

Analysis of Laterally Loaded Pile Groups in Improved Soft Clay

Amirata Taghavi, S.M.ASCE¹; and Kanthasamy K. Muraleetharan, F.ASCE²

Abstract: The use of p -multipliers in the analysis of the lateral loading behavior of pile groups is based on the concept of modifying the single-pile p - y curve to obtain the p - y curve for a pile in a group. The p -multipliers account for the reduced soil resistance mobilized at a given deflection as a result of overlapping of shear zones. Different factors can influence the p -multipliers; however, because of the lack of experimental data, most researchers and design guidelines consider only the normalized pile spacing in the direction of loading in their recommendations. In this study, the effects of pile spacing and clay stiffness on the p -multipliers were investigated using the results of two series of centrifuge tests. The soil profile consisted of four lightly overconsolidated clay layers overlying a dense sand layer. The pile groups had a symmetrical layout consisting of 2×2 piles spaced at center-to-center distances of 3.0 and 7.0 pile diameter (D) and were driven into improved (stiff clay) and unimproved soft clay. Ground improvement was done in situ using simulated cement deep soil mixing (CDSM). Computer analyses were performed to back-calculate the p -multipliers. There was very good agreement between the measured and computed responses for both the leading and trailing rows of piles in the unimproved and improved soft clay. The results reveal that increasing the clay stiffness and pile spacing in the direction of loading increase p -multipliers. No pile–soil–pile interaction was observed for the $7D$ spacing. The proposed set of p -multipliers for the soft clay was found to be in close agreement with some current guidelines, whereas other design guidelines appear to recommend relatively conservative values of p -multipliers for both soft and stiff clay (improved ground). DOI: [10.1061/\(ASCE\)GM.1943-5622.0000795](https://doi.org/10.1061/(ASCE)GM.1943-5622.0000795). © 2016 American Society of Civil Engineers.

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Introduction

Analyses of pile groups generally show that the lateral resistance in weak soils is not adequate and that there is a need for additional piles. Although additional piles can satisfy the design requirements, the solution is relatively expensive. Improving the weak soil surrounding the pile foundation is an alternative and more cost-efficient solution to restrict lateral displacement. There are different methods for predicting the behavior of pile groups under lateral loading. Fully coupled nonlinear computational methods can be used to model pile groups in improved or unimproved ground under lateral loading. Although these nonlinear analyses can provide useful insights, they are typically time-consuming and are not practical in day-to-day design applications. Several researchers and agencies have therefore proposed simpler methods to analyze closely spaced piles under lateral loading. Poulos (1971) used the theory of elasticity to take into account the effect of a pile on other piles in a group. Focht and Koch (1973) combined the nonlinear p - y model for an isolated single pile with the elastic continuum model of Poulos (1971) to predict the behavior of laterally loaded pile groups. In p - y models, p is the soil reaction per unit length, and y is the relative horizontal

displacement between the soil and the pile. Davisson (1970) considered the group effects by selecting an appropriate variation of subgrade modulus, k , by depth and by the direction of loading. Provided that the side-by-side distance between piles in a group is equal to or greater than $2.5D$, the subgrade modulus can be assumed to vary linearly from 1 to 0.25 k for the pile spacings from 8 to $3D$, respectively. The Canadian Geotechnical Society (1978) recommended Davisson's method to analyze pile groups under lateral loading. The Japan Road Association (1976) also used the subgrade modulus reduction procedure but was less conservative in computing reduction factors for closely spaced pile groups. Ooi and Duncan (1994) used a group amplification procedure in which the lateral deflections and maximum bending moments of single piles were multiplied by amplification factors to estimate those values for group piles.

Brown et al. (1988) used p -multipliers (f_m) to modify the p - y curve of a single pile and obtain these curves for piles of a group in sand. Single piles were modeled as beam elements interacting with surrounding soil through p - y springs. These p - y springs can be made nonlinear to account for the nonlinear behavior of soils. A p - y curve of a single pile can then be squashed to account for the reduced soil resistance of a pile in a pile group. Fig. 1 illustrates the concept of using a p -multiplier to squash a single-pile p - y curve and obtain the p - y curve for a pile in a group. Brown et al. (1988) pointed out that more research is needed to establish a methodology for predicting the variation of p -multipliers with soil properties, pile spacing and stiffness, depth, cyclic loading, and other factors.

The most important factor in determining p -multipliers is the pile spacing in the direction of loading (Brown et al. 1988; Brown et al. 1987; Chandrasekaran et al. 2010; Hannigan et al. 2006; Ilyas et al. 2004; Meimon et al. 1986; Reese and van Impe 2011; Rollins et al. 2006; Rollins et al. 1998; Rollins and Sparks 2002; Ruesta and Townsend 1997; Zhang et al. 1999). However, different researchers have mentioned or illustrated the effects of other

¹Graduate Research Assistant, School of Civil Engineering and Environmental Science, Univ. of Oklahoma, Norman, OK 73019 (corresponding author). ORCID: <http://orcid.org/0000-0001-9904-1918>. E-mail: ata@ou.edu

²Professor, School of Civil Engineering and Environmental Science, Univ. of Oklahoma, Norman, OK 73019. E-mail: muralee@ou.edu

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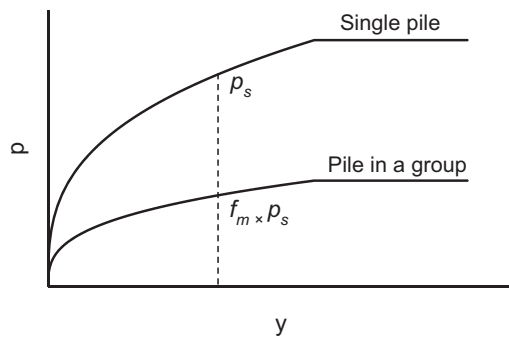


Fig. 1. Applying a p -multiplier (f_m) to a single-pile p - y curve to obtain the p - y curve for a pile in a group

parameters on p -multipliers. These parameters include pile spacing perpendicular to the direction of loading (Ashour and Ardalan 2011; Cox et al. 1984; Reese and van Impe 2011); type of soil and soil profile (Ashour and Ardalan 2011; Brown et al. 1988; Huang et al. 2001); pile arrangement in a group (Chandrasekaran et al. 2010; Ilyas et al. 2004); pile head fixity (Fayyazi et al. 2014); pile stiffness, width, and length (Brown et al. 1988; Rao et al. 1998); and pile installation method (Brown et al. 2001; Huang et al. 2001; Reese and van Impe 2011). Moreover, p -multipliers vary along the length of a pile and also depend on the amount of lateral deflection (Brown et al. 1987; Rollins et al. 2006). Most agencies and researchers (AASHTO 2012; FEMA 2012; Hannigan et al. 2006; Mokwa and Duncan 2005; Rollins et al. 2006; USACE 1993), however, have only considered the normalized pile spacing (S/D ; where S is center-to-center pile spacing and D is the outside diameter of a single pile) in the direction of loading in recommending p -multipliers for each row of piles. The p -multipliers suggested by different agencies and researchers for pile groups are presented in Table 1. Based on this table, there is significant uncertainty in determining the p -multipliers for pile groups. For example, the suggested p -multiplier for the leading row of piles (Row 1) ranges from 0.33 to 0.87 for the normalized spacing of 3.

Because of the lack of experimental data, analyzing pile groups in improved soils under lateral loading involves uncertainty, and in most cases designs are conservative. This study utilized the p - y curves and the p -multipliers to analyze the behavior of closely spaced pile groups in unimproved and improved soft clay. The soil profile consisted of four lightly overconsolidated clay layers overlying a dense sand layer. Centrifuge model tests were carried out on six pile groups in 2×2 configurations and spaced at center-to-center distances of 7.0 and 3.0D. Pile foundations were driven into the improved ground soon after the soft clay was improved in situ using simulated cement deep soil mixing (CDSM). The details of testing and results of these tests were reported by Taghavi (2015) and Taghavi et al. (2015).

Reported herein are the methods used to determine separate sets of p -multipliers for each row of the 2×2 pile groups as a function of pile spacing and clay stiffness. Measured and computed lateral load–deflection, maximum bending moment–load, and bending moment–depth responses of the pile groups are presented. The computed results were obtained with the finite-difference computer program *GROUP* using the p -multipliers developed during this study. For the sake of comparison and also calibration of parameters needed for the analyses, the measured single-pile results are also presented and compared with results computed by the finite-difference code in the computer program *LPILE*. Finally, the p -multipliers obtained in this study for stiff clay and soft clay for the 3 and 7D

Table 1. Recommended p -Multipliers for All Soils

Pile group	S/D	p -Multiplier (f_m)			Reference
		Row 1	Row 2	Row 3+	
2×2	3	0.87	0.62	—	Reese and van Impe (2011)
	7	1.0	1.0	—	
Any group	3	0.79	0.57	0.41	FEMA (2012); Rollins et al. (2006)
	7	1.0	1.0	0.92	
	3	0.8	0.4	0.3	AASHTO (2012); Hannigan et al. (2006)
	5	1.0	0.85	0.7	
	3	0.33	0.33	0.33	USACE (1993) ^a
	4	0.39	0.39	0.39	
	5	0.45	0.45	0.45	Mokwa and Duncan (2005)
	6	0.56	0.56	0.56	
	7	0.71	0.71	0.71	
	8	1.0	1.0	1.0	
	3	0.8	0.6	0.45	

^aThe USACE (1993) reported that group reduction factors are the same for all piles in a group.

center-to-center pile spacings are compared with previously recommended values. The centrifuge experiments and analyses described in this paper represent, to the authors' knowledge, the first study of pile–soil–pile interaction to explicitly include the effect of the undrained shear strength of clay on p -multipliers.

Centrifuge Tests

The quasi-static lateral load tests were carried out at a centrifuge acceleration of 30g in a flexible shear beam container (Kutter 1995) using the 9-m-radius centrifuge at the Network for Earthquake Engineering Simulation (NEES) at the University of California, Davis, facility. The container had internal dimensions of 1,722 (length) \times 686 (width) \times 700 mm (height). The scaling laws for centrifuge model tests can be found in Schofield (1981), Kutter (1992), and Garnier et al. (2007). All the results in this paper are presented in prototype scale unless otherwise stated. All the digital data from these centrifuge experiments (in model scale) are available through the NEES data repository (Taghavi et al. 2013).

Soil Profile

The total depth of four clay layers and the dense sand layer was 9.6 and 8.1 m, respectively (Fig. 2). The water level was at the ground surface. Except for the top clay layer, which had overconsolidation ratio (OCR) values varying from approximately 1.1 to 10 near the ground surface, the clay layers were lightly overconsolidated with OCR values between 1.1 and 2. The undrained shear strength profile of the unimproved clay was obtained based on the assumption of normalized behavior implied by the following equation (Ladd et al. 1977; Wroth 1984):

$$S_u/\sigma'_{v0} = (S_u/\sigma'_{v0})_{NC} \times (\text{OCR})^\Lambda \quad (1)$$

where σ'_{v0} = situ vertical effective stress; $(S_u/\sigma'_{v0})_{NC}$ = normally consolidated strength ratio; and Λ = plastic volumetric strain ratio. The OCR profile of the clay layer in the centrifuge model is shown in Fig. 3(a), and the values of other parameters in Eq. (1) are presented in Table 2. Isotropically consolidated undrained compression (CIUC) triaxial tests (Thompson 2011) were conducted to obtain these parameters. These parameters were then verified by a bounding surface elastoplastic constitutive model (Dafalias and

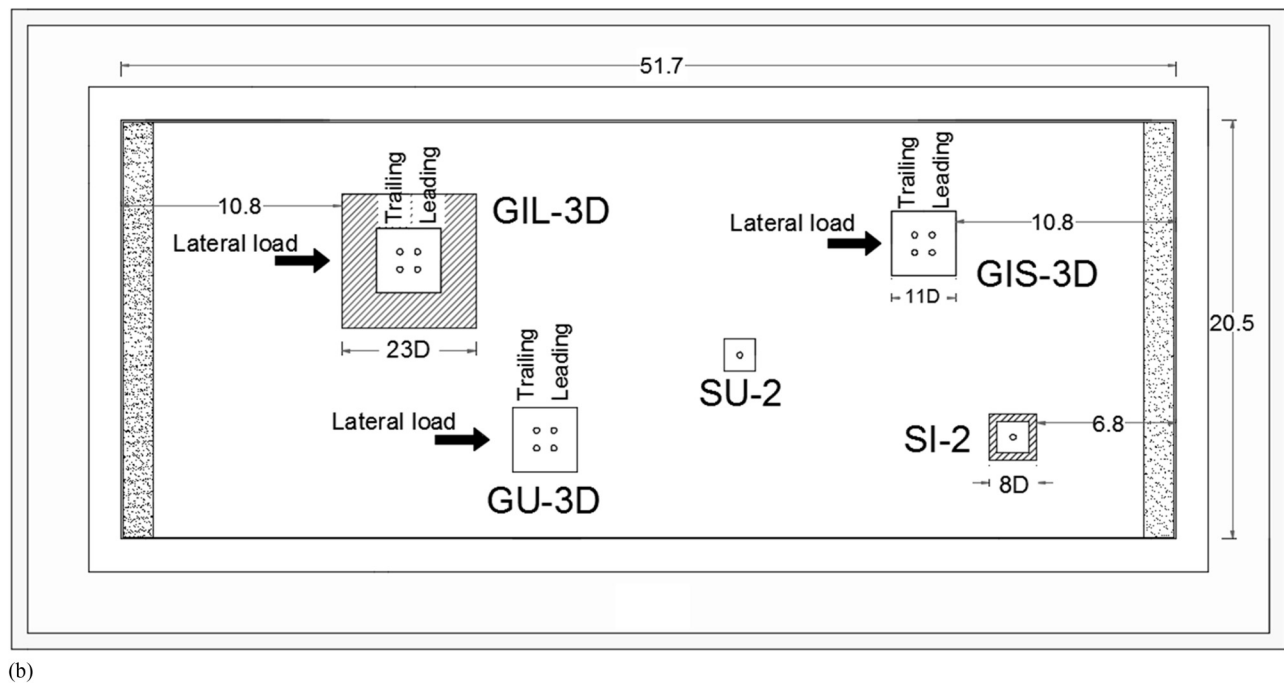
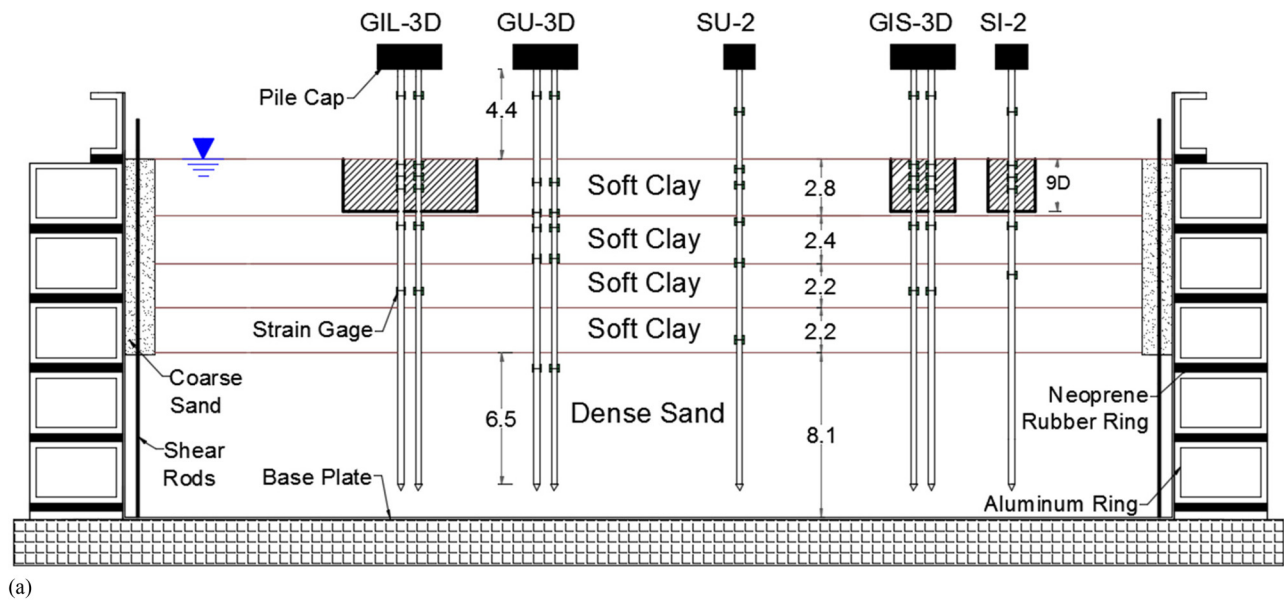


Fig. 2. Centrifuge model setup for single piles and 2×2 pile groups with 3D spacing: (a) elevation; (b) plan (all dimensions in meters)

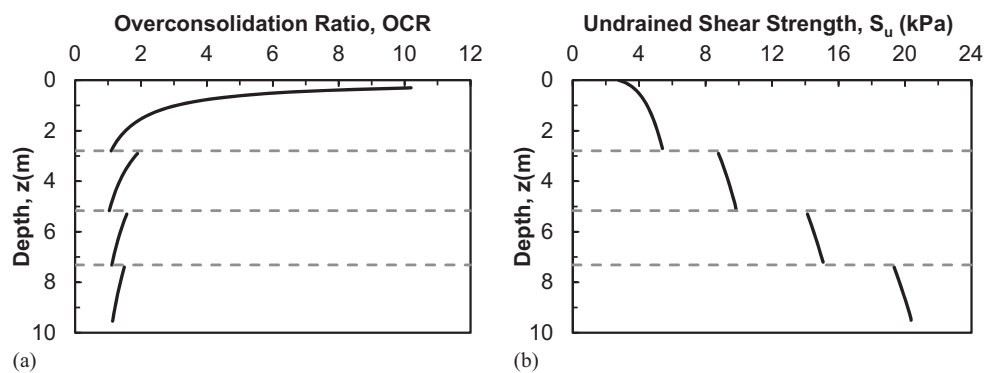


Fig. 3. Profiles: (a) OCR; (b) undrained shear strength (reprinted from [Taghavi et al. 2015](#), © ASCE)

Table 2. Properties of Soft Clay

Property	Parameter	Value
Liquid limit	LL	32
Plastic limit	PL	15
Specific gravity	G_s	2.7
Normally consolidated strength ratio	$(S_u/\sigma'_{v0})_{NC}$	0.22
Plastic volumetric strain ratio	Λ	0.80
Compression index	C_c	0.42
Swelling index	C_s	0.04

Herrmann 1986). The undrained shear strength profile calculated using Eq. (1) is depicted in Fig. 3(b). The dense Nevada sand ($G_s = 2.67$) layer was constructed at a void ratio of 0.57 (a relative density of 84%) using air pluviation.

To improve the soft clay after its consolidation in the hydraulic press, a laboratory equivalent of CDSM was used. Soft clay was excavated from the top in the centrifuge model prior to the preparation of the soilcrete. The soilcrete was then added in layers and lightly tamped and compacted. Before any tests were conducted, all of the CDSM blocks were cured in situ under water for 28 days. A similar method was used to prepare and cure improved clay samples in laboratory for unconfined compression strength (UCS) testing. Several samples were tested, and the average 28-day UCS of the samples was found to be 760 kPa. Additional details on preparation of soft clay and soilcrete are given by Taghavi et al. (2015) and Taghavi (2015). The CDSM procedure used in the centrifuge tests simulated cases in the field where wet soil mixing is utilized for ground improvement before installation of new pile foundations. It is obvious that the deep mixing method (DMM) or CDSM cannot be used for existing pile foundations. One of the most commonly used techniques to improve existing pile foundations is jet grouting, where the grout can be pumped between the piles (J. I. Baez, personal communication, 2010; Rollins et al. 2010a; Rollins and Brown 2011; Rollins et al. 2010b). All of these techniques aim to improve the stiffness and strength of a weak soil, and therefore the results of the centrifuge tests and analyses presented in this paper should be applicable to variety of ground improvement techniques.

Two separate centrifuge models were tested for 3 and 7D pile spacings. Schematics of the second centrifuge model ($S = 3D$) are given in Fig. 2, which includes the locations of selected instrumentation and important dimensions. Only the pile groups GU (group in the unimproved soil) and GIL (group in the large improved ground) and the single pile in the unimproved soil (SU) will be discussed in this paper. For the other two configurations (GIS and SI), the extent of ground improvement was not sufficiently large to consider the improved ground as a semi-infinite layer, an *LPILE* and *GROUP* assumption. The dimensions of the improved ground around GIL was $23 \times 23 \times 9D$ (depth) in both the tests. Because the soil within the upper 5–10D dictate the pile response under lateral loading (Brown et al. 1988; Taghavi et al. 2015), soft clay was improved to a 9D depth.

Piles

A hollow steel tube was used to fabricate the model piles. The piles were instrumented with pairs of strain gauges at six different levels to measure bending moment along the pile. The prototype values of the ultimate and yield flexural moments of the steel tube were close to those of the Caltrans steel pipe pile PP14. The steel conformed to ASTM A513 Grade 1010 specifications and, based on the 0.2% off-set criteria, had an average yield strength of 369 MPa (53,500 psi). The cross section of the piles had a second moment of area (I) of

$1.84 \times 10^{-4} \text{ m}^4$. The outside diameter of the piles was 0.29 m, and the wall thickness was 0.027 m. Coupon tests were performed, and the Young's modulus was obtained as 182 GPa. The length of the piles was 20.4 m, with the lower 16.0 m inserted into the ground. A thick solid aluminum was used to fabricate the pile caps. The caps were located at 4.4 m above the soil surface (free length), and the piles were tightly fixed through openings in the pile cap using several sets of screws. The pile heads were therefore assumed to be fixed in all pile-group analyses. The weight of the single-pile caps was 1/4 of the weight of pile-group caps. The piles were driven into the soil at 1g soon after the placement of the soilcrete. A system consisting of a guide rod, a hammer, and a cushion was used for pile driving. All piles were driven to the desired depth by dropping the hammer from a constant height and through the guide rod on the cushion.

The relative stiffness between the soil and the pile has an important effect in determining the lateral load behavior of pile foundations. The results of the current study and therefore the back-calculated p -multipliers are directly pertinent to the pile foundation systems with similar relative stiffness values. A dimensionless parameter, $(E \times I/A \times t)/S_u \times D$, can be used to capture the relative stiffness of the pile–soil system. The dimensionless-parameter values for pile–soft clay and pile–CDSM were 248×10^5 and 5×10^5 , respectively. The undrained shear strength ratio of improved to unimproved soft clay was 47 (380/8.17 kPa). Judgment and additional analyses are warranted in extrapolating the results and p -multipliers presented in this study for other values of relative stiffness and undrained shear strength ratio.

Quasi-Static Lateral Loading

After the CDSM blocks were cured for a minimum of 28 days, two series of quasi-static lateral load tests were performed on the 7 and 3D pile groups. To compare the results, similar lateral load tests were also done on companion single piles. Eighteen lateral load tests were conducted on 12 pile foundations in 18 days. Because the centrifuge model was damaged soon after the initial lateral load tests, only small-amplitude tests were done in the first series of tests (7D tests). In the second series of tests (3D tests), small-amplitude quasi-static lateral load tests were first conducted on the pile foundations. The pile foundations were then subjected to earthquake base excitations and finally to large-amplitude quasi-static lateral loads with deflections larger than the deflections experienced during seismic events. All tests were displacement controlled with a loading rate of 0.05 mm/s applied using a hydraulic actuator above the pile cap. The displacements and loads were measured using a ILVDT and a load cell, respectively.

Analyses of Single Piles and Pile Groups in Improved and Unimproved Soft Clay

The computer programs *LPILE* and *GROUP* were used to model the laterally loaded single piles and pile groups, respectively. The p - y curves of Matlock (1970), Reese et al. (1975, 1974) were used to model the p - y behavior of piles in soft clay, stiff clay (CDSM block) with free water, and sand, respectively. These curves are provided within *LPILE* and *GROUP* as default curves, and no modifications were made to these curves.

The Matlock (1970) p - y curve for soft clay varies with depth and mainly depends on the strain at one-half the maximum difference in

principal stresses (ε_{50}), undrained shear strength (S_u), and effective unit weight (γ'). To build the p - y curves for soft clay, a typical ε_{50} value of 0.02, as suggested by Reese and van Impe (2011), for soft clay was used. The undrained shear strength values presented in Fig. 3(b) were also used. The effective unit weight was determined for each layer of centrifuge model and is presented in Table 3.

The p - y curve of Reese et al. (1974) for sand mainly depends on the ultimate soil resistance and initial stiffness. The ultimate soil resistance depends on pile diameter and the internal friction angle of sand. The friction angle of dense Nevada sand was assumed to be 38 degrees, which is consistent with the values reported in the VELACS project (Arulmoli et al. 1992). The initial stiffness (k) was obtained by back-analysis of measured single-pile response.

Table 3. Soil Properties Used for the Analysis of the Unimproved Pile Group

Layer	Undrained shear strength (kPa)	Strain at 50% stress (ε_{50})	Friction angle ($^\circ$)	Soil modulus [k (MN/m ³)]	Submerged unit weight [γ' (kN/m ³)]
Soft clay (top)	4.40–8.71	0.02			8.17
Soft clay	8.71–9.96				8.68
	14.19–15.34				9.04
Soft clay (bottom)	19.59–20.76				9.27
Sand	—	—	38	34.0	10.43

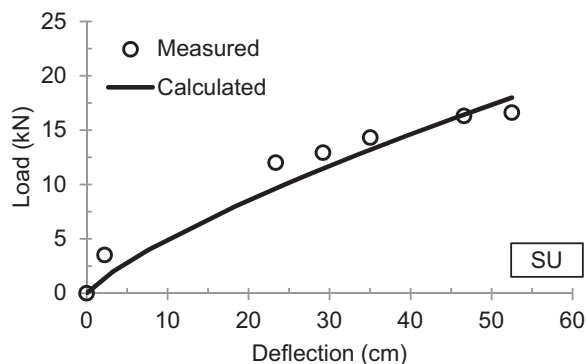


Fig. 4. Comparison of measured and *LPILE*-computed load-deflection curves for the single pile in unimproved soft clay

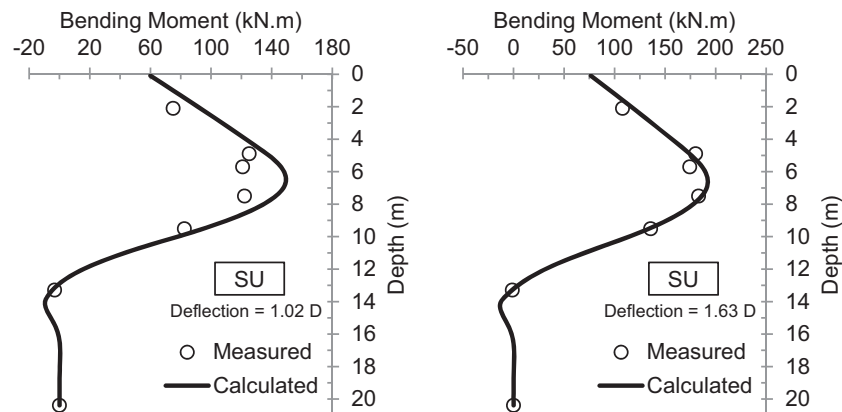


Fig. 5. Comparison of measured and *LPILE*-computed bending moment-depth curves for the single pile in unimproved soft clay for lateral deflections of 1.02 and 1.63D

The analyses were performed to obtain the best possible match between the calculated and measured response for the unimproved single pile. Fig. 4 depicts the *LPILE*-computed and measured lateral load-deflection responses for the unimproved single pile. Fig. 5 shows the computed and measured bending moment profiles of the unimproved single pile for two different deflection values. In this figure and other figures, depth refers to the distance from the pile head (just below the cap). The single-pile heads were free to rotate. Because of the loading setup, the point of application of the load was at a higher elevation, and therefore nonzero bending moments were obtained at zero depth, as shown in Fig. 5. In both the lateral load-deflection and bending moment-depth curves, there is a good agreement between computed and measured responses. The initial stiffness was back-calculated for the dense Nevada sand and was found to be 34.0 MN/m³. The final calibrated unimproved soil input data for the GROUP analyses are presented in Table 3.

The p - y curve of Reese et al. (1975) provided within the program *GROUP* for stiff clay in the presence of free water was used to model the p - y behavior of piles in the CDSM block. To build this curve in each desired depth, strain at one-half the maximum difference in principal stresses (ε_{50}), effective unit weight (γ'), undrained shear strength (S_u), and initial stiffness (k) are required. A typical value of ε_{50} for stiff clays (0.004), as suggested by Reese and van Impe (2011), was used for the improved clay, and the effective unit weight of the improved soil was assumed to be 8.17 kN/m³. As mentioned earlier, the undrained shear strength of the improved clay obtained from the uniaxial compression tests was 380 kPa. The initial stiffness (k) was obtained by back-analysis of the measured GIL-7D pile-group response. By assuming an initial value of 1.0 for p -multipliers, the analyses were done to achieve the best possible match between the predicted and measured response for the GIL-7D improved pile group at very small deflections to lower the axial response effects. Single piles were not used in this step because none of the CDSM layers in the tested improved single piles could be assumed to be semi-infinite, an *LPILE* assumption. To model the p - y behavior of piles in unimproved soft clay layers, the p - y curve of Matlock (1970) was used. The results of numerical modeling of the GIL-7D improved pile group are presented later and show that assuming a value of 1.0 for p -multipliers in the GIL-7D pile group is a valid assumption. Using this procedure, the initial stiffness (k) for the CDSM block was back-calculated as 460.0 MN/m³. This matches well with the k values available in literature (Reese and van Impe 2011; Reese et al. 2010). The input data for the CDSM improved soil is presented in Table 4.

Determination of p -Multipliers

After calibrating the soil properties as described previously, the GU-3D and GIL-3D pile-group results were used to back-calculate p -multipliers using *GROUP*. The p -multipliers were initially obtained by matching the measured and computed total lateral load–deflection responses. These p -multipliers were then adjusted for each row of pile groups in improved and unimproved pile groups by matching the measured and computed responses for each row.

The p -multipliers suggested by different researchers for pile groups in clay, including the present study, are presented in Table 5. Because different p -multipliers were obtained for pile groups with the same arrangement and spacing, this table illustrates that these two parameters are not the only governing factors in determining p -multipliers. The p -multipliers obtained for the leading and the trailing rows of a 2×2 improved pile group with 3D spacing in this study were 0.89 and 0.60, respectively. For the unimproved group, these values were 0.84 and 0.43, respectively.

Table 4. CDSM Properties Used for the Analyses of the Improved Pile Groups

Layer	Improved soil
Undrained shear strength (kPa)	380
Strain at 50% stress (ϵ_{50})	0.004
Soil modulus [k (MN/m ³)]	460.0
Submerged unit weight [γ' (kN/m ³)]	8.17

The set of (0.84, 0.43) obtained in this study for the 2×2 pile group with 3D spacing in soft clay is comparable with the suggested sets of FEMA (2012), Rollins et al. (2006), and Mokwa and Duncan (2005) for all soils (Table 1, Row 1 and Row 3+). Moreover, the set of (0.89, 0.60) obtained for the 2×2 pile group with 3D spacing in stiff clay (improved clay) is close to the suggested set by Huang et al. (2001) for pile groups driven in silt and silty clay (Table 5) and the values suggested by Reese and Van Impe (2001) for all soils (Table 1). The percentage differences between the p -multipliers of this study and those recommended by various agencies are presented in Table 6. As shown in Table 6, whereas FEMA recommendations for soft clay sites are reasonable, using the recommendations of FEMA (2012), AASHTO (2012), and the USACE (1993) will result in an overly conservative design of piles in improved soft clay sites. As mentioned earlier, p -multipliers should generally vary with lateral deflection and also by depth. However, this

Table 6. Percent Difference in p -Multipliers between This Study and Various Agency Recommendations

Agency	Stiff clay, $S/D = 3$		Soft clay, $S/D = 3$		Clay, $S/D = 7$	
	Leading	Trailing	Leading	Trailing	Leading	Trailing
FEMA (2012)	12	38	6	5	0	8
AASHTO (2012)	11	67	5	36	NA	NA
USACE (1993)	92	58	87	26	34	34

Table 5. Comparison of p -Multipliers Obtained from Lateral Load Tests on Pile Groups in Cohesive Soils

Pile group	S/D	p -Multiplier (f_m) in row					Type and strength (kPa)	Test type	Reference
		1	2	3	4	5			
2×2	3	0.89	0.60	—	—	—	Stiff (improved) clay, $S_u = 380$	Centrifuge	Present study
2×2	7	1.0	1.0	—	—	—			
2×2	3	0.84	0.43	—	—	—	Soft clay, $S_u = 4$ –10		
2×2	7	1.0	1.0	—	—	—			
3×4	3 ^a	0.89	0.61	0.61	0.66	—	Silt and silty clay, $S_u = 110$ –190	Full scale	Huang et al. (2001)
2×3	3 ^b	0.93	0.70	0.74	—	—			
3×3	3 ^c	0.7	0.6	0.5	—	—	Stiff clay, $S_u = 70$ –180	Full scale	Brown et al. (1987)
3×3	3 ^d	0.7	0.5	0.4	—	—			
3×3	5.65	0.95	0.88	0.77	—	—	Stiff clay, $S_u = 70$ –105	Full scale	Rollins et al. (2006)
3×4	4.4	0.90	0.80	0.69	0.73	—			
3×5	3.3 ^e	0.82	0.61	0.45	0.45	0.46			
3×5	3.3 ^f	0.82	0.61	0.45	0.45	0.51			
3×3	3	0.60	0.38	0.43	—	—	Silt and clay, $S_u = 25$ –60	Full scale	Rollins and Sparks (2002)
3×3	3	0.60	0.38	0.43	—	—	Clay and silt, $S_u = 25$ –50	Full scale	Rollins et al. (1998)
3×2	3	0.9	0.5	—	—	—	Silty clay, $S_u = 25$	Full scale	Meimon et al. (1986)
3×3	3	0.65	0.5	0.48	—	—	Soft clay, $S_u = 0$ –20	Centrifuge	Ilyas et al. (2004)
2×2	3	0.96	0.78	—	—	—			
1×2	3	0.8	0.63	—	—	—			
1×4	3	0.76	0.56	0.46	0.54	—	Soft clay, $S_u = 9$	Small scale at 1g	Chandrasekaran et al. (2010)
3×3	3	0.66	0.41	0.44	—	—			
2×2	3	0.74	0.48	—	—	—			
2×2	5	0.85	0.58	—	—	—			
1×2	3	0.81	0.70	—	—	—			

^aDriven group piles.

^bBored group piles.

^cLateral deflection = 3 cm.

^dLateral deflection = 5 cm.

^eLateral deflection < 5 cm.

^fLateral deflection > 5 cm.

consideration is beyond the scope of this research. As is shown later in the paper, the p -multipliers obtained in this study predicted the lateral load behavior of both improved and unimproved pile groups and were valid for all deflection ranges less than 35 cm. Additional validation of the p -multipliers obtained in this study using full-scale tests on similar pile configurations is desirable.

A comparison of the full-scale tests results of Rollins et al. (1998) and Rollins et al. (2006) show that they have obtained greater p -multipliers for the pile group in stiff clay than the group in soft to medium-stiff clay and silt (Table 5). These two pile groups both had a free-head condition and were tested under lateral loading in two sites located in Utah. The first pile group tested by Rollins et al. (1998) in soft to medium-stiff clay and silt had a 3×3 arrangement, $3D$ spacing in both the longitudinal and transverse directions, and a 0.315-m OD pile made from steel pipe filled with pea-gravel concrete, and they were driven to a depth of approximately 9.1 m. The second pile group in stiff clay tested by Rollins et al. (2006) had a 3×5 arrangement, $3.3D$ spacing in both the longitudinal and transverse directions, and a 0.324-m OD pile made from steel pipe, and they were driven to a depth of approximately 11.9 m. The key differences in these two pile groups that can contribute to the magnitude of p -multipliers are the clay stiffness and pile-group arrangement. Based on their test results and other results available in literature, Rollins et al. (2006) proposed equations for p -multipliers. Considering the fact that these equations are same for 3×3 and 3×5 pile groups, the difference in p -multipliers obtained from the aforementioned tests can be reasonably related to the difference in clay stiffness.

Furthermore, the suggestion of larger values of p -multipliers for pile groups in stiff clay than soft clay is consistent with the findings of Ashour and Ardalan (2011). They used the strain

wedge (SW) model technique (Ashour et al. 1998) to perform lateral load analyses on a 3×3 pile group with $3D$ spacing in both the longitudinal and transverse directions. They used two different clay profiles in their analyses and kept all other factors the same and obtained greater p -multipliers for the medium to stiff clay profile ($S_u = 72\text{--}144$ kPa) than the soft to medium-stiff clay profile ($S_u = 24\text{--}72$ kPa).

Comparing Measured and Computed Responses

The total lateral load–deflection curves of the unimproved and improved pile groups with 3 and $7D$ spacing are compared with the *GROUP*-computed curves in Figs. 6 and 7, respectively. Generally, the agreement between the measured and the *GROUP*-computed total load–deflection responses was found to be good; however, greater deviations were observed in the GU- $7D$ group. Some of this error may be a result of uncertainties in interpretation of load and deflection measurements and the inadequacy of the numerical model. The measured and computed bending moment–depth and load–maximum bending moment curves of the GU- $7D$ group (shown later), however, were in good agreement. Next, the responses for the leading and trailing row of piles are presented. Measured and computed load–deflection, maximum bending moment–load, and bending moment–depth responses are discussed in the following sections. These comparisons between measured and computed responses indicate that the lateral dimension of the CDSM layers in the GIL pile groups ($23D$) was long enough for these layers to be assumed as semi-infinite layers, an inherent assumption in *GROUP*.

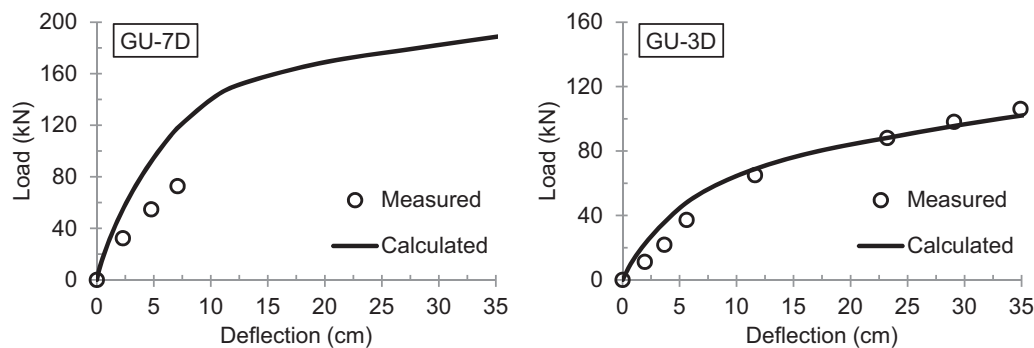


Fig. 6. Comparison of measured and *GROUP*-computed total load–deflection curves for the unimproved pile groups at 7 and $3D$ spacings

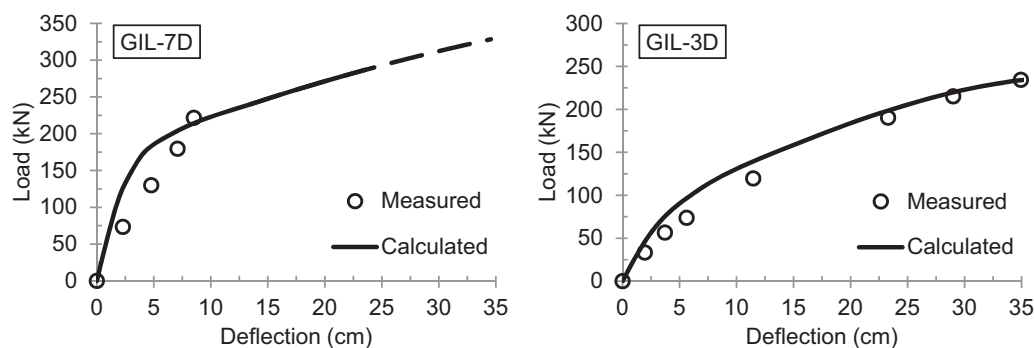


Fig. 7. Comparison of measured and *GROUP*-computed total load–deflection curves for the improved pile groups at 7 and $3D$ spacings

Load-Deflection Curves

By integrating soil reaction profiles, lateral loads carried by each pile can be deduced. Soil reaction (p) can be obtained by double differentiation of the bending moment using the weighted residual method. Readers are referred to Wilson (1998) and Brandenburg et al. (2010) for details on applying this method to double differentiate discrete bending moment profiles. Figs. 8 and 9 show the lateral loads carried by the leading and trailing row of piles of the GU pile groups at 3 and 7D spacing, respectively. Load-deflection curves for each row computed using *GROUP* with the p -multipliers developed in this study are also plotted in these figures for comparison. The agreement was generally found to be good. Figs. 10 and 11 depict similar plots but for the improved GIL pile groups with 3 and 7D spacing, respectively. The agreement was again found to be good, particularly considering the simplicity of the adjustment factor and different soils involved.

At a given deflection in the GIL-7D and GU-7D groups, both leading and trailing piles carried approximately the same amount of

loading. This shows that there were no group-interaction effects in the GU-7D and GIL-7D pile groups. At 3D spacing, however, the leading row of piles in GIL-3D and GU-3D carried larger loads than the trailing row of piles. At small deflections, the difference between lateral loads carried by leading and trailing piles was small, but the difference became larger as the deflections became larger. This illustrates that pile-soil-pile interactions increased as the deflection increased. The fact that the leading row of piles carries a larger portion of load than the trailing row of piles in closely spaced groups is in agreement with the findings of Rollins et al. (1998, 2006), Brown et al. (1987), and Meimon et al. (1986).

Maximum Bending Moment-Load Curves

Measured maximum bending moment-load curves for improved (GIL) and unimproved (GU) piles are shown in Figs. 12–15. The maximum bending moment is the largest moment along the length of each leading or trailing row of piles in each group, and the load is the total load carried by that pile group. Calculating maximum

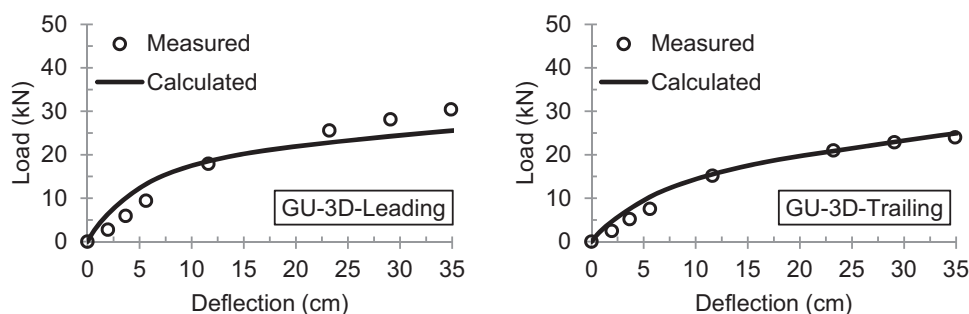


Fig. 8. Comparison of measured and *GROUP*-computed load-deflection curves for each row of the unimproved pile group at 3D spacing

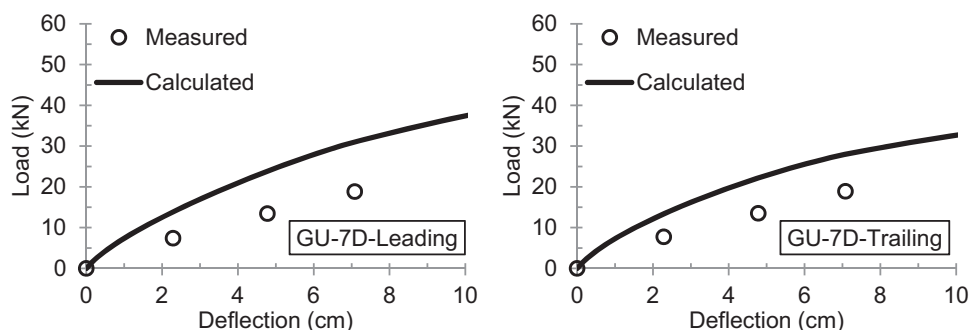


Fig. 9. Comparison of measured and *GROUP*-computed load-deflection curves for each row of the unimproved pile group at 7D spacing

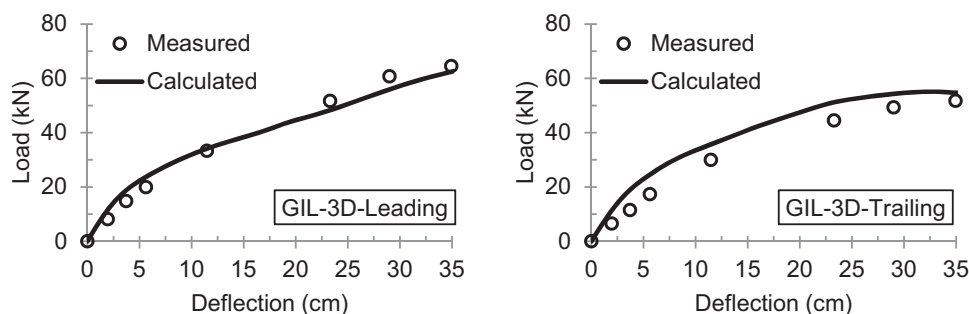


Fig. 10. Comparison of measured and *GROUP*-computed load-deflection curves for each row of the improved pile group at 3D spacing

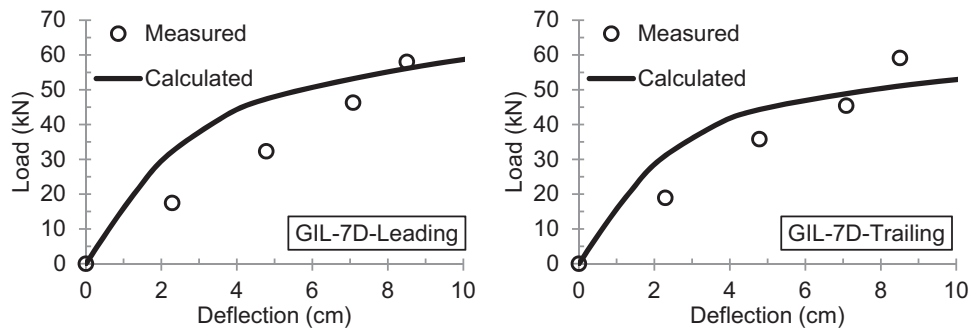


Fig. 11. Comparison of measured and *GROUP*-computed load-deflection curves for each row of the improved pile group at 7D spacing

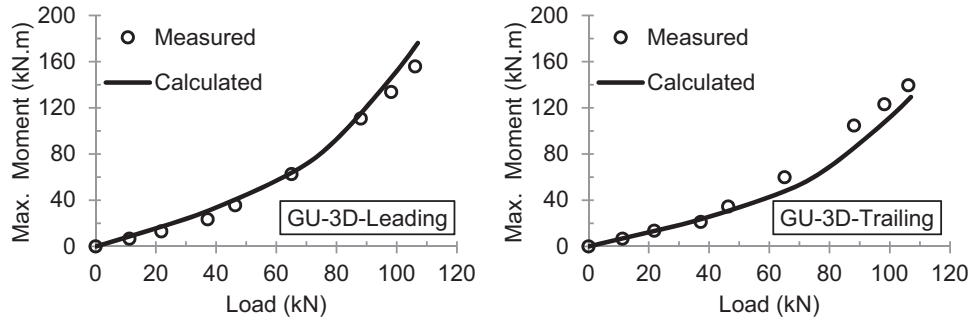


Fig. 12. Comparison of measured and *GROUP*-computed maximum bending moment-load curves for each row of the unimproved pile group at 3D spacing

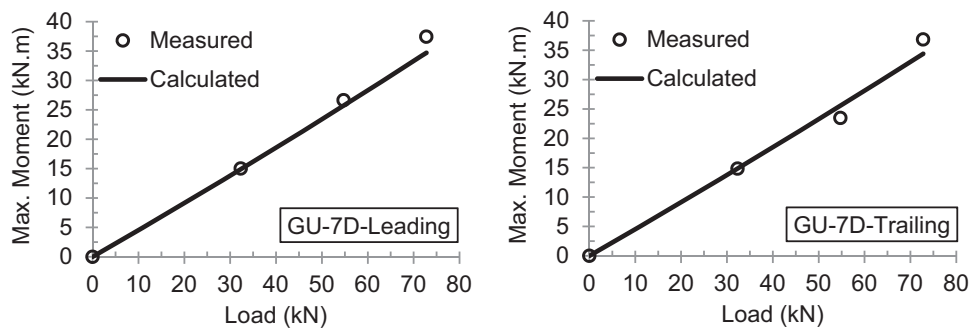


Fig. 13. Comparison of measured and *GROUP*-computed maximum bending moment-load curves for each row of the unimproved pile group at 7D spacing

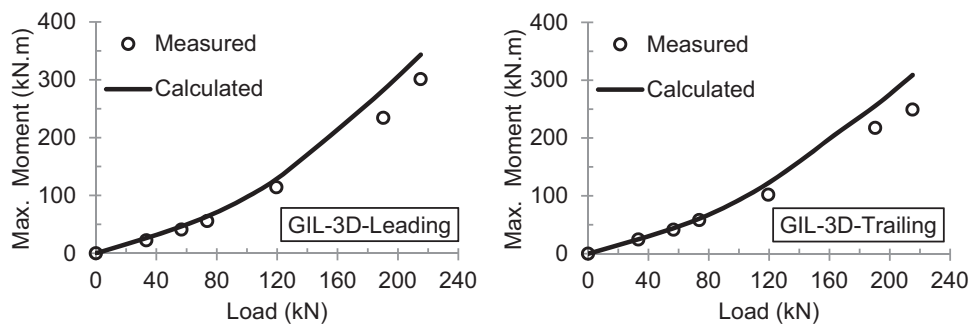


Fig. 14. Comparison of measured and *GROUP*-computed maximum bending moment-load curves for each row of the improved pile group at 3D spacing

bending moments and their resulting stresses is important because they often control the design of piles (e.g., number of piles). Maximum bending moment–load curves computed using *GROUP* with the p -multipliers developed in this study are also provided in Figs. 12 and 15 for comparison. The agreement between measured and computed maximum bending moment–load curves was found to be very good for all four pile groups.

Bending Moment–Depth Curves

For each row of the 7 and 3*D* pile groups in improved and unimproved soft clay, bending moment–depth curves are presented in Figs. 16–19. For a given deflection for the GU and GIL pile groups

with 7*D* spacing, the maximum moments for both the trailing and leading piles were close to one another. The maximum bending moment for GU (3 and 7*D*) and GIL (3 and 7*D*) piles was measured at depths of approximately 9–10 and 1–2*D* below the ground surface, respectively. Increasing applied deflections increased the bending moments in all pile groups. For a given deflection, the piles in unimproved clay experience lower bending moments than those in the improved clay. This is as expected because the piles in the improved clay would resist considerably more load compared with the piles in the unimproved clay at the same deflection, as indicated in Figs. 6 and 7. Figs. 16–19 show that a reduction in the pile spacing from 7 to 3*D* led to larger bending moments being mobilized in the leading row of piles than the trailing row at the same amount of

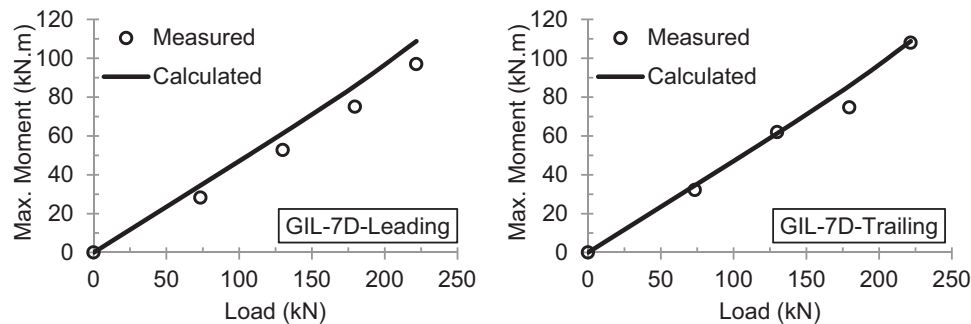


Fig. 15. Comparison of measured and *GROUP*-computed maximum bending moment–load curves for each row of the improved pile group at 7*D* spacing

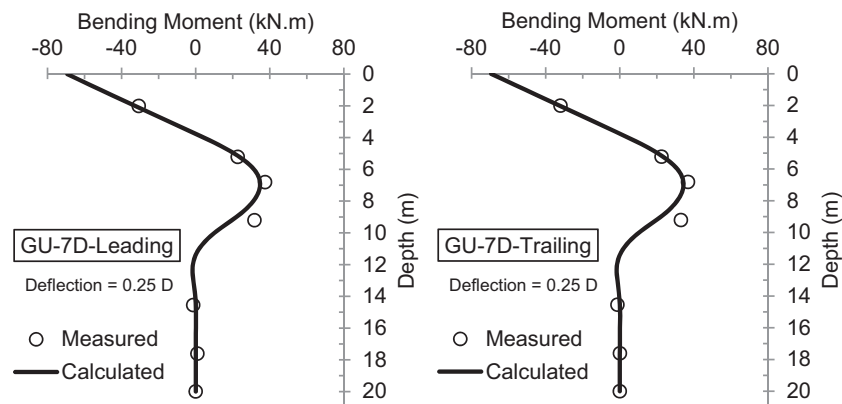


Fig. 16. Comparison of measured and *GROUP*-computed bending moment–depth curves for each row of the unimproved pile group at 7*D* spacing

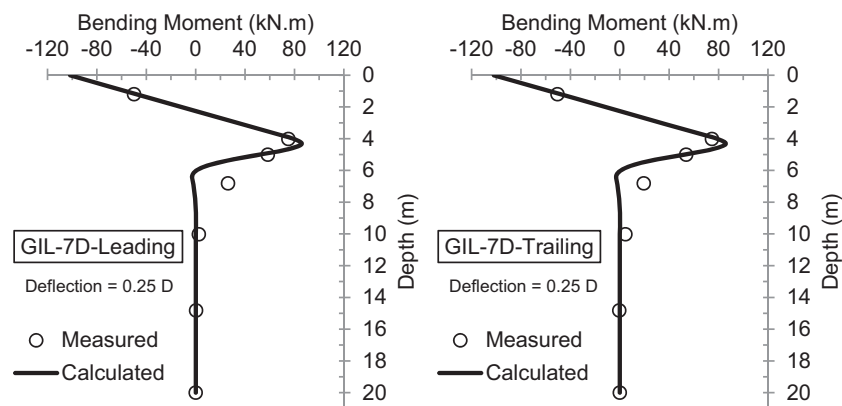


Fig. 17. Comparison of measured and *GROUP*-computed bending moment–depth curves for each row of the improved pile group at 7*D* spacing

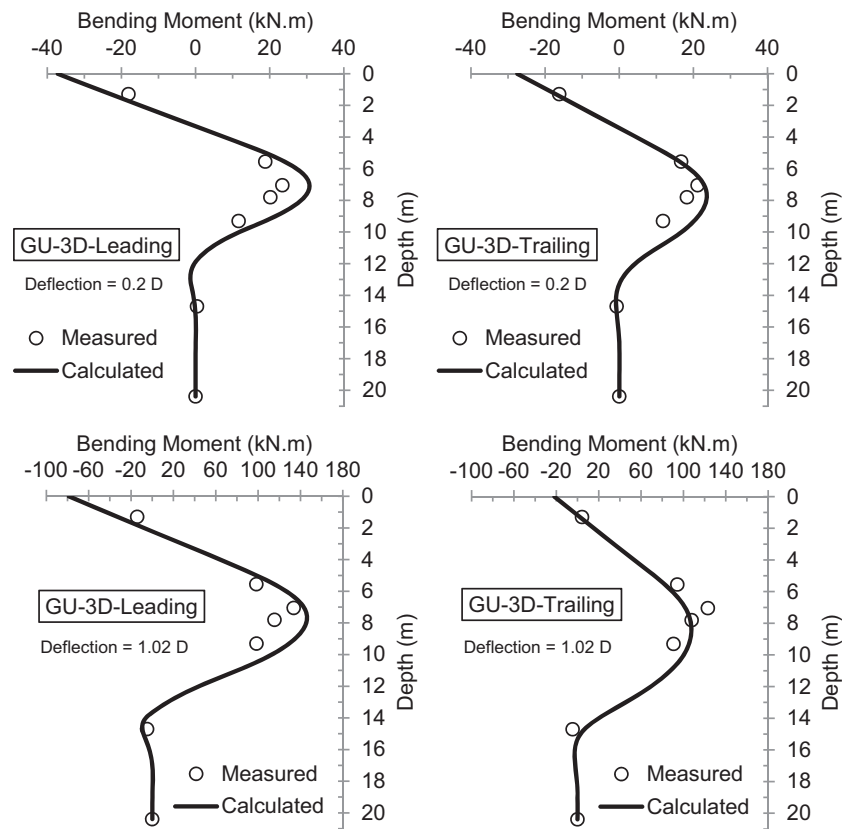


Fig. 18. Comparison of measured and *GROUP*-computed bending moment–depth curves for each row of the unimproved pile group at 3D spacing

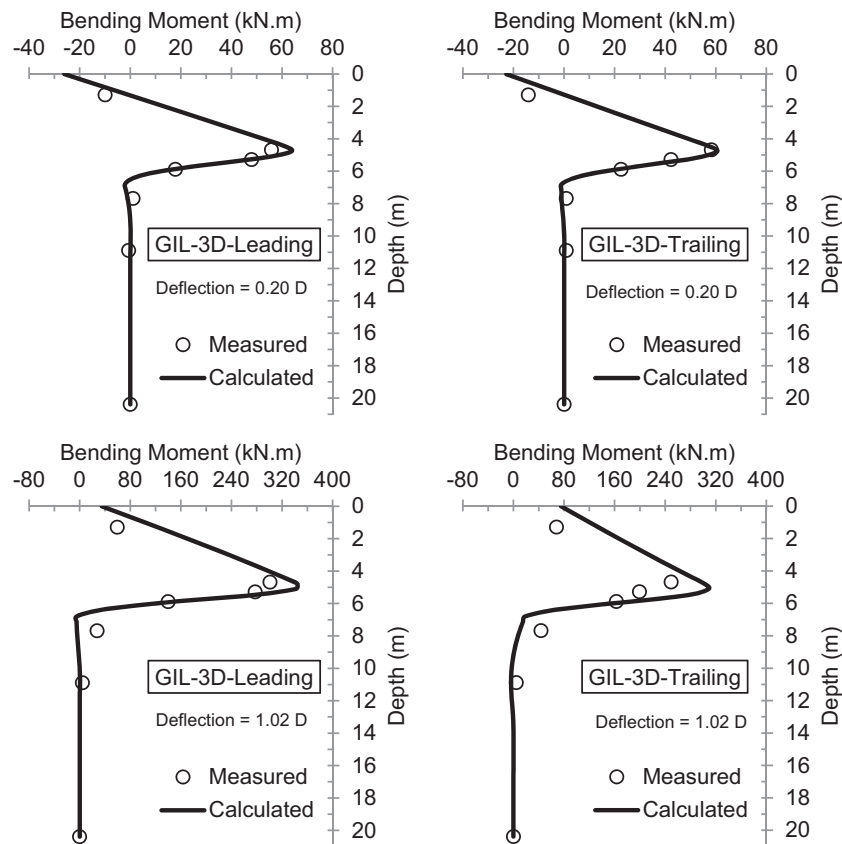


Fig. 19. Comparison of measured and *GROUP*-computed bending moment–depth curves for each row of the improved pile group at 3D spacing

deflection. Although this effect was more noticeable at larger deflections, it occurred even at smaller deflections for 3D spacing. The fact that the leading row of piles carried higher loads than the trailing row of piles for 3D spacing is an indication of pile–soil–pile interactions.

Bending moment–depth curves computed using the program *GROUP* with the p -multipliers developed in this study are also plotted in Figs. 16–19 for comparison purposes. These comparisons show that the implemented p -multipliers in *GROUP* are capable of successfully predicting the shape of the curve and the depth to the maximum bending moment for all pile groups in improved and unimproved soft clay.

Conclusions

Centrifuge model tests were performed to determine the p -multipliers for pile groups in CDSM improved and unimproved soft clay under lateral loading. The p -multipliers account for the reduced soil resistance resulting from the overlapping of shear zones in the piles of a group. The p -multipliers squash the p - y curve of an isolated single pile to obtain those for a pile in a group. The centrifuge model consisted of four lightly overconsolidated clay layers overlying a dense layer of Nevada sand. The pile groups had a symmetrical layout consisting of 2×2 piles spaced at center-to-center distances of 3 and 7D and were driven into the improved (stiff clay) and unimproved soft clay. Ground improvement was done in situ using simulated CDSM. Centrifuge test results and analyses revealed the following:

1. The computer programs *LPILE* and *GROUP*, utilizing the p -multipliers developed in this study, were able to simulate the lateral load behavior of single and group piles satisfactorily.
2. The p -multipliers for the leading row of piles were greater than those for the trailing row of piles, indicating that more overlapping between shear zones occurred for the trailing row of piles.
3. Increasing the center-to-center pile spacing from 3 to 7D increased the p -multipliers. No pile–soil–pile interaction was observed in pile groups with 7D spacing in both improved and unimproved soft clay.
4. The p -multipliers for stiff clay were greater than those for soft clay. This finding is in agreement with the previous strain wedge model analyses and also with the p -multipliers obtained from different full-scale test results of similar pile groups. Assuming the same p -multipliers for all clays and neglecting clay stiffness will result in a relatively conservative design of pile groups in stiff clay.
5. The p -multipliers obtained for the pile group in soft clay were found to be in good agreement with those suggested by FEMA (2012), Rollins et al. (2006), and Mokwa and Duncan (2005) for all soils.
6. The p -multipliers obtained in this study for the 2×2 pile groups in stiff clay were found to be higher than those recommended by AASHTO (2012), FEMA (2012), and the USACE (1993) and most other recommendations for all soils. The p -multipliers recommended by these agencies appear to be relatively conservative. The stiff clay p -multipliers obtained in this study, however, were found to be comparable with those suggested by Reese and Van Impe (2001) for all soils and those obtained by Huang et al. (2001) in a stiff silt and silty clay site.

Acknowledgments

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