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Pricing Risk-Adjusted Deposit Insurance: An Option-Based Model

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ABSTRACT

This paper presents a methodology for arriving at empirical estimates of deposit insurance premiums from market data by using isomorphic relationships betweeen equity and a call option, and insurance and a put option. The data utilizes the market value of equity to solve for the asset value and its volatility. Market perceptions of FDIC bailout policies are explicitly modeled so as to eliminate the bias in inverted values of assets and their volatility. Sensitivity analyses are performed to show that rank orderings based on premiums are robust to changes in specification, thus facilitating allocation of aggregate premium across banks.

WHILE ECONOMISTS HAVE LONG argued in favor of risk-adjusted deposit insurance as both more equitable and more efficient than the current system of flat-rate premiums, various recent developments have further contributed to an increasing dissatisfaction with the current system. First, both the banking industry and the government seem to be tending to the view that deregulation of the banking industry would be necessary in order to meet more sophisticated future demands on the industry as well as desirable as a policy means of stimulating greater competition among banks. Moreover, a sudden rise in the incidence of bank failures,[1] and the vulnerability of the U.S. banks to the so-called international debt crisis have served to bring to the fore concern about the health of the banking industry.

In the absence of deposit insurance, riskier banks will be able to attract deposits only at higher rates, and these higher costs of funding serve as built-in market-regulated incentives to limit excessive risk-taking by banks. As introduction of deposit insurance makes deposits equally risk-free across banks, these incentives disappear, and regulation and close supervision of the banking industry must necessarily replace them as deterrents to excessive risk-taking. Thus, when insurance is offered at a flat premium, regulation is designed to ensure that the risk posed to the insurer—both asset *and* financial risk—is appropriately uniform

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[1] "During the 20 years preceding 1975, the number of failures averaged only 4.2 per year, but this increased to 10 for the period 1975 through 1981. [In 1982] there were 34 commercial bank and eight mutual savings bank failures—highest rate since 1940." *Deposit Insurance in a Changing Environment*, FDIC [6, p. II-3].

so that it corresponds with uniformity on the premium side. Risk-adjusted deposit insurance, on the other hand, can be readily seen to reintroduce incentives to limit excessive risk-taking, thus combining the benefits of deposit insurance (avoiding bank runs) with those of deregulation (higher competition).

The chief argument against risk-adjusted deposit insurance has been that its implementation will be infeasible, as it calls for accurately quantifying the riskiness of each insured bank in terms of observable and realistic data. In the absence of such quantification, the FDIC's role would become necessarily judgemental, running counter to the very spirit that deregulation seeks to foster.

This paper attempts to demonstrate that the problem of empirical estimation of risk and the deposit insurance premium is tractable when time series data on the market value of the bank's equity and the book value of its debt are available. In keeping with this purpose, we have restricted ourselves to a model for which computer-accessible data is readily available.

The model utilized in this paper is an application of the Black-Scholes option pricing model to the valuation of corporate liabilities. Deposit insurance transforms heretofore risky debt to a riskless obligation; the value of this insurance is a put option on the assets of the bank. Thus, as shown by Merton [13], "fair" deposit insurance exacts this value of the put via the insurance premium. The current model is designed to arrive at a point estimate of the value of this limited-liability put and to infer the appropriate deposit premium from the put's value.

Early literature on the pricing of deposit insurance, e.g., Scott and Mayer [17] and Humphrey [9], relied upon historical loss experience as a guide to the fair value of the premium. Implicit in this approach is an analogy with the insurance industry in general, where premiums are largely based on the probability of loss as measured by historical frequencies of loss-entailing events. There are theoretical as well as practical problems with this approach. First, as Horvitz [8] has pointed out, the analogy is not quite valid because of the unique nature of deposit insurance, embedded as it has to be in the federal regulatory framework where the insurer is concerned to a greater degree with *prevention* of the event it insures against than with compensation after the event. Such preventive measures—including cash assistance and merger subsidization—forestall the failure event and consequently result in a historical loss experience substantially below the regulation-free *ex ante* rate. Second, stationarity of the underlying distribution of the loss-generating event is far less likely than it would be for, say, the fire insurance industry, as the regulatory climate and bank investment policies have changed dramatically since the mid-thirties. Finally, on a practical level, historical loss rate can be measured only for the banking industry as a whole, and accordingly the approach does not readily lend itself to estimates of risk-adjusted premiums for *each* bank on an *individual* basis.

Moreover, the main concern of much of the previous literature has been with the question of whether the *flat* premium which is at present charged by the FDIC represents a *fair* value of the insurance. For instance, Buser, Chen, and Kane [2] argue that the *explicit* premium that the FDIC charges is deliberately underpriced, and capital adequacy and other regulations serve as an additional implicit premium.[2] On the other hand, Marcus and Shaked [11] (henceforth MS), following an option-based model similar to ours, found empirical evidence of substantial overpricing on the part of the FDIC.

Our approach differs from that of MS in several aspects. MS focus their attention on whether the existing flat premium is fair. On the other hand, we emphasize that the option-based approach lends itself more readily to *cross-sectional* comparisons of risk across banks. This conclusion follows from the explicit modeling of the FDIC policies of coming to the aid of distressed banks in various ways. Bank equity holders are not unmindful of these policies. Consequently, ignoring these assistance measures would understate the cost of deposit insurance and may account for the MS conclusion of FDIC premium overpricing. Our emphasis on cross-sectional comparisons follows from the impact of assistance measures, periodicity of audit, and other intractable aspects of modeling market perceptions, which, taken together, detract from the reliability of the absolute magnitude of insurance premiums. However, as we document in the sensitivity analyses that we perform, the rank orderings of banks remain robust to changes in specifications, thus facilitating allocation of a predetermined aggregate premium across banks on the basis of the relative premiums that would emerge from the invariance of rankings. Thus, one important distinction lies in the interpretation that can be ascribed to the empirical results.

Moreover, our approach is also distinct from that of MS in methodological detail. To begin with, MS assume that the price of insurance as a put is determined by the *pre*-insurance value of assets, which serves as the underlying security for the put option. Therefore, they need an equation relating the value of the assets before insurance to that after insurance, and in postulating such an equation they argue that insurance, being renegotiable, is a one-period contract and that *all* of the increment on account of the purchase of insurance accrues to the value of the bank, thus ignoring the issue of incidence of the accretion altogether.[3] In contrast, we observe that it is the *future* stochastic behavior of the assets, and therefore the value of the assets *after* insurance, that impinges upon the price of the insurance, and accordingly we make no assumption about how the value of the assets before and after insurance are related. Second, in arriving at the maturity value of debt (which serves as the striking price), MS have used Regulation Q rates, which take effect *after* the regulations are imposed

[2] Sharpe [18] examined capital adequacy in the context of a state preference model and recommended risk-adjusted capital adequacy standards, given that the insurance premiums are currently levied on a flat basis. In fact, one of the advantages of the option-based methodology is that it permits the simultaneous consideration of the deposit premium and capital adequacy issues. The regulating agency can use either tool to exact an appropriate deposit premium: (1) it can increase the per-dollar deposit premium; or (2) it can require the bank to increase its equity values, thus reducing the value of the limited-liability put. Options analysis provides a method of computing the required equity capital injection designed to reduce the put value to that exacted by the deposit insurance premium. Thus, under a flat deposit premium regime, banks displaying higher risk levels would be required to maintain higher capital adequacy standards.

[3] See Section I.A for a detailed analysis of the effect of deposit insurance on the value of the bank's assets.

directly as a result of deposit insurance. In contrast, as subsequently discussed, we assume that in the current deregulated environment, banks pay market riskless interest rates on their deposits. The relationship that MS use between the volatility of returns on equity and that of returns on total assets has a constant continuous dividend factored into it, which ignores the fact that equity, being the recipient of dividends, is fully dividend protected.[4] Further, in arriving at the pre-insurance value of the assets, MS assume that the market value of the debt is equal to its face value. This may partly account for their low estimates of the deposit insurance premium since, with the treatment of debt as riskless, the post- and pre-insurance values of the assets are artificially inflated, thus reducing the value of the limited-liability put. In contrast, by simultaneously solving two equations—the first relating equity as a call to the post-insurance value, and the second relating the variance of the equity and that of the value—not only were we able to *solve* for the underlying *market* value of assets, but in the process we were also able to explicitly build into our model the bailing-out effect implied by the FDIC policies of Direct Assistance and Purchase and Assumption. Thus, in the context of our model, given the role of the FDIC as that of preventing bank runs by containing disruptive effects of an individual bank failure, it is somewhat meaningless to talk of a fair premium without first considering the extent of FDIC bail-out effort. We model this effort explicitly in our model below.

In addition, another reason why both the historical loss rate estimates earlier discussed and MS estimates of the FDIC premium are low is that these estimates have been arrived at in circumstances where the risk of the insured banks was being kept at artificially low levels by the very regulations that the risk-adjusted insurance seeks in large measure to do away with. However, even when the bulk of regulations designed to control the quality of assets is dismantled by introduction of risk-adjusted premiums, the bailing-out policies will survive since these stem from a policy of avoiding economy-wide bank runs. Thus, the FDIC will still be expected to fulfill the social welfare purpose of bank-run avoidance. Therefore, we will still need to model the bailing-out effect for estimating the risk-adjusted premium. Naturally, deregulation brought about in the wake of risk-adjusted deposit insurance will change the risk-taking behavior of the banks, in turn changing the riskiness of the assets, and finally, the insurance premium based on the risk of the assets.

This paper is organized as follows. Section I presents the analytical derivation of the model. Section II describes the data and presents point estimates of deposit insurance premium; this section also contains a number of sensitivity analyses. Section III concludes by suggesting some interesting extensions of the current analysis.

[4] There is a subtle asymmetry between the valuation of equity as a call option and insurance as a put option which MS have not fully appreciated. Whereas equity must be modeled as a dividend-protected call by virtue of its receiving dividends, the put option is indeed written on the assets *less* the dividends paid out. This modeling of the put option follows from the insurer's inability to recapture the dividends once they have been paid out. MS correctly modeled the put option, but in relating the equity volatility to asset volatility [their Equation (3)], they did not properly account for the dividend-protection aspect of equity.

I. The Model

A. Analytic Derivation

As shown in Merton [13], insuring a single, homogeneous-term debt issue against default of payment of principal and interest is equivalent to acquiring (from the insurer) a European put option on the value of the bank before deposit insurance.[5] In this isomorphic relationship, the maturity of the put option is the same as that of the debt issue, and the striking price is equal to the maturity value of the debt. It was further convincingly argued in that paper that this isomorphic relationship could reasonably be applied to deposit insurance even though the assumption of a single homogeneous-term debt issue was not strictly valid for banks issuing mostly demand deposits. This was done by reinterpreting the maturity of the debt as the length of time until the next audit of the bank's assets by the insurer. Further, if we make the standard assumptions of the Black-Scholes option pricing model (see, e.g., Merton [12]), then we have an analytic representation of the value of deposit insurance.

Our model begins with the following notation:

V = the unobserved post-insurance value of the bank's assets
B_1 = the face value of the insured deposits
B_2 = the face value of all debt liabilities other than the insured deposits
$B \equiv B_1 + B_2$ = face value of total debt liabilities
σ_V = the instantaneous standard deviation of the rate of return on the value of the bank's assets
T = time until next audit of the bank's assets
δ = dividend per dollar of value of the assets, paid n times per period.

Now, assuming all pre-insurance debt to be of equal seniority, holders of deposits would be entitled to either the future value of their deposits, or to a prorated fraction of the value, should the value be less than total debt. In other words, they will receive

$$\min\left\{FV(B_1), \frac{V_T B_1}{B_1 + B_2}\right\}$$

upon maturity of the debt, where $FV(\cdot)$ denotes the future value operator, and V_T is the terminal value of the bank's assets. Thus, the maturity value of deposit insurance is given by

$$\max\left\{0, FV(B_1) - \frac{V_T B_1}{B_1 + B_2}\right\}.$$

Then, following Merton [13], the value of the insurance is equivalent to the value of a put, written with a striking price equal to *total* debt, and then scaled down

[5] As noted in the introduction, an attempt to evaluate the put option from observable equity requires the estimation of the put from *post*-deposit insurance values. This distinction from Merton's modeling will be made explicit in the text.

by the proportion of demand deposits to total debt, B_1/B.[6] And, therefore, the *per dollar* deposit insurance premium, denoted d, is then given by

$$d = N(y + \sigma_V\sqrt{T}) - (1 - \delta)^n(V/B)N(y) \quad (1)$$

where

$$y \equiv \frac{\ln[B/V(1 - \delta)^n] - \sigma_V^2 T/2}{\sigma_V\sqrt{T}}$$

and $N(\cdot)$ is the cumulative density of a standard normal random variable.

It may first be noted that the per-dollar deposit insurance premium does not depend directly on the risk-free rate of interest. In the Black-Scholes option pricing expression, the risk-free rate of interest enters only in the factor with which the striking price is discounted. Alternatively, it is only the *present value* of the striking price that is relevant to Black-Scholes option pricing, and since the face value of the debt, B, *is* the present value of the striking price in our context, the risk-free interest rate will not appear in our computation of per-dollar deposit insurance premium.[7] Naturally, the interest rate can indirectly affect the cost of deposit insurance via its effects on two of the premium's direct determinants: the value of the assets, V, and its associated volatility, σ_V.

Second, at the time of entering into the insurance contract, the insurer is concerned with the future stochastic behavior of the assets—in other words, with the behavior of assets after the insurance—because, once the insurance is issued, the FDIC incurs out-of-pocket expenses only if the terminal value of the bank's assets *after* insurance, V_T, is less than B. Moreover, within the regulatory framework, submission to which is associated with purchase of insurance, banks choose a risk-taking behavior that characterizes the parameters of the stochastic process for the assets from then on into the future. As the insuring agency is

[6] To see this, note that

$$P = B_1N(y + \sigma_V\sqrt{T}) - \frac{(1 - \delta)^n VB_1}{B_1 + B_2}N(y)$$

where

$$y \equiv \frac{\ln\left[\frac{B_1}{(1 - \delta)^n VB_1/(B_1 + B_2)}\right] - \sigma_V^2 T/2}{\sigma_V\sqrt{T}}$$

$$= \frac{\ln[B/V(1 - \delta)^n] - \sigma_V^2 T/2}{\sigma_V\sqrt{T}}.$$

Defining $d \equiv P/B_1$ yields Equation (1) in the text.

[7] We are implicitly assuming that all debt is issued at the risk-free rate of interest. The assumption is doubtless valid for the insured deposits, which for most banks account for a substantial portion of the total debt. Since the remaining debt is not riskless, we should ideally compound it by the actual rate paid and discount it back by the risk-free rate. Empirically, the effect of not doing this will be the understatement of d. However, the premium over the risk-free rate for the remaining small portion of the total debt will have a negligible effect on the value of deposit insurance, particularly as option values are not very sensitive to small changes in the interest rate parameter.

aware of this, it prices its insurance policy according to σ_V, where σ_V is the volatility of the assets *after* insurance.

Letting V' denote the value of the assets *before* insurance, the value of the assets *after* insurance, V, will be given by

$$V = V' + P(V) - C$$

where $P(V)$ stands for the accretion to value on account of the insurance,[8] and C denotes the reduction in the value due to competition. Competition in the industry may lead banks to pass some or all of the accretion in value to their clientele. Deposit insurance *per se* cannot erode banks' assets. Whatever bank-specific managerial inefficiencies exist, they are not attributable to deposit insurance. Thus, at the extreme, the entire value of the insurance will be eliminated by competition; thus $P(V) \geq C$. Now, if $P(V) > C$, then banks are retaining at least a part of the insurance subsidy, whereas if $P(V) = C$, then all of the subsidy is being eliminated by competition to the eventual benefit of the depositors or of the borrowers, depending on the relative extent of competition in the two markets.[9] Thus, whether $C = 0$ or $P(V) - C = 0$, the possible (cross-sectional) mispricing of deposit insurance leads to distortion in borrowing markets, lending markets or both. Under the assumption of one-period[10] put valuation for $P(V)$, the proper method of eliminating these distortions is by charging d as given in equation (1), since with proper pricing no accretion occurs in the value and $V = V'$.

In conclusion, recognition of the fact that it is the post-insurance value that bears upon the price of insurance circumvents the question of the extent and incidence of the subsidy. While the nature and degree of competition in the banking industry will determine whether, and to what extent, the bank itself, or the clientele of the bank, eventually benefits from the mispricing subsidy, the relationship between the value of the assets before and after the insurance does not affect the price of insurance which depends only on the value after the insurance.

Finally, the per-dollar price of the insurance as given by (1) depends on total debt, B, rather than on the insured debt, B_1. As discussed above, we have motivated equation (1) by an appeal to equal seniority of debt, which entails

[8] The value of P follows from the (subsidized) risk-free interest rate, r, which the banks pay to holders of demand deposits. In the absence of deposit insurance, banks would pay a promised rate $R > r$. Thus, for a promised face value, F, and a one-period maturity, the value of deposit insurance is $F[\exp(-r) - \exp(-R)]$. Moreover, as newly issued deposits are automatically insured, this amounts to a tangible savings in cost of funds.

[9] Kane [10] argues that the subsidy was passed on to mortgage borrowers and less developed countries in the seventies, and to money market account holders in the early eighties when the sum of implicit and explicit returns on these accounts exceeded the Treasury bill or money market fund returns. More recently, this spread of explicit return in excess of money market returns has dwindled. He goes on to conjecture that the subsidy may also explain restored popularity of fixed-rate mortgages and a smaller premium on these than is justified by their prepayment and rollover options.

[10] Even though the insurance is renewed in most cases, the renewal is by no means unconditional. Between 1971 and 1982, the FDIC issued 71 termination-of-insurance notices. The FDIC also has an opportunity to renegotiate its terms by imposing capital adequacy and portfolio restrictions, and thus we may think of the insurance as a one-period put.

proration of assets should the value fall below the total debt. However, in practical terms, an assumption of equal seniority is not strictly necessary, because we may validly argue that given existing bailing-out practices of the FDIC, *de facto* insurance extends to all liabilities of an insured bank. This then constitutes an alternative motivation for the valuation of d in Equation (1).

For instance, FDIC's Purchase and Assumption procedure consists of replacing a bank's "bad assets" by cash before all liabilities—including uninsured deposits—of the distressed bank are assumed by a new or existing bank. The FDIC report [6] states that as a consequence of this procedure "no general creditor incurs any loss." Also, the report goes on to state: "On a few occasions the FDIC has provided direct assistance to banks that were open but would otherwise have failed. Recently it has also provided direct assistance to facilitate open-bank mergers of failing savings banks. In these transactions, like Purchase and Assumptions, *all* depositors are made whole [emphasis added]." Either procedure has the merit of helping accomplish one of the primary objectives of Federal Deposit Insurance, namely, that of providing monetary stability by preventing or minimizing wider secondary disruptive effects of a bank failure. Thus, while the contractual obligations of the FDIC are limited to insured deposits, *de facto* insurance coverage extends to all debt liabilities, insured as well as uninsured. This is implicitly acknowledged in the report, which states (p. xv): "[A reduction in de facto insurance coverage] could be accomplished if the FDIC were simply to abandon use of the P&A and direct assistance procedures, and follow a policy henceforth of only paying depositors in failed insurance banks the amounts of their deposits up to the statutory ceiling of $100,000."

As discussed earlier, the chief obstacle to empirical application of the model lies in the fact that neither the true value of the firm, V, nor its instantaneous volatility, σ_V, can be empirically observed.

In the context of our model, the equity of a firm can be represented as a call option on the value of the assets of the firm with the same maturity as that of the debt of the firm and with a striking price equal to the maturity value of the debt (see Black and Scholes [1]). Thus, with the assumption of Black-Scholes option pricing, and letting E stand for the equity of the bank,

$$E = VN(x) - BN(x - \sigma_V\sqrt{T}) \tag{2}$$

where

$$x \equiv \frac{\ln(V/B) + \sigma_V^2 T/2}{\sigma_V\sqrt{T}}$$

and

$$\sigma_E = \frac{V\left(\dfrac{\partial E}{\partial V}\right)}{E}\sigma_V \tag{3}$$

where σ_E is the instantaneous standard deviation of the return on E.[11]

[11] In the framework of a similar model, Christie [4] examined the relationship between a nonstationary σ_E and leverage and other explanatory variables. We defer to Section II.D a discussion of the nonstationarity of the equity return series (arising from modeling equity as a call on the asset value, V, the time series for which is assumed to be stationary).

It may be noted that in contrast with the valuation of the per-dollar put in Equation (1), Equation (2) models equity as a fully dividend-protected call because being the *recipient* of dividends, equity is in fact dividend-protected.

B. Modeling Direct Assistance and Purchase and Assumption

As earlier quotations from the FDIC report bear out, the FDIC does not as a first recourse step in to liquidate a bank's assets when it observes that its net worth has been fully eroded and that the value has fallen below the total debt. Rather, it tries to revive the concerned bank either by direct infusion of funds, or by what amounts to it, i.e., a temporary reprieve from closure. It is reasonable to suppose, however, that there will be a hypothetical limit beyond which erosions in value, should they occur, would make the revival efforts excessively costly, and therefore beyond which dissolution of assets would be the only feasible alternative. Let this hypothetical limit be expressed as a percentage of the total debt of the bank, i.e., as ρB where $\rho \leq 1$. Therefore, if the value of the bank happens to fall between ρB and B, the insuring agency infuses up to $(1 - \rho)B$ to make the value equal to B, while should the value fall below ρB, it steps in to dissolve the assets of the bank.

While ρ can conceivably be estimated from past histories of failure, or near-failure where direct assistance or Purchase and Assumption options were resorted to, essentially ρ is a policy parameter and is accordingly difficult to estimate empirically. However, the insuring agency will be in a position to decide upon its value by balancing the additional risk it exposes itself to against the objective of preventing wider secondary and possibly disruptive repercussions of a bank failure. Pyle [15], however, has observed that it may not be possible to decide on ρ *ex ante*. This is because its value depends on the nature and scope of disruption that a particular closure may bring about in its wake, which in turn will depend on the bank concerned, and perhaps on the economic conditions obtaining at the time the FDIC is actually confronted with the closure decision.

With this modified closure condition,[12]

$$E = VN(x) - \rho BN(x - \sigma_V\sqrt{T})$$

where now

$$x \equiv \frac{\ln(V/\rho B) + \sigma_V^2 T/2}{\sigma_V\sqrt{T}} \tag{2'}$$

and

$$\sigma_V = \frac{\sigma_E E}{VN(x)}, \tag{3'}$$

and given the solution pair (V, σ_V), we now arrive at the risk-adjusted deposit insurance premium, d.

The FDIC's obligation is modeled as writing a European put option, with an exercise price of $FV(B)$, where $FV(\cdot)$ denotes the future value operator at $T \equiv$

[12] This modified closure rule chiefly distinguishes our model from Merton's, whose theoretical article did not attempt an analysis of closure rules simulating FDIC behavior.

1. This put option is contrasted to equity holders' call options, with $K = \rho FV(B)$. The asymmetry between the two is deliberate; when $\rho FV(B) \leq V_T < FV(B)$, the FDIC provides payment of $FV(B) - V_T$, but equity holders retain ownership of the firm.

The modeling implicit in this analysis is an approximation to the FDIC's option of closing down the banking operation *at any time* $t \leq T$, if $V_t < FV_t(B)$, where $FV_t(\cdot)$ denotes the future value operator for any time $t \leq T$.[13] In this case, optimal behavior—equivalent to loss-minimizing FDIC action—would imply bank closure when $V_t = FV_t(B)$. The rationale for such optimality is simple: at best, the FDIC can gain nothing; at worst, it might *lose* $\max\{0, V_t - FV_t(B)\}$. Thus, it should take whatever action possible to ensure against a positive loss. This is done by exercising its bank closure option at $V = FV_t(B)$.

As posited in the text, the FDIC is loath to invoke bank closure for political, statutory, and institutional reasons. In this study, then, the FDIC's policy is modeled as approximated by a "European put policy": closing the bank if $V_T = \rho FV(B)$, while providing direct assistance if $\rho FV(B) \leq V_T \leq FV(B)$. In this context, Campbell and Glenn [3] have noted that it is improper to invert market data for true value since market perceptions are valuations of contingent claims, and therefore market prices depend on the legal rather than economic definition of bankruptcy. Also, Horvitz [8] has observed that the outcome of allowing economically insolvent institutions to keep functioning cannot be factored into risk-adjusted pricing. Our modeling can therefore be viewed as an attempt to overcome these difficulties. The effect of FDIC policies is to allow operation with a negative economic net worth up to a certain point. ρ defines this point in units of B.

II. Point Estimates of Deposit Premium

A. Data

The empirical study was performed for a sample of 43 banks (listed in Table I) for which a full set of data is available on the Daily Return CRSP tape *and* on the Quarterly Compustat Tape for Banks for 1983.[14] Total domestic deposits, and the sum of total domestic deposits and total borrowings, were used as the empirical counterparts of B_1 and B, respectively.

For holding companies in our sample, the data that we have on equity and capital structure relate to the entire holding company. Therefore, to the extent that our model fails to take into consideration that not all of the assets held by the holding company are available to the creditors of the subsidiaries, the estimates of the insurance premium that we obtain will be biased. If the equity

[13] For a constant interest rate r, $FV_t(B) = B\exp(rt)$ and $FV(B) \equiv FV_1(B) = B\exp(r)$.

[14] Equimark Corporation turned out to have an estimate of variance much larger than the rest of the sample, which tended to have a disproportionate effect on the industry-wide estimates of the deposit insurance premium. Also, the volatility of equity returns for Bancal Tri State Corporation registered a sharp decline, from 74 percent in the third quarter of 1983 to 5 percent in the fourth quarter of 1983. Much of this unusual fall may have been due to absence of trading. Accordingly, we decided to exclude these two from the sample.

ratio is higher for the holding company than for its subsidiary banks, then the insurance premium will be downward-biased. In the following analysis, we are calculating the premium for the holding companies under the assumption that the assets and liabilities issued directly by the holding company are not significant in magnitude relative to those of the subsidiary.

B. Empirical Findings

Equations (2′) and (3′) of the previous section can be solved simultaneously for two unknowns, V and σ_V, by a numerical routine[15] for each observed E and σ_E (where the latter is estimated from daily return time series for the concerned quarter). Then, given the solution pair (V, σ_V), an estimate of the deposit insurance premium can be computed using equation (1).

We followed this procedure for each of the sample banks on a quarterly basis over the study period. Again, we set the value of T at 1 year, implicitly assuming that, in purchasing deposit insurance, banks buy a net put every quarter with a maturity of 1 year, and that the debt is rolled over every quarter so that its maturity at the beginning of each quarter is 1 year.[16]

In our current analysis, we have assumed a fixed, known, cross-sectionally constant ρ. However, as we further discuss in Section III, uncertainty in ρ can be analyzed using Fischer's [7] model. For the present study, we experimented with various values of ρ between 95 and 98 percent, which appeared at first glance a not unreasonable range for its value. A value of 0.97 yielded[17] an aggregate deposit premium weighted average[18] of a little less than 1⁄12 percent.[19]

[15] We used subroutine ZSCNT of the International Mathematical and Statistical Library (IMSL). In a trial run, we found that the convergence is not sensitive to the initial estimates of the solution. The initial estimate that we used for the value, V, was the sum of the market value of equity and face value of the debt, while that for σ_V was σ_E scaled down by the leverage ratio.

The values of $N(\bullet)$ were obtained using the polynomial approximation

$$N(z) = 1 - \frac{1}{\sqrt{2\pi}}[\exp(-z^2/2)](a_1k + a_2k^2 + a_3k^3 + a_4k^4 + a_5k^5) \quad \text{for} \quad z > 0$$

where

$$k = \frac{1}{1 + pz}$$
$$p = 0.2316419; \quad a_1 = 0.31938153; \quad a_2 = -0.356563782;$$
$$a_3 = 1.781477937; \quad a_4 = -1.821255978; \quad a_5 = 1.330274429;$$

and obtaining $N(z)$ for $z < 0$ by symmetry, $N(0)$ being exactly equal to one-half. See, e.g., Cox and Rubinstein [5].

[16] See Section II.C.

[17] Spearman's Rank Correlation between ranks with $\rho = 0.97$, and those with ρ between 0.98 and 1.00, varies from 0.967 to 0.829, respectively. The fact that rank ordering, although still largely unchanged, is somewhat less robust to changes in ρ than it is to other changes in specification is due to the impact that a change in ρ has on *both* the effective leverage and estimates of asset volatility.

[18] Each bank's deposit premium was weighted by B_1, so that the FDIC's weighted average deposit premium equaled 1⁄12 of 1 percent.

[19] In fact, after properly accounting for the credit rebates, the FDIC premium averaged close to 1⁄30th of 1 percent over the period from 1974 to 1983. The FDIC gives back 60 percent of its profits as rebates.

Table I

Risk-adjusted Deposit Premium, 1983

Ranked in Descending Order of 1983 Average Annual Deposit Premium ($\rho = 0.97$)

	Quarter I, 1983				Quarter IV, 1983					
Bank Name	Market Value of Assets	Face Value of Total Debt	σ_V	Deposit Premium	Market Value of Assets	Face Value of Total Debt	σ_V	Deposit Premium	Average Annual Deposit Premium	Average Annual σ_E
First Pennsylvania Corp.	4048	4094	1.03	1.2107	3857	3866	0.99	0.5122	0.7241	46.2
Crocker National Corp.	18543	18507	0.61	0.2789	15247	15195	1.43	0.5601	0.2666	26.6
Banctexas Group, Inc.	1761	1688	2.85	0.1069	1969	1905	4.57	0.6836	0.2398	46.5
Money Management Corp.	268	266	1.28	0.2486	303	297	1.69	0.1365	0.2007	36.4
Continental Illinois Corp.	22968	22767	1.43	0.3452	22289	22073	0.98	0.1593	0.1944	26.4
Wells Fargo and Co.	20859	20802	1.10	0.4121	22200	21911	1.08	0.0870	0.1838	28.1
Marine Midland Banks, Inc.	11065	10943	1.32	0.2050	12083	11969	1.05	0.1515	0.1405	27.5
First Chicago Corp.	19362	19056	1.96	0.2919	19202	18706	1.51	0.0399	0.1375	34.1
Manufacturers Hanover Corp.	31022	30445	1.43	0.1050	34626	34313	1.09	0.2002	0.1269	27.4
Harris Bankcorp., Inc.	5346	5285	1.23	0.1746	5788	5487	0.87	0.0000	0.1234	29.2
First City Bancorp. of Texas	12975	12696	1.83	0.1510	13929	13732	1.40	0.1679	0.1186	31.9
Bankamerica Corp.	68185	67002	1.60	0.1676	74642	73714	0.99	0.0895	0.1035	27.1
Interfirst Corp.	16354	15759	3.20	0.2621	17377	16770	2.10	0.0634	0.0985	35.8
General Bancshares Corp.	1493	1444	1.56	0.0135	1749	1689	3.68	0.3674	0.0965	28.1
First Interstate Bancorp.	33767	33337	1.54	0.2388	37039	36405	0.95	0.0245	0.0856	26.9
First Wisconsin Corp.	4260	4084	1.47	0.0015	4335	4291	1.13	0.1599	0.0748	25.3
Chase Manhattan Corp.	35686	34912	1.60	0.0992	36184	35674	0.99	0.0694	0.0577	24.4
Bankers Trust NY Corp.	22292	21818	1.75	0.1326	20996	20266	1.56	0.0113	0.0568	29.0
First National State Bancorp.	4163	4083	1.17	0.0463	5814	5745	0.66	0.0225	0.0540	21.3
Southwest Bancshares, Inc.	6127	6003	1.54	0.1085	6929	6705	1.54	0.0161	0.0508	28.7
Citicorp.	61786	58432	3.26	0.0784	66129	63407	2.32	0.0477	0.0440	33.2
Norwest Corp.	16040	15743	1.40	0.0940	17773	17353	1.18	0.0176	0.0425	25.6
First Virginia Banks, Inc.	1967	1833	2.39	0.0022	2258	2113	2.26	0.0021	0.0422	31.9
Sterling Bancorp.	491	461	2.39	0.0079	541	505	3.91	0.0936	0.0391	33.5
Chemical NY Corp.	32001	31046	1.77	0.0469	32754	31718	1.71	0.0311	0.0270	27.1
First Wyoming Bancorp.	584	557	2.08	0.0155	706	674	2.39	0.0342	0.0245	30.1

Security Pac Corp.	27736	26992	1.57	0.0407	29699	28682	1.55	0.0109	0.0162	24.3
Mellon National Corp.	14405	13910	1.83	0.0344	19864	19122	1.75	0.0148	0.0157	25.1
Bank of Virginia Co.	3056	2991	1.16	0.0257	3283	3181	1.27	0.0052	0.0153	21.6
NCNB Corp.	9041	8723	1.66	0.0150	10301	9890	1.68	0.0067	0.0129	25.6
Barnett Banks of Florida, Inc.	7742	7540	1.43	0.0271	9119	8771	1.13	0.0002	0.0109	21.8
Bank of Boston Corp.	10040	9580	2.18	0.0204	10690	10231	1.92	0.0132	0.0106	25.2
Texas Comm. Bancshares, Inc.	14120	13386	2.54	0.0228	16308	15439	1.87	0.0015	0.0092	27.4
Mercantile Texas Corp.	8742	8241	2.16	0.0036	10598	10235	1.22	0.0018	0.0088	24.4
Citizens First Bancorp.	878	815	2.01	0.0001	1096	1061	1.68	0.0251	0.0071	20.4
First Atlanta Corp.	4238	3954	2.29	0.0012	5016	4819	1.89	0.0165	0.0060	26.5
Fleet Financial Group, Inc.	3425	3285	1.26	0.0006	4204	3967	0.91	0.0000	0.0056	19.3
Bank of New York, Inc.	8885	8323	1.97	0.0004	8689	8458	1.27	0.0147	0.0045	22.1
Norstar Bancorp., Inc.	3855	3650	1.77	0.0017	5123	4814	1.32	0.0000	0.0016	21.3
Republic NY Corp.	5229	4864	2.70	0.0050	4225	3851	2.21	0.0000	0.0015	23.5
Irving Bank Corp.	10189	9695	1.69	0.0016	10326	9693	1.25	0.0000	0.0004	17.3
Wachovia Corp.	5945	5522	2.45	0.0016	6807	6299	1.70	0.0000	0.0004	19.2
Morgan, J. P. and Co., Inc.	31183	29062	1.99	0.0003	28913	26981	1.71	0.0000	0.0001	18.4
Weighted average				0.1380				0.0784	0.0808	

Note: σ_V = annualized standard deviation of rate of return on assets; σ_E = annualized standard deviation of rate of return on equity.

The results for $\rho = 0.97$ are contained in Table I, which lists each bank's average d for 1983. It may be noted that this value of ρ gives an aggregate weighted average of 0.0808. Variation across banks indicates the extent to which certain banks in our sample can be said to have been subsidizing the higher risk that the FDIC was exposed to because of certain others.

Figure 1 displays the empirical distributions of d for the fourth quarter of 1983. It can readily be seen that the distribution is quite skewed; most banks' deposits are relatively "safe," and the flat deposit premium implies that this majority is subsidizing a "risky" minority of the financial institutions analyzed here.

It is reasonable to expect that, taking into account costs of regulation and other such considerations as historical loss experience for the entire industry, the insurer will be able to arrive at an industry aggregate of the total deposit insurance premium. The analysis in Table I can then be interpreted as suggesting how it can be allocated across banks. Moreover, the *magnitudes* of the premiums presented in Table I are sensitive to changes in the value of ρ as well as to other modifications discussed in the following subsections. However, as we have documented in these subsections, the *ranks* of the institutions prove relatively robust to such changes.

C. The Issue of Maturity

In the previous sections, where empirical results were presented, we assumed that the value of T both in equation (1) and in equations (2′) and (3′) was one year. It must at the outset be pointed out that while the two maturities, that of the equity construed as a call on the value, and that of the deposit insurance as a put on a per-dollar basis, are conceptually different (the former referring to the

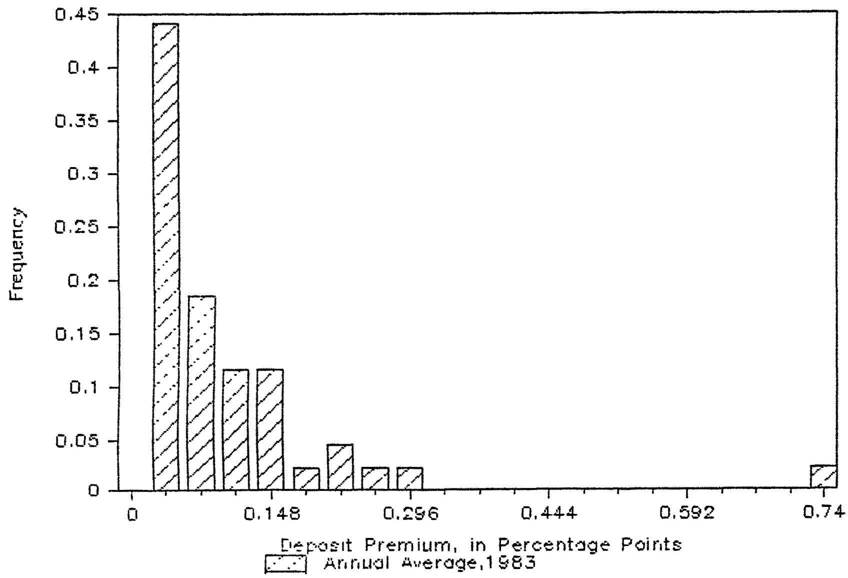

Figure 1. Distribution of Deposit Premiums

maturity of the debt, while the latter being more naturally associated with the periodicity of audit by the insurer), the two are intricately linked. In fact, previous authors have not even attempted to explicitly distinguish between the two. While Merton [13] argues for interpreting the maturity of the put which the insurance represents as the length of time until next audit, Marcus and Shaked [11] have assumed it to be one year, arguing that it is the approximate periodicity of examination in reality.

Moreover, in the context of our model, it is essential that *stockholders* perceive the time to maturity to be equal to one year. The simultaneous estimation procedure inverts E to solve for V, *given* $T = 1$. If the equity holders' perceived T differs from unity, then V will be misestimated, resulting in a mispricing of deposit insurance. This assumption lies at the very heart of our analysis; if equity holders perceive the bank's common to be different from a call option, then the appropriate risk-adjusted d will be misestimated.[20] We argue below that rational investors would intimately link the debt maturity to the audit periodicity. Ideally, it would be desirable to obtain a *third* equation, one which would permit the simultaneous estimation of V, σ_V, and T.

The reason why the two maturities cannot be separated in the context of banks is that the insured deposits account for a large part of the banks' debt, and new deposits made with a bank before the expiration of the insurance are automatically covered by the insurance. At the time of the audit, therefore, if the FDIC decides to dissolve the bank, all depositors are paid off, and it is therefore at the time of the audit that the boundary value assumed by the total assets of the bank impinges on the value of the stockholders' investment. In other words, the boundary condition for the value of the equity as a call, i.e., $\text{Max}[0, V_T - \rho B]$, comes into effect at the time of the audit. It is therefore reasonable to argue that the time until next audit should be the proper value of maturity in both sets of equations.

We analyzed the sensitivity of our estimates of industry-wide weighted average of deposit insurance premium to changes in the value of T. Table II shows the behavior of this weighted average when T is varied between ¼ and 5 years. It may be noted that, while the industry average of the premium rises with the assumed value of the maturity, the ranking of the banks does not change dramatically as evidenced by high values of Spearman's rank correlation coefficient. However, the rank correlation with the $T = 1$ premiums gets progressively weaker as we extend the maturity.

Closely related with the issue of maturity is the problem of the proper way of annualizing the deposit insurance premium. Whatever may be the unobserved periodicity of audit, the premiums are at present paid annually, and the annual per-dollar premium of 1/12th of 1 percent is the only standard of comparison that we have. Inherent in this issue are all the complexities of term structure and risk-adjusted discounting. We are aware of the problematic nature of interpreting

[20] For example, we have previously argued that Marcus and Shaked [11] erroneously used book value of debt rather than market values. This assumption has the effect of artificially inflating V and reducing σ_V, since E is an empirical datum unchanged by their assumption. Thus, their assumption would result in a downward-biased estimate of d.

Table II

Industry-wide Weighted Average Deposit Insurance Premium

Time in Years (T)	Non-annualized Deposit Insurance Premium (d)	Spearman's Rank Correlation Coefficient with Ranks when $T = 1$
¼	0.0084	0.9909
1	0.0808	1.0000
2	0.2591	0.9926
3	0.4956	0.9795
4	0.7801	0.9601
5	1.1093	0.9428

the results in Table II. The problem arises from the fact that these one-time premiums provide payment for periods in excess of one year, and comparability would require some form of annualization. While we do not explicitly perform this annualization, the data in Table II convincingly document the sensitivity of the insurance premium to the time parameter T.

Further, we tried to gather data on the periodicity of audit. One major difficulty encountered is that it is not quite clear what exactly constitutes an audit. First of all, the authority to conduct on-site examinations is shared among the FDIC, the Federal Reserve, and the chartering authorities (the U.S. Comptroller of the Currency and State Banking Commissions). Therefore, given coordination and community of purpose among these agencies, an examination by any one of them should constitute an audit. Then, examinations differ according to the end with which they are performed. For instance, of 12,977 examinations performed by the FDIC in 1983, only 4,352 were what they termed safety and soundness examinations, while the remaining related to civil rights compliance, trust departments, etc.[21] Further, as documented by the FDIC Annual Report, 1983, the FDIC has lately adopted the policy of identifying problem areas from the financial analysis performed on the quarterly bank reports and addressing these specific problems "in short-term visits to the banks in place of more frequent full-scale examinations." As these short-term visits, should conditions warrant, could conceivably trigger a full-scale examination, it may be argued that the periodicity of audit should be related to the periodicity of such visits. By the same token, since these short-term visits are themselves occasioned by the analysis based on the quarterly reports, it may be argued in turn that the effective periodicity is a quarter. However, the quarterly report contains largely unaudited data. Therefore, apart from the fact that differing accounting practices make the information contained therein disparate across banks, there also arises the problem of deliberate or inadvertent misrepresentations, and of what disincentives against such misrepresentation at present exist.

In sum, the reasons why we assumed T to be one year were, first, that it helped

[21] FDIC Annual Report 1983, pp. 4–5.

us circumvent the problem of annualization in that we could directly arrive at the *annual* deposit insurance premium, and second, that the data on examinations are not easily available, and such data as are available are not unambiguous in their implication for audit periodicity. Moreover, our overall objective for this study was to demonstrate that it was feasible to distinguish among banks and that risk-adjusted deposit insurance premium could be reasonably estimated even within the constraints of readily computer-accessible data.

D. Non-Stationarity of the Variance of Equity

According to the basic assumptions of our model, the value of the bank follows a stochastic process with a stationary σ_V, which, together with the interpretation of equity as a call on the value, implies that σ_E is not stationary over time. Using a value of σ_E computed from the time series of daily returns on equity over a given quarter in simultaneously solving (2′) and (3′) was therefore at best an empirical approximation.[22] We accordingly found it necessary to check how our estimates of σ_V, arrived at through a simultaneous solution approach, compared with an alternative procedure (described below) which uses the information contained in the daily time series of equity returns without assuming stationarity of σ_E over the quarter.

Starting from the value of the equity at the end of the quarter, and discounting it backward successively with the daily time series of returns on equity, we can generate a time series of equity values, say E_t, for each day t of a given quarter. Then by assuming an arbitrary initial value for σ_V, say σ_0, we can generate a daily time series for value, say V_t, by inverting equation (2′) and compute the standard deviation of instantaneous returns on value from the generated time series V_t, which can then be used to revise the initial arbitrary σ_0. The process is iterated with the revised initial value for σ_V until a convergence occurs in the sense that what was used as the initial estimate for σ_V to generate the time series V_t does in fact turn out to be the standard deviation of the instantaneous returns in that time series.[23]

The methodology described in the previous paragraph assumed that, on each day t, E_t is valued as a European call option with a maturity of $T = 1$. To a certain extent, this valuation is inconsistent with rational expectations; the procedure is valuing equity as a one-year European call option with a series of

[22] As a matter of fact, volatility of equity return over any discrete interval $(t, t + \Delta t)$ is given by $(\sigma_V/V) \int_t^{t+\Delta t} E(s)\, ds/N[x(s)]$, where the functions E and $N(x)$, both having time as an argument, cannot be taken outside the integral.

[23] The algorithm for the iterative solution for σ_V is given as follows:

1. Set $\sigma_V = \sigma_0$.
2. Given E_t, generate V_t for each day t in quarter.
3. Calculate $SD[\ln(V_t/V_{t-1})]$.
4. Revise σ_0 until convergence is achieved.

The revised estimate for σ_V used in each successive iteration was halfway between the initial estimate used in the previous iteration and the standard deviation of the instantaneous returns in the time series of value generated by the previous estimate. The convergent solutions for σ_V are invariant to the initial estimate; only the number of iterations needed for convergence goes up if the initial estimate turns out to be too far from the convergent solution.

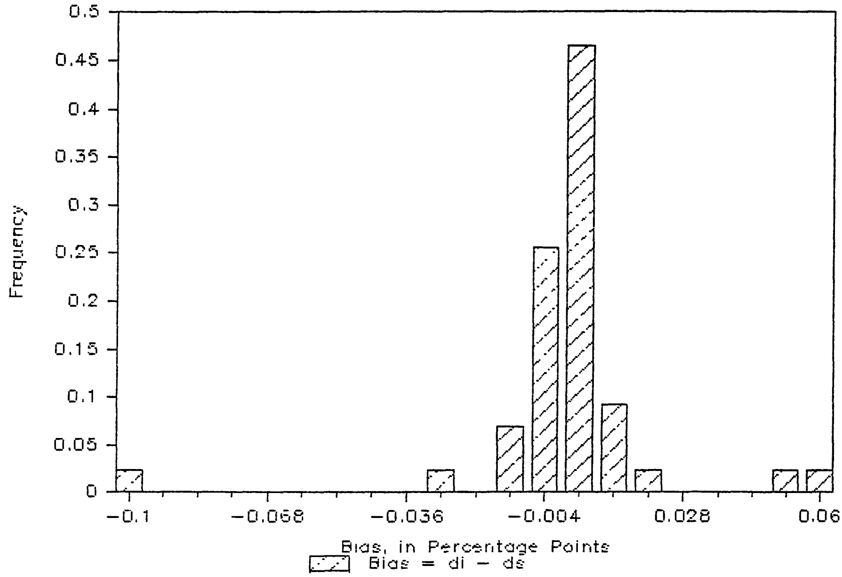

Figure 2. Estimation Bias in Deposit Premium

options to extend the maturity by one day. We argue that the error in valuation is not large. Note that the extension of maturity does not transfer much wealth to depositors, since banks may adjust the interest rate in accordance with changes in the risk-free interest rate. This motivates the conclusion that the one-year European call option is a "reasonable" valuation method.

We performed the above procedure for all the 43 banks in our sample for the last quarter of 1983. If we denote by σ_i the estimate of σ_V obtained from the iterative procedure described above, and by σ_s that obtained from the simultaneous equations approach, then the extent of percentage difference between the two, computed as $(\sigma_s - \sigma_i)/\sigma_i$, ranges between −10 and 7.5%. Despite the size of these differences, they do not greatly affect the industry-wide estimate of the deposit insurance premium, which goes down from 0.078 for the last quarter of 1983 if we use σ_s and its associated V's, to 0.074 if we use σ_i and its associated V's. Figure 2 depicts the distribution of differences in the insurance premium between the two approaches over the sample.[24] Further, Spearman's rank correlation coefficient between the ranks assigned to the banks on the basis of insurance premiums computed with σ_i and σ_s turns out to be as high as 0.9935, demonstrating the robustness of ranking.

E. Sensitivity to Specification of Stochastic Interest Rates

One of the standard assumptions of the Black-Scholes option pricing framework is that the interest rate changes over the life of the option are nonstochas-

[24] Either of the above procedures, however, only yields a point estimate for σ_V. Ideally, we should like to form a confidence interval for estimates of the insurance premiums. Performing this procedure would indicate the sensitivity of the model to misestimation of the parameters. See Pyle [14] for a discussion of the effects of mismeasurement.

tic.[25] Changes in the value of the banks' assets induced by stochastic changes in the interest rate are, however, captured in our specific application of the Black-Scholes model, since all factors driving V, the value of the assets, are presumed to be embodied *in toto* in the stochastic process postulated for the changes in value, dV, thus obviating enumeration and separate consideration of various sources of risk. However, since the value of the assets is not directly observed, but instead inferred from the market value of the equity, we found it of interest to see whether the estimates of the pair (V, σ_V), and ultimately those of the insurance premiums, are sensitive to a change in specification of the stochastic interest rate model, in which equity as a call is a function not only of the value of the assets, V, but also of another state variable, the discount factor, $D(T)$—the price of a pure discount loan T years from maturity, upon which it pays unity.

If we assume a lognormal diffusion process for $D(T)$ as well, with an instantaneous variance of $\sigma_D^2(T)$, an instantaneous covariance between the returns on V and D of $\sigma_{VD}(t)$, and serial independence between the returns on the two assets, then following the generalization of the Black-Scholes model by Merton [12], Equation (2′) for equity as a call continues to hold with $\sigma^2 T \equiv \int_0^T [\sigma_V^2 + \sigma_D^2(t) - 2\sigma_{VD}(t)]\, dt$ replacing $\sigma_V^2 T$.

Since the price of equity as a call now depends on V and D, we have, by applying Itô's Lemma to $E = E(V, D, t)$,

$$dE = E_t dt + ½[E_{VV}(dV)^2 + 2E_{VD}(dV)(dD) + E_{DD}(dD)^2] + E_V dV + E_D dD$$

and, as the first two terms are deterministic, the variance of the instantaneous return dE/E is

$$\sigma_E^2 dt = \left[\left(\frac{VE_V}{E}\right)^2 \sigma_V^2 + \left(\frac{DE_D}{E}\right)^2 \sigma_D^2 + 2\left(\frac{VE_V}{E}\right)\left(\frac{DE_D}{E}\right) \sigma_{VD}\right] dt$$

and, similarly $\text{cov}\left(\frac{dE}{E}, \frac{dD}{D}\right)$ is

$$\sigma_{ED} dt = \left[\left(\frac{VE_V}{E}\right) \sigma_{VD} + \left(\frac{DE_D}{E}\right) \sigma_D^2\right] dt$$

where subscripts to E denote partial derivatives.

Therefore, we can express σ_V^2 and σ_{VD} in terms of observable counterparts σ_E^2 and σ_{ED}, and obtain

$$\sigma_V^2 + \sigma_D^2 - 2\sigma_{VD} = \left(\frac{E}{VE_V}\right)^2 \sigma_E^2 + \left(1 + \frac{DE_D}{VE_V}\right)^2 \sigma_D^2 - 2\left(\frac{E}{VE_V}\right)\left(1 + \frac{DE_D}{VE_V}\right) \sigma_{ED}.$$

Further, since, as shown by Merton [12]

$$E = VE_V + DE_D,$$

[25] Since most of the large banks are effectively immunized, it may be argued that interest rate changes have relatively little effect on the value of their assets.

we have

$$\sigma_V^2 + \sigma_D^2 - 2\sigma_{VD} = \left(\frac{E}{VE_V}\right)^2 (\sigma_E^2 + \sigma_D^2 - 2\sigma_{ED}). \tag{4}$$

In contrast, for nonstochastic interest rates, we had

$$\sigma_V^2 = \left(\frac{E}{VE_V}\right)^2 \sigma_E^2. \tag{5}$$

Merton's specification for the dynamics of D leaves us free to assume any form of functional dependence of σ_D and σ_{VD} on the maturity T without impairing the validity of the model. On the day of maturity, however, both of these must equal zero. For the purpose of empirical estimation, we therefore assumed that $\sigma_D(t) = \sigma_D$ and $\sigma_{VD}(t) = \sigma_{VD}$ before maturity, where σ_D and σ_{VD} are constants,[26] so that

$$\sigma^2 = \sigma_V^2 + \sigma_D^2 - 2\sigma_{VD}.$$

In either equation (4) or equation (5), $E_V = N(x)$, where the variance parameter used for computing x is different in the two cases. However, for small values of the variance (which enters the denominator of x), $N(x)$ approaches unity in either case so that the difference between the two instantaneous variance parameters is $(E/V)^2(\sigma_D^2 - 2\sigma_{ED})$. Further, since most of the banks are very highly leveraged, E/V tends to be very small, $(E/V)^2$ smaller still, and because $(\sigma_D^2 - 2\sigma_{ED})$ is not expected to be very large in magnitude either, for all practical purposes, we would expect the product $(E/V)^2(\sigma_D^2 - 2\sigma_{ED})$ to be almost negligible. Analytically, therefore, we would not expect a change in specification to stochastic interest rates to make a perceptible degree of difference in our estimates of the deposit insurance premium.

Constructing pure discount loans from the daily series of 12-month Commercial Deposit rates available on the Berkley Options Data Base for the year 1979[27] to estimate σ_D^2 and σ_{ED}, we repeated the empirical analysis for the stochastic interest rate model. The comparative results are presented in Table III. As can be readily seen, the foregoing analysis is borne out, and the estimates of the variance parameter and the deposit insurance premium for all the 32 banks in the sample are almost identical.

The foregoing empirical results should not be interpreted to mean that, whether on account of effective immunization or otherwise, the variance of the commercial banks' assets remains unaffected by explicit consideration of interest rate changes as a source of risk. As mentioned earlier, σ_V^2 as a measure of risk captures the magnitude of the factors bringing about a change in value, *including* interest rate changes. However, one possible manner in which we can assess the relative contribution of interest rate risk to overall variance of the assets is through

[26] Assuming that the *instantaneous* standard deviation is constant implies that the variance of the return over time from t until maturity at T is given by $(T - t)\sigma_D^2$, which decreases with time and reaches 0 at maturity. A similar argument applies to σ_{VD}. Further, as in the deterministic analysis for estimating σ_E^2, so in this section σ_E^2 and σ_{ED} were estimated as if dE/E were a stationary process, which ignores the dependence on time induced by the presence of (E/VE_V). In Section II.D, we found that this approximation does not seriously bias the results.

[27] This is the most recent year for which such data are available on the data base.

Table III

Contrast of Deposit Premiums under Deterministic and Stochastic Interest Rates

$\rho = 0.97$

Average Annual Values, 1979

Bank Name	Deterministic Rates: Deposit Premium	Deterministic Rates: σ_V	Stochastic Rates: Deposit Premium	Stochastic Rates: σ
Bank of New York, Inc.	0.0157	1.576	0.0156	1.580
Bank of Virginia Co.	0.2583	0.752	0.2580	0.750
Bankers Trust NY Corp.	0.0318	1.143	0.0299	1.149
Chemical NY Corp.	0.0050	1.103	0.0045	1.127
First Pennsylvania Corp.	0.4659	0.940	0.4673	0.945
First Virginia Banks, Inc.	0.0749	1.554	0.0752	1.554
First Wisconsin Corp.	0.0037	1.031	0.0037	1.027
General Bancshares Corp.	0.0997	0.619	0.0980	0.615
Harris Bankcorp., Inc.	0.1116	0.914	0.1107	0.914
Marine Midland Banks, Inc.	0.7571	0.765	0.7568	0.764
Morgan, J. P. and Co., Inc.	0.0004	1.352	0.0003	1.351
Southwest Bancshares, Inc.	0.0006	1.326	0.0007	1.320
Bank of Boston Corp.	0.0017	1.427	0.0015	1.439
Bankamerica Corp.	0.0027	1.439	0.0022	1.429
Chase Manhattan Corp.	0.1052	0.946	0.1022	0.943
Citicorp	0.0129	1.818	0.0116	1.818
Continental Illinois Corp.	0.0100	1.035	0.0095	1.047
Crocker National Corp.	0.2441	0.654	0.2473	0.662
First Chicago Corp.	0.0931	1.357	0.0921	1.355
First City Bancorp. of Texas	0.0000	2.049	0.0000	2.081
First Interstate Bancorp.	0.0469	1.061	0.0456	1.059
First National State Bancorp.	0.1328	0.770	0.1373	0.787
Fleet Financial Group, Inc.	0.0973	1.113	0.0961	1.114
Interfirst Corp.	0.0002	3.673	0.0001	3.699
Irving Bank Corp.	0.0034	0.986	0.0029	0.983
Manufacturers Hanover Corp.	0.0474	0.818	0.0461	0.822
Mercantile Texas Corp.	0.0014	2.113	0.0013	2.103
Norwest Corp.	0.0028	1.229	0.0030	1.238
Republic NY Corp.	0.0000	2.779	0.0000	2.783
Texas Community Bancshares, Inc.	0.0001	2.611	0.0001	2.627
Wachovia Corp.	0.0111	1.146	0.0112	1.155
Wells Fargo and Co.	0.0636	0.860	0.0622	0.857
Weighted average	0.0643	1.312	0.0637	1.315

Note: σ_V = annualized standard deviation of rate of return on assets; σ = annualized volatility "priced" under stochastic interest rate model.

studying the behavior of σ_V^2 in different interest rate regimes, i.e., over different periods with significantly different interest rate volatilities. Although we would ideally wish to hold non-interest rate effects constant, a preliminary probe into this aspect was made possible by the fact that, in our study, the interest rate risk, as measured by σ_D, jumped from 1.43% in the first quarter of 1979 to 4.32% in the third quarter of 1979, a three-fold increase. Table IV shows how σ^2 and σ_V^2 changed for each bank from the first quarter to the third quarter of 1979.

Table IV

Comparisons of Volatilities under Deterministic and Stochastic Interest Rates

$\rho = 0.97$; Year = 1979

	Quarter I $\sigma_D = 1.43\%$			Quarter III $\sigma_D = 4.33\%$		
	Deterministic		Stochastic	Deterministic		Stochastic
	σ_V	σ	σ_V	ρ_V	ρ	ρ_R
Bank of New York, Inc.	1.04	1.05	1.67	2.17	2.16	4.74
Bank of Virginia Co.	0.70	0.70	1.60	0.59	0.60	4.21
Bankers Trust of NY Corp.	0.58	0.59	1.42	1.23	1.30	4.04
Chemical NY Corp.	1.37	1.39	1.77	1.00	1.09	3.96
First Pennsylvania Corp.	0.55	0.53	1.63	0.76	0.78	4.18
First Virginia Banks, Inc.	1.48	1.50	1.84	1.57	1.54	4.62
First Wisconsin Corp.	1.19	1.20	1.73	1.03	1.05	4.25
General Bancshares Corp.	0.53	0.52	1.60	0.71	0.72	4.27
Harris Bankcorp., Inc.	0.73	0.72	1.60	0.99	1.01	4.26
Marine Midland Banks, Inc.	0.69	0.69	1.61	0.41	0.42	4.25
Morgan, J. P. and Co., Inc.	1.43	1.45	1.82	1.10	1.15	4.15
Southwest Bancshares, Inc.	1.42	1.40	2.11	1.01	1.04	4.20
Bancal Tri-State Corp.	0.78	0.78	1.59	0.93	0.97	4.07
Bank of Boston Corp.	1.52	1.52	1.99	1.29	1.35	4.12
Bankamerica Corp.	1.39	1.40	1.86	1.41	1.43	4.28
Chase Manhattan Corp.	0.76	0.76	1.58	0.85	0.91	4.06
Citicorp	1.65	1.66	2.05	1.70	1.77	4.11
Continental Illinois Corp.	1.15	1.16	1.79	0.86	0.94	4.01
Crocker National Corp.	0.56	0.57	1.41	0.73	0.77	4.10
First Chicago Corp.	1.16	1.16	1.84	1.36	1.36	4.43
First City Bancorp. of Texas	2.08	2.11	2.31	2.21	2.37	4.00
First Interstate Bancorp.	1.18	1.18	1.74	0.74	0.77	4.17
First National State Bancorp.	0.86	0.88	1.41	0.67	0.72	4.10
Fleet Financial Group, Inc.	1.19	1.19	1.76	1.04	1.08	4.15
Interfirst Corp.	3.23	3.29	3.17	3.92	3.93	5.48
Irving Bank Corp.	0.85	0.85	1.63	1.12	1.13	4.28
Manufacturers Hanover Corp.	0.64	0.65	1.46	0.84	0.88	4.10

Mercantile Texas Corp.	2.00	1.99	2.46	2.41	2.44	4.61
Norwest Corp.	1.26	1.29	1.57	1.25	1.27	4.28
Republic NY Corp.	2.49	2.50	2.78	2.37	2.38	4.57
Texas Commerce Bancshares, Inc.	2.19	2.16	2.68	2.31	2.45	4.19
Wachovia Corp.	0.97	0.97	1.66	1.16	1.19	4.23
Wells Fargo and Co.	0.80	0.81	1.57	0.84	0.85	4.26

Note: σ_D = annualized standard deviation of rate of return on riskless bond; σ_V = annualized standard deviation of rate of return on assets; σ = annualized volatility "priced" under stochastic interest rate model = $\sigma_V^2 + \sigma_D^2 - 2\sigma_{VD}$, where σ_{VD} is the annualized covariance between the rates of return on V and D.

A comparison of the first two columns of Table IV with columns 4 and 5 indicates that there was not, in general, a significant change in estimated volatilities from the first to the third quarter of 1979. These results are in general agreement with those of Pyle and Morrison [16], who in a simulated experiment had found that interest rate risk makes only a small contribution to the overall risk of a bank.

III. Conclusions and Extensions

In sum, we were able to demonstrate feasibility of estimating risk-adjusted premiums working only with the market evaluated data on equity. Our approach yields a *rank ordering* of the banks on the basis of their risk to the insuring agencies. Alternatively, the model may be viewed as yielding an allocation rule for apportioning a given premium across the banking sector. This approach has the merit of not using data provided by bank management or that collected as a part of the FDIC audit, use of which would have introduced the complex issue of incentives for revelation.

The present approach admits at least four extensions. First, it can be used, with only small modifications, to arrive at the capital adequacy standard that an individual bank should be required to maintain, given the volatility of its assets and the flat premium of 1/12th of 1 percent. It should be noted that the insuring agencies can implement risk-specific capital adequacy standards, since setting of these standards is within their discretionary powers. The second deals with uncertain FDIC-Federal Reserve regulatory intervention. Specifically, how large a cash infusion will the federal authorities expend rather than closing down the bank? In the context of our model, there is uncertainty with respect to the exercise price.[28] Fischer [7] has modeled call option pricing under exercise price uncertainty, and the application of his model could be particularly illuminating.

The third approach for enhancing the scope of our model involves an attempt to proxy for the values of E and σ_E when these are unavailable. Using a "pure-play" approach permits such an estimation, yielding the "fair" risk-adjusted premium for banks that do *not* have publicly traded equity.

The fourth extension involves a more detailed modeling of FDIC regulatory policy. If the FDIC was following a policy of monitoring both the book *and* market values of equity, and invoking bank closures if *both* equity values become negative, then stockholders' equity equals the maximum of the book value of equity or the market value of the firm after payment of debt. Stulz [19] has modeled the value of "Options on the Minimum or Maximum of Two Risky Assets." This model could also yield interesting estimates of risk-adjusted deposit premiums.

[28] Note that in Section II, we assumed away this uncertainty by positing a fixed, known ρ, where $(1 - \rho)$ is the cash infusion (expressed as a percentage of total deposits B) which the FDIC would incur to "avoid" bank closure.

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