first examine the impact of VMI on the retailer’s performance by comparing its profits under RMI and VMI.

5.1. Impact on Retailer

By equations (1) and (8), the retailer profits under RMI and VMI are, respectively,

$$\begin{aligned} \pi_r^R &= (p-w)\mu + [(1-\delta)(p-w) + \gamma_s s] [q^R - E(q^R-D)^+ - \mu] - \gamma_h h E(q^R-D)^+, \\ \pi_r^V &= (p-w)\mu + [(1-\delta)(p-w) + \gamma_s s] [q^V - E(q^V-D)^+ - \mu]. \end{aligned}$$

Define the value of VMI to the retailer as

$$\Delta_r \equiv \pi_r^V - \pi_r^R.$$

Let γ_h^r and γ_s^r be the unique roots of the following equation, respectively:¹

$$\begin{aligned} [(1-\delta)(p-w) + \gamma_s s] [q^V - E(q^V-D)^+ - q^R + E(q^R-D)^+] \\ + \gamma_h h E(q^R-D)^+ = 0. \end{aligned} \quad (10)$$

PROPOSITION 1. (i) The retailer benefits from VMI if γ_h is greater than γ_h^r as defined in equation (10), and loses from VMI otherwise. That is, $\pi_r^V \geq \pi_r^R \iff \gamma_h \geq \gamma_h^r$; (ii) The retailer benefits from VMI if γ_s is less than γ_s^r as defined in equation (10), and loses from VMI otherwise. That is, $\pi_r^V \geq \pi_r^R \iff \gamma_s \leq \gamma_s^r$.

By proposition 1.i, the impact of VMI on the retailer profit is characterized by the critical value γ_h^r , defined

in equation (10), for the holding cost increment. VMI benefits the retailer when the holding cost increment from the manufacturer to retailer is greater than the critical value γ_h^r , while it hurts the retailer when it is less than the critical value γ_h^r . To understand the intuition, note that VMI has a direct effect and an indirect effect on the retailer profit and that the total effect determines whether VMI benefits the retailer. On one hand, VMI has a positive direct effect on the retailer profit because VMI relieves the retailer from the burden of owning inventory. On the other hand, VMI has an indirect effect on the retailer profit in that it changes the incentives for setting the stocking level as the manufacturer under VMI owns the inventory and makes the stocking decision instead of the retailer under RMI. The sign and size of the indirect effect of VMI on the retailer profit depends on the relative change in the stocking levels from RMI to VMI, which is ultimately driven by the holding cost increment as well as the shortage cost increment. Specifically, when holding cost increment from the manufacturer to retailer is very small, the stocking level q^R under RMI is significantly higher than the stocking level q^V under VMI, which implies that VMI leads to lower sales and higher shortage cost. Consequently, the indirect effect of VMI on the retailer profit is negative, and dominates the positive direct effect of VMI. As a result, the retailer loses from VMI. As the holding cost increment rises, the stocking level under RMI decreases and the stocking level under VMI remains the same; consequently, the negative indirect effect of VMI becomes less severe, and will be offset by its positive direct effect when the holding cost increment reaches the critical value γ_h^r . As the holding cost increment becomes even bigger, the total effect of VMI turns positive, benefiting the retailer. A similar argument will lead to Proposition 1.ii. Next, we examine the impacts of different problem parameters by characterizing how they affect the critical values γ_h^r and γ_s^r .

PROPOSITION 2. (i) γ_h^r increases in h but decreases in s ; (ii) γ_s^r decreases in h but increases in s .

We know that γ_h^r is the critical value above which the retailer benefits from VMI and γ_s^r is the critical value below which the retailer benefits from VMI. Consequently, Proposition 2 implies that the retailer is more likely to benefit from VMI when its shortage cost is high. To understand the intuition, recall that in order for the retailer to benefit from VMI, the positive direct effect of VMI must at least cancel out its potential negative indirect effect. From equations (3) and (6), it is not straightforward to see how changes in the shortage cost affect the indirect effect of VMI on the retailer profit as the shortage cost affects both the manufacturer and

retailer critical ratios in the same direction. Note that the difference between the two critical ratios is

$$\beta_m - \beta_r = h(1 - \delta)(1 + \gamma_h) \times \frac{w - w_o}{[(1 - \delta)(w - c) + h + s][(1 - \delta)(p - w) + \gamma_h h + \gamma_s s]}. \quad (11)$$

Recall that at γ_h^r or γ_s^r , the indirect effect of VMI must be negative in order to cancel out its positive direct effect on the retailer profit, which means that $q^R > q^V$. Therefore, $\beta_m < \beta_r$ when $\gamma = \gamma_h^r$ or $\gamma = \gamma_s^r$. Then, it immediately follows from Lemma 1 that the wholesale price w that satisfies equation (10) must be lower than w_o , i.e., $w < w_o$. Thus, the difference between the two critical ratios increases as the shortage cost s increases. This implies that the stocking level under VMI increases at a greater rate than that under RMI, as s increases. Consequently, the negative indirect effect of VMI on the retailer profit becomes less severe as s increases: The retailer is more likely to benefit from VMI when its shortage cost is high. What is surprising, though, is the impact of inventory holding cost on whether the retailer benefits from VMI. Recall that in our model an increase in h implies more inventory holding cost incurred by the manufacturer and the retailer. Because the retailer does not bear the burden of owning inventory under VMI, one would expect that the retailer would benefit from VMI when its inventory holding cost is high. However, Proposition 2 suggests that the value of VMI to the retailer is not necessarily monotone increasing in its inventory holding cost. As the inventory holding cost increases, the value of VMI to the retailer may decrease, and finally becomes negative. This implies that γ_h^r increases but γ_s^r decreases in the inventory holding cost. Consequently, *VMI is more likely to hurt the retailer when its inventory holding cost is high*. To understand the intuition, note that $\beta_m - \beta_r$ decreases in h for $w < w_o$. Consequently, as h increases, the stocking level under VMI decreases at a greater rate than that under RMI. This implies that the potential negative indirect effect of VMI becomes more severe as the inventory holding cost increases, and dominates the positive direct effect on the retailer profit. As a result, the retailer is more likely to benefit from VMI when its inventory holding cost is low.

5.2. Impact on Manufacturer

In this subsection, we examine the impact of VMI on the manufacturer's performance. Following the line of argument to derive equations (2) and (5), we obtain the manufacturer profits under RMI and VMI:

$$\pi_m^R = (w - c)\mu + [(1 - \delta)(w - c) + s][q^R - E(q^R - D)^+ - \mu], \quad (12)$$

$$\pi_m^V = (w - c)\mu + [(1 - \delta)(w - c) + s][q^V - E(q^V - D)^+ - \mu] - hE(q^V - D)^+. \quad (13)$$

It follows from equations (12) and (13) that the value of VMI to the manufacturer is $\Delta_m \equiv \pi_m^V - \pi_m^R = [(1 - \delta)(w - c) + s][q^V - q^R - E(q^V - D)^+ + E(q^R - D)^+] - hE(q^V - D)^+$. Define $\gamma_h^m \in (\gamma_h^r, +\infty)$ and $\gamma_s^m \in (0, \gamma_s^r)$ as the unique roots of the following equation, respectively:²

$$[(1 - \delta)(w - c) + s]\{q^V - E(q^V - D)^+ - q^R + E(q^R - D)^+\} - hE(q^V - D)^+ = 0. \quad (14)$$

PROPOSITION 3. (i) The manufacturer benefits from VMI if γ_h is greater than γ_h^m as defined by equation (14), and loses from VMI otherwise. That is, $\pi_m^V \geq \pi_m^R \iff \gamma_h \geq \gamma_h^m$; (ii) The manufacturer benefits from VMI if γ_s is less than γ_s^m as defined by equation (14), and loses from VMI otherwise. That is, $\pi_m^V \geq \pi_m^R \iff \gamma_s \leq \gamma_s^m$.

As on the retailer profit, VMI has a direct effect and an indirect effect on the manufacturer profit as well. And, the total effect of VMI on the manufacturer profit determines whether the manufacturer benefits from VMI. Clearly, the direct effect of VMI on the manufacturer profit is negative, because it adds the burden of owning inventory to the manufacturer. On the other hand, the indirect effect of VMI on the manufacturer profit can be positive, depending on how the RMI and VMI stocking levels compare. As γ_h increases, q^R decreases and q^V remains unchanged; consequently, the indirect effect of VMI on the manufacturer profit becomes more positive. When γ_h gets sufficiently big, the positive indirect effect will dominate the negative direct effect, and the manufacturer will thus benefit from VMI. A similar argument will lead to Proposition 3.ii. Proposition 3 may explain the recent industry trend that some manufacturers are offering VMI as a service to their downstream customers (Herndon 2015, FKMachinary 2016, Myriad 2016). It is easy to establish that $\gamma_h^m > \gamma_h^r$. To see the intuition, note that VMI relieves the retailer from the burden of owning inventory and transfers such a burden to the manufacturer. Therefore, VMI brings a positive direct effect on the retailer profit but a negative direct effect on the manufacturer profit. Meanwhile, VMI yields the same indirect effect on the retailer profit as well as on the manufacturer profit. As previously explained, the indirect effect of VMI gets more positive as γ_h increases. Consequently, in order for VMI to deliver the positive indirect effect on the

manufacturer profit to cancel out its negative direct effect, γ_h^m must be sufficiently big. A similar argument will lead to the relationship: $\gamma_s^m < \gamma_s^r$.

It follows that γ_h^r and γ_h^m partition the feasible continuum for the holding cost increment into three regions: the small region in which $\gamma_h \leq \gamma_h^r$, the medium region in which $\gamma_h^r < \gamma_h \leq \gamma_h^m$, and the big region in which $\gamma_h > \gamma_h^m$. Combining Propositions 1 and 3, we see that the holding cost increment from the manufacturer to retailer is key in determining how VMI affects the profits of both the manufacturer and retailer: Both firms are better off with VMI when the holding cost increment is big (bigger than γ_h^m), the retailer is better off while the manufacturer is worse off with VMI when the holding cost increment is medium (between γ_h^r and γ_h^m), and both firms are worse off when the holding cost increment is small (smaller than γ_h^r). On the other hand, the shortage cost increment from the manufacturer to retailer is also important in determining how VMI affects the profits of both the manufacturer and retailer: Both firms are better off with VMI when the shortage cost increment is small (smaller than γ_s^m), the retailer is better off while the manufacturer is worse off with VMI when the shortage cost increment is medium (between γ_s^m and γ_s^r), and both firms are worse off when the shortage cost increment is big (bigger than γ_s^r).

PROPOSITION 4. (i) γ_h^m decreases in h but increases in s ; (ii) γ_s^m increases in h but decreases in s .

The impacts of shortage cost and inventory holding cost on the value of VMI to the manufacturer are different from those on the value of VMI to the retailer. In particular, the manufacturer is more likely to benefit from VMI for a lower shortage cost. To understand the intuition, recall that at γ_s^r the indirect effect of VMI must be positive in order to cancel out its negative direct effect on the manufacturer profit, which indicates that $q^R < q^V$ when $\gamma = \gamma_h^m$ or $\gamma = \gamma_s^m$. Thus, the wholesale price w that satisfies equation (14) must be higher than w_o according to Lemma 1. Consequently, as s increases, the critical ratio difference defined in equation (11) decreases, which implies that the stocking level under VMI decreases at a greater rate than that under RMI. As a result, the positive indirect effect of VMI becomes less significant and is dominated by its negative direct effect on the manufacturer profit. Therefore, the manufacturer is less likely to benefit from VMI when its shortage cost is high. But when the inventory holding cost increases, the critical ratio difference increases, which implies that the potential positive indirect effect of VMI becomes more significant and outweighs its negative direct effect on the manufacturer profit. Therefore, as for the retailer, the inventory holding cost has an unexpected impact for

the manufacturer: VMI is more likely to benefit the manufacturer when its inventory holding cost is high.

6. Model Extension: Endogenous Pricing Case

In this section, we consider the scenario in which the wholesale and retail prices are determined endogenously. In particular, we model the interaction between the manufacturer and retailer as a Stackelberg game as in Mishra and Raghunathan (2004). Under both RMI and VMI, the manufacturer moves first to set the wholesale price, and the retailer responds with the retail price. Once the wholesale and retail prices are determined, the retailer (manufacturer) chooses the stocking level under RMI (VMI), as in the main body of the paper.

6.1. Full Backlog

In this subsection, we consider the full backlog case, as in Mishra and Raghunathan (2004). That is, we set $\delta = 1$, which means that unsatisfied demand is fully backlogged. Such an assumption allows us to develop analytical insights regarding the impacts of VMI. To model the dependence of demand on the retail price, we assume that demand is random and price dependent. Let $D = \varepsilon - p$ with ε as a random variable. Furthermore, we assume that the distribution function of ε is $F_\varepsilon(\cdot)$ with $E[\varepsilon] = A$, and the failure rate of ε is an increasing function.

With this demand specification, we can rewrite the retailer profit under RMI (defined in equation 1) as follows:

$$\pi_r^R(w, p, q) = (p - w)(A - p) + \gamma_s s[p + q - E(q - \varepsilon + p)^+ - A] - \gamma_h hE(q - \varepsilon + p)^+. \quad (15)$$

(Note that we write the retailer profit as a function of w , p , and q to emphasize its dependence on the wholesale price w , retail price p , and stocking level q .)

By equation (3), the retailer's optimal stocking level q^R under RMI satisfies

$$F_\varepsilon(q^R + p) = \frac{\gamma_s s}{\gamma_h h + \gamma_s s}. \quad (16)$$

Consequently, for a given pair of wholesale price w and retail price p , the retailer's best profit under RMI is

$$\pi_r^R(w, p) = (p - w)(A - p) + \gamma_s s[p + q^R - E(q^R - \varepsilon + p)^+ - A] - \gamma_h hE(q - \varepsilon + p^R)^+.$$

Note that by equation (16), $q^R + p$ is independent of the retail price. Consequently, the retailer profit

function $\pi_r^R(w, p)$ is concave in p . Furthermore, the retailer's best responding retail price $p^R(w)$ is

$$p^R(w) = \frac{A + w}{2}. \quad (17)$$

Taking into consideration the retailer's responding retail price (characterized by equation 17) and responding stocking level (see equation 16), the manufacturer sets the wholesale price to maximize its profit:

$$\pi_m^R(w) = (w - c)(A - p^R(w)) + s[q^R + p^R(w) - E(q^R + p^R(w) - \varepsilon)^+ - A].$$

Applying equations (16) and (17) to the above equation, we obtain the equilibrium wholesale price w^R under RMI:

$$w^R = \frac{A + c}{2}. \quad (18)$$

To characterize the equilibrium outcome under VMI, we first use equation (5) to rewrite the manufacturer profit under VMI as follows:

$$\pi_m^V(w, p, q) = (w - c)(A - p) + s[p + q - E(q - \varepsilon + p)^+ - A] - hE(q - \varepsilon + p)^+.$$

By equation (6), the manufacturer's optimal stocking level q^V under VMI satisfies

$$F_\varepsilon(q^V + p) = \frac{s}{h + s}. \quad (19)$$

Consequently, the retailer profit under VMI for a given pair of wholesale price w and retail price p is

$$\pi_r^V(w, p) = (p - w)(A - p) + \gamma_s s[q^V + p - E(q^V - \varepsilon + p)^+ - A].$$

Note that by equation (19), $q^V + p$ is independent of the retail price. Consequently, the retailer profit function $\pi_r^V(w, p)$ is concave in p . Furthermore, the retailer's best responding retail price $p^V(w)$ is

$$p^V(w) = \frac{A + w}{2}. \quad (20)$$

Taking into consideration the retailer's responding retail price $p^V(w)$ by equation (20) and knowing its own stocking level decision by equation (19), the manufacturer sets the wholesale price to maximize its profit:

$$\pi_m^V(w) = (w - c)(A - p^V(w)) + s[q^V + p^V(w) - E(q^V + p^V(w) - \varepsilon)^+ - A] - hE(q^V + p^V(w) - \varepsilon)^+.$$

Using equations (19) and (20), we can obtain the equilibrium wholesale price w^V under VMI:

$$w^V = \frac{A + c}{2}. \quad (21)$$

Comparing equation (18) with equation (21), we see that the equilibrium wholesale prices are identical under RMI and VMI. It then follows from equations (17) and (20) that the equilibrium retail prices are identical under RMI and VMI as well. Consequently, in the case of full backlog assessing the impacts of VMI with endogenous prices boils down to assessing the impacts of VMI with exogenous prices, studied in the base model. One key insight from the base model is that whether a supply chain member benefits from VMI depends on how the holding cost increment or shortage cost increment compares with two corresponding critical values. Furthermore, the retailer (manufacturer) is more likely to benefit from VMI when its inventory holding cost is low (high) or when its shortage cost is high (low). All these insights continue to hold in the endogenous pricing case with full backlog, as demonstrated by the following Propositions 5–8.

PROPOSITION 5. Let $\bar{\gamma}_h^r \in (0, \gamma_s)$ and $\bar{\gamma}_s^r \in (\gamma_h, +\infty)$ be the unique roots of the following equation, respectively:

$$\gamma_s s[(p^V + q^V) - (p^R + q^R) - E(p^V + q^V - \varepsilon)^+ + E(p^R + q^R - \varepsilon)^+] + \gamma_h h E(p^R + q^R - \varepsilon)^+ = 0. \quad (22)$$

Then, $\pi_r^V \geq \pi_r^R \iff \gamma_h \geq \bar{\gamma}_h^r$, or, $\gamma_s \leq \bar{\gamma}_s^r$.

PROPOSITION 6. (i) $\bar{\gamma}_h^r$ increases in h but decreases in s ; (ii) $\bar{\gamma}_s^r$ decreases in h but increases in s .

PROPOSITION 7. Let $\bar{\gamma}_h^m \in (\gamma_s, +\infty)$ and $\bar{\gamma}_s^m \in (0, \gamma_h)$ be the unique roots of the following equation, respectively:

$$s[(p^V + q^V) - (p^R + q^R) - E(p^V + q^V - \varepsilon)^+ + E(p^R + q^R - \varepsilon)^+] - h E(p^R + q^R - \varepsilon)^+ = 0. \quad (23)$$

Then, $\pi_m^V \geq \pi_m^R \iff \gamma_h \geq \bar{\gamma}_h^m$, or, $\gamma_s \leq \bar{\gamma}_s^m$.

PROPOSITION 8. (i) $\bar{\gamma}_h^m$ decreases in h but increases in s ; (ii) $\bar{\gamma}_s^m$ increases in h but decreases in s .

Note that $\bar{\gamma}_h^r < \bar{\gamma}_h^m$ and $\bar{\gamma}_s^r < \bar{\gamma}_s^m$. Combining Propositions 5 and 7, we see that as in the base model, both the holding cost increment and the shortage cost increment are key in determining how VMI affects the profits of both the manufacturer and retailer. Specifically, both

firms are better off with VMI when the holding cost increment is big (bigger than $\bar{\gamma}_h^m$), the retailer is better off while the manufacturer is worse off with VMI when the holding cost increment is medium (between $\bar{\gamma}_h^r$ and $\bar{\gamma}_h^m$), and both firms are worse off when the holding cost increment is small (smaller than $\bar{\gamma}_h^r$). On the other hand, both firms are better off with VMI when the shortage cost increment is small (smaller than $\bar{\gamma}_s^m$), the retailer is better off while the manufacturer is worse off with VMI when the shortage cost increment is medium (between $\bar{\gamma}_s^r$ and $\bar{\gamma}_s^m$), and both firms are worse off when the shortage cost increment is big (bigger than $\bar{\gamma}_s^r$). Propositions 6 and 8, taken together, characterize the impacts of different problem parameters on the four critical values. Consistent with the base model, the retailer (manufacturer) is more likely to benefit from VMI when its inventory holding cost is low (high) or when its shortage cost is high (low).

6.2. Partial Backlog

In this subsection, we consider a more general case where only a portion of unsatisfied demand is backlogged (i.e., partial backlog). As such, δ is set to be any value between 0 and 1. We continue to assume $D = \varepsilon - p$ with ε being a random variable and its cumulative distribution function is again characterized by $F_\varepsilon(\cdot)$ with $E[\varepsilon] = A$.

For $0 \leq \delta \leq 1$, the retailer profit and manufacturer profit under RMI are expressed as follows:

$$\begin{aligned} \pi_r^R(w, p, q) = & (p - w)(A - p) + [(1 - \delta)(p - w) \\ & + \gamma_s s][p + q - E(q - \varepsilon + p)^+ - A] \\ & - \gamma_h h E(q - \varepsilon + p)^+, \end{aligned} \quad (24)$$

and

$$\begin{aligned} \pi_m^R(w, p, q) = & (w - c)(A - p) + [(1 - \delta)(w - c) + \gamma_s s] \\ & [p + q - E(q - \varepsilon + p)^+ - A]. \end{aligned} \quad (25)$$

It can be shown that for a given pair of wholesale price w and retail price p , the retailer profit under RMI is concave in q . Thus, the retailer's optimal stocking level q^R under RMI satisfies

$$F_\varepsilon(q^R + p) = \frac{(1 - \delta)(p - w) + \gamma_s s}{(1 - \delta)(p - w) + \gamma_s s + \gamma_h h}. \quad (26)$$

Then, for the given pair of wholesale price w and retail price p , the retailer's best profit under RMI is

$$\begin{aligned} \pi_r^R(w, p) = & (p - w)(A - p) + [(1 - \delta)(p - w) + \gamma_s s] \\ & [p + q^R - E(q^R - \varepsilon + p)^+ - A] \\ & - \gamma_h h E(q^R - \varepsilon + p)^+. \end{aligned}$$

Note that in order to assess the impacts of VMI on the supply chain member profits, we first need to characterize the equilibrium wholesale and retail prices under RMI and VMI. As can be seen from equation (26), $q^R + p$ is no longer independent of the retail price in the partial backlog case. Consequently, it becomes quite a challenge to analytically characterize the best retail price p even for an exogenous wholesale price w . Therefore, we use Mathematica to numerically compute each firm's equilibrium decisions and performance under RMI.

To characterize the equilibrium outcome under VMI, we first present the manufacturer profit under VMI as follows:

$$\pi_m^V(w, p, q) = (w - c)(A - p) + [(1 - \delta)(w - c) + s] [p + q - E(q - \varepsilon + p)^+ - A] - hE(q - \varepsilon + p)^+. \quad (27)$$

Because the manufacturer profit under VMI is concave in q for a given pair of wholesale price w and retail price p , the manufacturer's optimal stocking level q^V under VMI is determined by

$$F_\varepsilon(q^V + p) = \frac{(1 - \delta)(w - c) + s}{(1 - \delta)(w - c) + s + h}. \quad (28)$$

Hence for the given pair of wholesale price w and retail price p , the retailer profit under VMI is

$$\pi_r^V(w, p) = (p - w)(A - p) + [(1 - \delta)(p - w) + \gamma_s s] [q^V + p - E(q^V + p - \varepsilon)^+ - A].$$

Note that by equation (28), $q^V + p$ is independent of the retail price. Consequently, $\pi_r^V(w, p)$ is then concave in p and the retailer's best responding retail price $p^V(w)$ satisfies

$$A - 2p^V + w + (1 - \delta)[p^V + q^V - E(p^V + q^V - \varepsilon)^+ - A] = 0. \quad (29)$$

Anticipating the retailer's responding retail price by equation (29) and knowing its own stocking level decision by equation (28), the manufacturer sets the wholesale price to maximize its profit as

$$\pi_m^V(w) = (w - c)(A - p^V) + [(1 - \delta)(w - c) + s] [p^V + q^V - E(p^V + q^V - \varepsilon)^+ - A] - hE(p^V + q^V - \varepsilon)^+.$$

However, it is quite difficult to analytically characterize the equilibrium wholesale price. Therefore, we again use Mathematica to numerically compute each firm's equilibrium decisions and performance under VMI.

Finally, numerical results for RMI and VMI enable us to examine under what conditions each firm benefits from VMI in the endogenous pricing case with partial backlog. The numerical study was conducted for uniform demand distribution. Using built-in functions of Mathematica, we developed codes that numerically compute each supply chain member's equilibrium decisions and evaluate the profits of the retailer and the manufacturer under the two inventory management regimes. Then, for given problem parameter values, we plotted the critical values of γ_h and γ_s for the manufacturer and the retailer with respect to the inventory holding cost and shortage cost. Figures 1 and 2 present eight such plots in total. Recall that $\bar{\gamma}_h^r$ ($\bar{\gamma}_h^m$) is defined as the critical value above which the retailer (manufacturer) benefits from VMI and below which the retailer (manufacturer) loses from VMI in section 6.1; Similarly, $\bar{\gamma}_s^r$ ($\bar{\gamma}_s^m$) is defined as the critical value below which the retailer (manufacturer) benefits from VMI and above which the retailer (manufacturer) loses from VMI. We make the following observations from these two figures:

Observation 1. (i) There exists a $\bar{\gamma}_h^r$ ($< \gamma_s$) such that the retailer benefits from VMI if $\gamma_h \geq \bar{\gamma}_h^r$. (ii) $\bar{\gamma}_h^r$ increases in h but decreases in s ;

Observation 2. (i) There exists a $\bar{\gamma}_h^m$ ($> \gamma_s$) such that the manufacturer benefits from VMI if $\gamma_h \geq \bar{\gamma}_h^m$. (ii) $\bar{\gamma}_h^m$ decreases in h but increases in s ;

Observation 3. (i) There exists a $\bar{\gamma}_s^r$ ($> \gamma_h$) such that the retailer benefits from VMI if $\gamma_s \geq \bar{\gamma}_s^r$. (ii) $\bar{\gamma}_s^r$ decreases in h but increases in s ;

Observation 4. (i) There exists a $\bar{\gamma}_s^m$ ($< \gamma_h$) such that the manufacturer benefits from VMI if $\gamma_s \geq \bar{\gamma}_s^m$. (ii) $\bar{\gamma}_s^m$ increases in h but decreases in s .

In summary, we numerically show that the key results developed in the full backlog case continue to hold qualitatively in the partial backlog case, and establish the robustness of the key insights developed in the base model.

Figure 1 Critical Values of $\bar{\gamma}_h^r$ and $\bar{\gamma}_h^m$ with Respect to Inventory Holding Cost and Shortage Cost in the Endogenous Pricing Case for A One-Manufacturer-One-Retailer Supply Chain ($c = 50$, $\delta = 0.9$, $\gamma_s = 1$, $\varepsilon \sim U[100, 200]$) [Color figure can be viewed at wileyonlinelibrary.com]

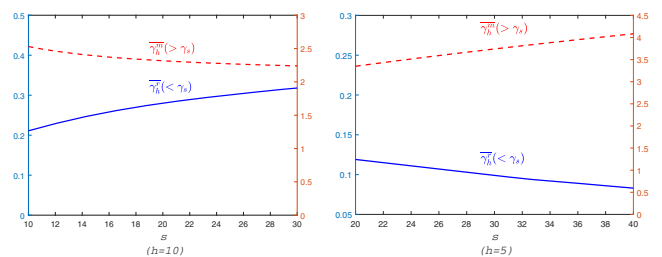
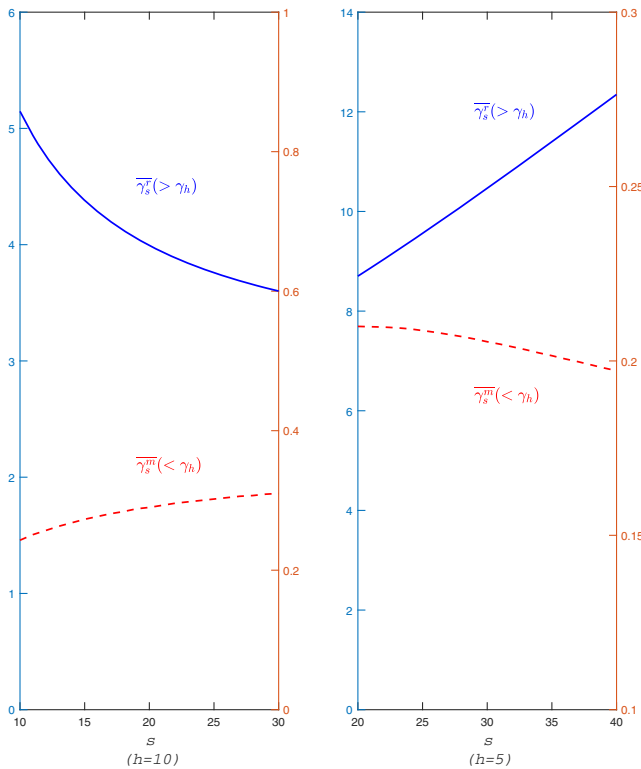


Figure 2 Critical Values of $\bar{\gamma}_s^r$ and $\bar{\gamma}_s^m$ with Respect to Inventory Holding Cost and Shortage Cost in the Endogenous Pricing Case for A One-Manufacturer-One-Retailer Supply Chain ($c = 50$, $\delta = 0.9$, $\gamma_h = 1$, $\varepsilon \sim U[100, 200]$) [Color figure can be viewed at wileyonlinelibrary.com]



7. Conclusion

In this paper, we have examined the effects of VMI in a supply chain with one manufacturer and one retailer. Under RMI, the retailer chooses the stocking level to maximize its average profit; under VMI the manufacturer chooses the stocking level to maximize its own average profit. Using this base model, we develop four main findings:

- Whether a supply chain member benefits from VMI depends on how the holding cost increment from the manufacturer to retailer compares with two critical values. When the holding cost increment is big, VMI benefits both the manufacturer and retailer; when it is medium, VMI benefits the retailer but hurts the manufacturer; when it is small, VMI hurts both the manufacturer and retailer.
- The effects of inventory holding cost on the values of VMI to the manufacturer and retailer contradict our intuition. In particular, the retailer is more likely to benefit from VMI when its inventory holding cost is low, and the manufacturer is more likely to benefit from VMI when its inventory holding cost is high.

- Whether a supply chain member benefits from VMI also depends on how the shortage cost increment from the manufacturer to retailer compares with two other critical values. When the shortage cost increment is small, VMI benefits both the manufacturer and retailer; when it is medium, VMI benefits the retailer but hurts the manufacturer; when it is big, VMI hurts both the manufacturer and retailer.
- The effects of shortage cost on the values of VMI to the manufacturer and retailer are aligned with our intuition. Specifically, the retailer is more likely to benefit from VMI when its shortage cost is high, and the manufacturer is more likely to benefit from VMI when its shortage cost is low.

Our results offer practical guidelines for VMI implementation in a supply chain to leverage the value of information technology. In particular, supply chain partners should find common grounds and start implementing VMI with products for which the downstream firm's inventory holding cost is sufficiently large compared with the upstream firm's, or the downstream firm's shortage penalty cost is small relative to the upstream firm's. Among these candidate products, the downstream firm should pursue implementing VMI for products with low inventory holding costs or high shortage penalty costs while the downstream firm should do so for products with high inventory holding costs or low shortage penalty costs.

To explore the applicability and scope of our key insights, we study a one-manufacturer-one-retailer supply chain with endogenous wholesale and retail prices. We analytically show that when the unsatisfied demand is fully backlogged, the equilibrium wholesale and retail prices are identical under RMI and VMI. Consequently, our analysis and insights derived from the base model continue to hold in the endogenous pricing case with the assumption of full backlog. For the endogenous pricing case with partial backlog, we numerically demonstrate that our key insights developed in the base model continue to hold qualitatively.

A natural extension of this research is to study the impacts of VMI on a supply chain with two competing manufacturers and one common retailer. The two manufacturers are competing with each other because the two brands are (partially) substitutable driven by customer search (Jiang and Anupindi 2010). That is, when the brand from one manufacturer is out of stock, the unsatisfied customers may choose to substitute for the alternative brand. In Appendix S1, we show that the key insights developed for the one-manufacturer-one-retailer supply chain continue to hold qualitatively for the

two-manufacturer-one-retailer supply chain. While studying the supply chain with two competing manufacturers, we assume that the two manufacturers are symmetric in their economic parameters and demand distributions to keep the analysis tractable and insights transparent. Because we are examining the competition between two substitutable brands, the symmetric parameter assumption should be a good approximation to reality. However, it is possible that the demand distributions for the two brands are different. Studying the value of VMI to different supply chain members under such a scenario will be an interesting future research. In practice, a manufacturer may strategically set a wholesale price to better align its interest with the retailer's. Studying the equilibrium result in terms of wholesale price and stocking level in such a setting would also be a promising future research.

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Notes

¹The existence and uniqueness of γ_h^r and γ_s^r are established in the proof of Proposition 1 in Appendix S1.

²The existence and uniqueness of γ_h^m and γ_s^m are established in the proof of Proposition 3 in Appendix S1.

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Supporting Information

Additional supporting information may be found online in the supporting information tab for this article:

Appendix S1: Proof of Results.

Appendix S2: Model Extension: Two-Manufacturer Case with Exogenous Prices.

Appendix S3: Model Extension: Two-Manufacturer Case with Endogenous Prices.