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Cost optimization of project schedules under constrained resources and alternative production processes by mixed-integer nonlinear programming

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Abstract

Purpose – The purpose of this paper is cost optimization of project schedules under constrained resources and alternative production processes (APPs).

Design/methodology/approach – The model contains a cost objective function, generalized precedence relationship constraints, activity duration and start time constraints, lag/lead time constraints, execution mode (EM) constraints, project duration constraints, working time unit assignment constraints and resource constraints. The mixed-integer nonlinear programming (MINLP) superstructure of discrete solutions covers time–cost–resource options related to various EMs for project activities as well as variants for production process implementation.

Findings – The proposed model provides the exact optimal output data for project management, such as network diagrams, Gantt charts, histograms and S-curves. In contrast to classic scheduling approaches, here the optimal project structure is obtained as a model-endogenous decision. The project planner is thus enabled to achieve optimization of the production process simultaneously with resource-constrained scheduling of activities in discrete time units and at a minimum total cost.

Practical implications – A set of application examples are addressed on an actual construction project to display the advantages of proposed model.

Originality/value – The unique value this paper contributes to the body of knowledge reflects through the proposed MINLP model, which is capable of performing the exact cost optimization of production process (where presence and number of activities including their mutual relations are dealt as feasible alternatives, meaning not as fixed parameters) simultaneously with the associated resource-constrained project scheduling, whereby that is achieved within a uniform procedure.

Keywords Optimization, Process, Scheduling, Project management, Construction planning, Novel model

Paper type Research paper

Nomenclature

Indices:

i	activity, $i \in I(p)$
j	succeeding activity, $j \in J(i,p)$
k	activity execution mode, $k \in K(i,p)$
p	production process, $p \in P$
r	resource, $r \in R(i,p)$
t	working time unit, $t \in T$

Parameters:

aw_t	available working time unit
$dc_{i,k,p}$	discrete direct cost option related to activity execution mode in an alternative production process

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$dd_{i,k,p}$	discrete duration option related to activity execution mode in an alternative production process	Cd	direct project cost
$D_{i,p}^{\mathrm{LO}}$	lower bound on activity duration in an alternative production process	Ci	indirect project cost
$D_{i,p}^{\mathrm{UP}}$	upper bound on activity duration in an alternative production process	C_i	activity direct cost
M	large-value parameter	Cp	project penalty cost
$q_{i,k,p,r}$	discrete option for resource quantity related to activity execution mode in an alternative production process	Ct	total project cost
$Qi_{t,r}^{\max}$	maximum allowed time-dependent increment resource quantity	D_i	activity duration
$Qc_{t,r}^{\max}$	maximum allowed time-dependent cumulative resource quantity	Dp	project duration
$S_{i,p}^{\mathrm{LO}}$	lower bound on activity start time in an alternative production process	$L_{i,j,p}$	lag/lead time between interdependent activities in an alternative production process
$S_{i,p}^{\mathrm{UP}}$	upper bound on activity start time in an alternative production process	S_i	activity start time
$W_{i,t}$	activity working time units	$Qc_{t,r}$	cumulative quantity of resource in a working time unit
Variables:		$Qi_{t,r}$	increment quantity of resource in a working time unit
B	project bonus	ya_p	binary decision variable for optimal selection of a production process
		$yd_{i,k}$	binary decision variable for optimal selection of an activity execution mode
		$yw_{i,t}$	binary decision variable for optimal selection of activity working time units

Introduction

In an industrial environment such as the construction sector, the success of a project is highly dependent on the contractor's ability to establish the optimal production process and allocate the appropriate resources with which the agreed work can be carried out. This is usually achieved on the basis of previous practice, experience, knowledge and intuition. From the viewpoint of project scheduling, a network diagram with precedence relations and schedules, like Gantt charts, histograms, S-curves, etc., can be determined after the production process with associated resources has been identified. Afterwards, the optimization of project schedules is most commonly performed on the basis of a fixed network diagram that reflects the selected production process. However, a production process established in this manner does not necessarily represent the optimal solution, and the minimization of total project costs can be achieved only within its frame. Thus, there exists the potential to additionally minimize the total project cost by taking into account the fact that the same project could be successfully completed in different ways, e.g. by considering alternatives for the production process.

When a production process is intuitively chosen and set as unchangeable, cost-optimal project schedules can be achieved by accelerating those activities on critical paths in a fixed network diagram that exert the most influence on the reduction of total project cost. If alternatives for a production process are available and are required to be considered in minimizing cost, the optimization of project schedules no longer needs to be performed on the basis of a fixed network diagram, but rather on its variants. In this way, planning of a production process, which is mainly concerned with technological aspects of resources, and project scheduling, which deals primarily with resource availability and timing of activities, can be addressed simultaneously as a single optimization problem. This, however, increases the necessity for additional variables that support a higher number of model-endogenous decisions and makes the cost optimization of project schedules under constrained resources and alternative production processes (APPs) a challenging task.

For optimal scheduling of projects with fixed process plans and constrained resources, a wide range of contributions has been published over the decades of research, and comprehensive overviews of achievements can be found in the literature, see e.g. Hartmann and Briskorn (2010), Habibi *et al.* (2018). The time-cost trade-off problem (Feylizadeh *et al.*, 2018; Mahmoudi and Feylizadeh, 2018; Ballesteros-Perez *et al.*, 2019) and the resource-constrained project scheduling problem (Kasravi *et al.*, 2018; Hosseinian *et al.*, 2019; Servranckx and Vanhoucke, 2019) as well as their extensions have been mainly addressed in publications. However, recent research works are also focused on resource leveling under float loss impacts (El-Sayegh, 2018) and project scheduling problems with time, cost and risk trade-off (Tran and Long, 2018). Besides, the fact that process planning and scheduling are often complementary in industrial environments was also recognized among researchers, and early ideas about how to treat them in an integrative manner were introduced by Chryssolouris *et al.* (1985). Moreover, it seems that alternative process plans have attracted attention among the scientific community active in the field of scheduling, and contributions to said area can also be found in the relevant publications.

For instance, Beck and Fox (2000) introduced the idea of an activity's probability of existence on an expanded job-shop task by solving the alternative resource and process plan scheduling problem. Sormaz and Khoshnevis (2003) presented procedure steps and experimental verification for generation of a process network that was able to serve as an alternative process plan. Wu *et al.* (2010) introduced the constraint-based simulation approach for input data preparation by assigning construction patterns to the components. The proposed simulation method was demonstrated on a bridge construction project, where three different production processes were available for its completion: the balanced cantilever method, formwork carriage and *in situ* casting.

Rajabinasab and Mansour (2011) proposed a flexible job-shop problem and took into account dynamic events such as stochastic job arrivals, uncertain processing time and unexpected machine breakdowns. Čapek *et al.* (2012) studied a resource-constrained scheduling problem with alternative process plans in a controlled environment of wire harness production, where only one process plan was to be selected and executed from a set of alternatives. Kellenbrink and Helber (2015) researched a scheduling problem with a flexible process structure related to the aircraft turnaround procedure and suggested a genetic algorithm for its solution.

Tao and Dong (2017) considered the resource-constrained project scheduling problem with alternative activity chains in order to minimize the project duration. They proposed an approach for converting a classic project network into an AND–OR network and solved the optimization problem by a simulated annealing algorithm. Burgelman and Vanhoucke (2018) addressed the resource-constrained project scheduling problem with multiple alternatives for the execution of activities and showed the applicability of the proposed optimization approach on a project of removing asbestos from the building. They successfully solved the optimization problem, in which the number of weighted execution alternatives in the project scheduling phase was maximized, and have demonstrated that even a small increase of mentioned number may result in a high degree of flexibility.

A literature review revealed that the optimization problems of simultaneous production process planning and scheduling have been addressed in environments with predominantly repetitive tasks but more rarely in those that are particularly project-oriented. Moreover, there is room for a contribution in this particular field from the viewpoint of cost optimization by exact methods. However, cost optimization of project schedules can present complex, nonlinear discrete optimization problems in many actual situations. To achieve the exact optimal solutions of such problems, mixed-integer nonlinear programming (MINLP) approaches have been proposed and encouraging results can be found in literature (e.g. Klanšek and Pšunder, 2012; Al Haj and El-Sayegh, 2015; Klanšek, 2016).

For example, following the fact that the total cost distribution is often found as an issue of high significance in project scheduling (Cheng *et al.*, 2011), Klanšek (2016) developed and applied a MINLP model for nonlinear discrete optimization of project schedules with cost restrictions. The proposed MINLP model enabled exact optimal scheduling of project activities to be executed simultaneously with scheduling of restricted costs for each discrete working time unit, but with a fixed process plan and an associated network diagram. As regards project scheduling under increased number of activities, alternatives and/or modes, Cajzek and Klanšek (2016) have demonstrated that highly combinatorial nonlinear discrete problems can be solved to exact optimality in reasonable process time by the MINLP approach.

This study attempts to fill a gap in knowledge about the models capable of performing the exact cost optimization of production process (where presence and number of activities including their mutual relations are dealt as feasible alternatives, meaning not as fixed parameters) simultaneously with the associated resource-constrained project scheduling, whereby that should be achieved within a uniform procedure. Therefore, a new MINLP model for cost optimization of project schedules under constrained resources and APPs is proposed in this paper. This model contains a cost objective function, generalized precedence relationship constraints, activity duration and start time constraints, lag/lead time constraints, execution mode (EM) constraints, project duration constraints, working time unit assignment constraints and resource constraints. The MINLP superstructure of discrete solutions covers time–cost–resource options related to different EMs for project activities, as well as variants for production process implementation. The proposed model enables incorporation of a broad range of nonlinear expressions and provides the exact optimal output data for project management, such as network diagrams, Gantt charts, histograms and S-curves. In contrast to classic scheduling approaches, here the optimal project structure is obtained as a model-endogenous decision. The project planner is thus enabled to achieve optimization of the production process simultaneously with resource-constrained scheduling of activities in discrete time units and at a minimum total cost. An additional feature of the model is that it allows an operative determination of the optimal production process with an associated network diagram and accompanying updated schedules in the case of a total failure of a bottleneck resource, which is difficult to replace during the project execution. A set of application examples are addressed on an actual construction project to display the advantages of the proposed model.

MINLP model formulation

Cost objective function

Optimization of project schedules can generally be performed under a variety of relevant criteria. However, the minimization of total cost can probably be identified as an optimization criterion that is commonly applied in the scheduling of industrial projects. In this sense, the objective function was included in the model as:

$$\text{Minimize} \quad Ct = Cd + Ci + Cp - B, \tag{1}$$

where the Ct defines the total project cost, Cd and Ci stand for direct and indirect project costs, Cp determines the project penalty cost, and B denotes the project bonus.

The above cost objective function is well known and here it was mainly set in accordance with Klanšek (2016). Nevertheless, it is important to emphasize that part of the total cost, i.e. the direct cost of activities, is formulated differently here, since optimization of project schedules under APPs requires additional decision variables as well as the inclusion of new model features. For that purpose, the set P is declared to establish APPs $p \in P$ under which the project can be successfully completed, while the set of activities I is determined to take into account the fact that each APP $p, p \in P$, requires its own specific activities $I \in I(p)$. The set of feasible EMs for project activities was thus integrated in the model as $k \in K(i, p)$.

Vectors of binary 0–1 decision variables $\mathbf{ya} = \{ya_p\}$ and $\mathbf{yd} = \{yd_{i,k}\}$ were further determined to perform optimal selection of a project's production process and EMs for its activities among feasible alternatives, respectively. That was achieved by assigning a binary 0–1 variable to each generated alternative where the obtained value of 0 rejects the alternative while the value of 1 selects it. In this way, the MINLP algorithm is enabled to simultaneously identify the project's optimal production process with a specific combination of activities and their mutual relations among the APPs as well as the optimal engagement of time and resources from the generated EM options. It was assumed that only one production process could be selected for the project and that only one EM could be chosen for each activity (these conditions will be formulated and explained later). All this allowed freedom in handling the direct cost of project activities, either by continuous functions or by discrete terms. Therefore, the direct project cost was defined in the following form:

$$Cd = \sum_{p \in P} \sum_{i \in I(p)} \sum_{k \in K(i,p)} ya_p \left[C_i(D_i) + yd_{i,k}\, dc_{i,k,p} \right], \tag{2}$$

where C_i (D_i) represents continuous dependences between direct costs and the durations of project activities, while $yd_{i,k}\, dc_{i,k,p}$ denotes the discrete ones which were accomplished by the generated superstructure of direct cost options $dc_{i,k,p}$. These dependences mainly arise from the scope of resources allocated to project activities, and it should be stressed here that continuous as well as discrete cost-duration relations were set as mutually exclusive for the same activity.

As regards to indirect cost Ci, it usually depends upon the project duration and includes initial costs, business operating and overhead costs as well as equipment operating costs. Project penalty Cp and bonus B depend on the type of contract, for example firm-fixed-price contracts, fixed-price incentive fee contracts or cost-plus-award-fee contracts. The mentioned items of cost objective function can be formulated and included into the model as it was proposed by Cajzek and Klanšek (2016).

Generalized precedence relationship constraints

Project activities are commonly required to be executed by taking into account the precedence relations between them. In this connection, the proposed model covers well-known generalized precedence relations (GPRs) like Finish-to-Start (FS), Start-to-Start (SS), Finish-to-Finish and Start-to-Finish, by incorporating principles of the activity-on-node approach. However, the specificity of the model is its capacity to include the set of APPs $p \in P$, by which the project can be appropriately completed.

The selection of the optimal production process from a variety of alternatives is performed by binary decision variables ya_p, such that, if $ya_p = 1$, then the APP $p, p \in P$, is selected, and if $ya_p = 0$, then the APP $p, p \in P$, is rejected. Since each production process p, $p \in P$, demands its own specific activities i, $i \in I(p)$, which need to be directly followed by their successors $j \in J(i, p)$ in a manner that is characteristic for that process, the set of GPR constraints was determined as follows:

$$S_i + D_i + L_{i,j,p} - S_j \leqslant \leq M\left(1 - ya_p\right) \quad p \in P \;\; i \in I(p) \;\; j \in J(i,p) \;\; (i,j,p) \in FS, \tag{3}$$

$$S_i + L_{i,j,p} - S_j \leqslant M\left(1 - ya_p\right) \quad p \in P \;\; i \in I(p)\, j \in J(i,p) \;\; (i,j,p) \in SS, \tag{4}$$

$$S_i + D_i + L_{i,j,p} - S_j - D_j \leqslant M\left(1 - ya_p\right) \quad p \in P \;\; i \in I(p) \;\; j \in J(i,p) \;\; (i,j,p) \in FF, \tag{5}$$

$$S_i + L_{i,j,p} - S_j - D_j \leqslant M\left(1 - ya_p\right) \quad p \in P \;\; i \in I(p) \;\; j \in J(i,p) \;\; (i,j,p) \in SF, \tag{6}$$

where S_i and D_i denote the start time and duration of project activity in APP, while $L_{i,j,p}$ represents the lag/lead time between any activity i, $i \in I(p)$ and its succeeding activity j, $j \in J(i,p)$ in that specific process p, $p \in P$.

Here, S_i and D_i were set in the model as positive variables, while $L_{i,j,p}$ was allowed to take either positive (lag time), zero or negative (lead time) value. The right-hand side of the inequality constraints defined by Equations (3)–(6) contains the expression M $(1-ya_p)$ in which M represents a large-value parameter. This expression operates in the following way: if APP p, $p \in P$, is selected ($ya_p = 1$), then the right-hand side of the precedence relationship constraint that appears in that process attains a 0 value, and that constraint becomes relevant for the optimization result; otherwise, if APP p, $p \in P$, is rejected ($ya_p = 0$), then the right-hand side of this constraint takes a large value of M, which makes that constraint irrelevant.

Activity duration and start time constraints

As previously mentioned, implementation modes of project activities $i \in I(p)$ in APPs $p \in P$ were determined by the set $k \in K(i, p)$. In this respect, the superstructure of discrete duration options for project activities was formed using the integer constants $dd_{i,k,p}$. Since the generated range of integer constants $dd_{i,k,p}$ represents various feasible discrete solutions for the associated variable D_i, the procedure for selecting the optimum one was executed by the binary variables ya_p and $yd_{i,k}$ applying the following equations:

$$D_i = \sum_{p \in P} \sum_{k \in K(i,p)} ya_p \, yd_{i,k} \, dd_{i,k,p} \quad i \in I(p), \tag{7}$$

$$\sum_{p \in P} ya_p = 1, \tag{8}$$

$$\sum_{k \in K(i,p)} yd_{i,k} = 1 \quad i \in I(p). \tag{9}$$

In accordance with Equation (7), each discrete option $dd_{i,k,p}$ is chosen to be the discrete solution for activity duration D_i as soon as both the project's production process ($ya_p = 1$) and the activity implementation mode ($yd_{i,k} = 1$) have been determined. Equation (8) ensures that only one production process can be selected for the project while Equation (9) specifies that only one EM can be chosen for each activity required in a given process.

Bounds on the duration and start time of project activities were defined as follows:

$$\sum_{p \in P} D_{i,p}^{\mathrm{LO}} \, ya_p \leqslant D_i \leqslant \sum_{p \in P} D_{i,p}^{\mathrm{UP}} \, ya_p \quad i \in I(p), \tag{10}$$

$$\sum_{p \in P} S_{i,p}^{\mathrm{LO}} ya_p \leqslant S_i \leqslant \sum_{p \in P} S_{i,p}^{\mathrm{UP}} ya_p \quad i \in I(p), \tag{11}$$

where $D_{i,p}^{\mathrm{LO}}$ and $S_{i,p}^{\mathrm{LO}}$ denote the lower bounds on durations and start times of project activities in APP p, $p \in P$, while $D_{i,p}^{\mathrm{UP}}$ and $S_{i,p}^{\mathrm{UP}}$ represent their associated upper bounds.

Lower and upper bounds are required to be generated as input data. However, the binary decision variables ya_p are assigned to activate them as soon as APP is selected and to deactivate them if the particular process is rejected. Considering that the project can be executed by selecting only one production process among several alternatives, the constraints defined by Equations (10) and (11) operate as follows: if APP p, $p \in P$, is chosen ($ya_p = 1$), then bounds on activity duration and start time are established as $D_i^{\mathrm{LO}} \leqslant D_i \leqslant D_i^{\mathrm{UP}}$ and $S_i^{\mathrm{LO}} \leqslant S_i \leqslant S_i^{\mathrm{UP}}$; otherwise, if APP p, $p \in P$, is rejected ($ya_p = 0$), then D_i and S_i are both equal to 0. Note here that the said lower and upper bounds of activities that do not appear in the APP are generated as 0 values. This is consistent with the assumption that an activity specific to the rejected production process should not be included in the one selected.

Nevertheless, it should be mentioned that the data collection for activity start times may sometimes not be simple in real-world situations (e.g. if it is required to collect them based on SMEs opinion). Still, the main contractor can retain considerable freedom in setting bounds on start times of its own activities while for those of subcontractors it is necessary to reach suitable agreements on that issue.

Lag/lead time constraints

As was shown with precedence relations and activities, lag/lead times $L_{i,j,p}$ can also be specific to each APP. Moreover, APPs for the same project can even contain the same activities, which may nevertheless require consideration of different precedence relations and lag/lead times. Therefore, the selection of lag/lead times between related activities is performed by the expression:

$$L_{i,j,p} = \sum_{k \in K(i,p)} ya_p \, yd_{i,k} \, dl_{i,j,k,p} \; p \in P \quad i \in I(p) \quad j \in J(i,p),$$

$$(i,j,p) \in \mathrm{FS} \vee \mathrm{SS} \vee \mathrm{FF} \vee \mathrm{SF}, \tag{12}$$

where $dl_{i,j,k,p}$ represents the discrete lag/lead time options generated in the model as a set of integer constants.

The above formulation enables the lag/lead time to be related not only to the production process but also to the activity EM. That specific model feature can be suitably employed in cases when a change in the preceding activity EM also influences the change of lag/lead time required to fulfill the precedence relation to a succeeding activity. Note here that lag times are generated as positive parameters $L_{i,j,p}$, while lead times are determined by negative ones.

Execution mode constraints

The EM of an activity may sometimes influence the manner in which another activity should be carried out within the project. For example, technological characteristics of the production process may require that the selected EM for a certain activity demands a specific way of implementing another project activity. To cover such requirements of production processes, the following EM constraints were determined in the model:

$$ya_p \left(yd_{i,k} - yd_{j,k} \right) = 0 \quad p \in P \quad i \in I(p) \quad j \in J(i,p) \quad k \in K(i,p) \quad (i,j,p) \in \mathrm{EM}. \tag{13}$$

The given equality condition influences the selection of EM $k \in K(i, p)$ for activities $i \in I(p)$ and $j \in J(i, p)$, for which it was established that their manner of performance is interdependent within an APP $p \in P$. In this way, when a particular production process is chosen ($ya_p = 1$) for the project, the above constraint is fulfilled only when the same EM k, $k \in K(i,p)$, is either selected ($yd_{i,k} = yd_{j,k} = 1$) or rejected ($yd_{i,k} = yd_{j,k} = 0$) for those activities $i \in I(p)$ and $j \in J(i,p)$ that are found to be mutually dependent in the given sense.

Project duration and working time unit assignment constraints

Project duration and working time unit assignment constraints of the model proposed here were mainly formulated as it was suggested by Klanšek (2016). As formulated in the said reference, the activities were equally constrained here to be implemented between the project's start and completion times, i.e. within the project duration Dp. With regard to the working time unit assignment constraints, an illustrative example showing the relations among the set of working time units $t \in T$, available working time units aw_t, activity working time units $W_{i,t}$, activity start time S_i, activity duration D_i and assignment binary variables $yw_{i,t}$, can be found in the mentioned reference. However, bounds on activity working time units $W_{i,t}$ were defined here in a different manner. Indeed, each assigned working time unit $W_{i,t}$ needs to be situated between the start and completion times of activity required in the APP so the following bounds were set to be fulfilled:

$$\sum_{p \in P} S_{i,p}^{\mathrm{LO}} ya_p \leqslant W_{i,t} \leqslant S_i + D_i - \sum_{p \in P} S_{i\alpha,p}^{\mathrm{LO}} ya_p \quad t \in T \quad i, i\alpha \in I(p), \tag{14}$$

where $S_{i\alpha,p}^{\mathrm{LO}}$ represents lower bounds on start times of initial activities $i\alpha$, $i\alpha \in I(p)$. Thus, if APP p, $p \in P$, is selected ($ya_p = 1$), then bounds on the activity working time units $W_{i,t}$ are established as $S_{i,p}^{\mathrm{LO}} \leqslant W_{i,t} \leqslant S_i + D_i - S_{i\alpha,p}^{\mathrm{LO}}$, i.e. as bounds on the activity start (the first term) and completion (the last term) times; otherwise, if APP p, $p \in P$, is rejected ($ya_p = 0$), then $W_{i,t}$ are equal to 0.

Resource constraints

Project execution needs to be supported by resources allocated to activities in any APP. However, each APP p, $p \in P$, can require a specific set of activities $i \in I(p)$ that may need to be carried out by different resources. Therefore, various types of resources were related to project activities in APPs by the special set $r \in R(i, p)$. Since the model allows project implementation under APPs $p \in P$, while associated activities $i \in I(p)$ can be performed in optional modes $k \in K(i,p)$ supported by corresponding resources $r \in R(i, p)$, the superstructure of various integer constants was created to cover the required quantities of various resources $q_{i,k,p,r}$.

However, real-life projects, especially those in industry, often face a variety of resource limitations. For implementation of project scheduling under such circumstances, it is important to take into account the distribution of incremental and cumulative quantities of resources across units of working time on the project. Therefore, any increment in resource requirement can be allocated to each working time unit by the following equation:

$$Qi_{t,r} = \sum_{p \in P} \sum_{i \in I(p)} \sum_{k \in K(i,p)} ya_p yd_{i,k} yw_{i,t} q_{i,k,p,r} \quad t \in T \quad r \in R(i,p), \tag{15}$$

where $Qi_{t,r}$ denotes the quantity increment in the resource r, $r \in R(i, p)$, for the working time unit t, $t \in T$.

As soon as the incremental quantity of resource r, $r \in R(i, p)$, is assigned to the working time unit t, $t \in T$, the associated cumulative quantity of that resource, labeled $Qc_{t,r}$, can be calculated by the following formula:

$$Qc_{t,r} = Qi_{t,r} + Qc_{t-1,r} \quad t \in T \quad r \in R(i,p). \tag{16}$$

Considering that Equations (15) and (16) allow resource scheduling to be executed in each unit of working time on the project, various time-dependent resource restrictions can be included in the optimization model if necessary. For example, when increments in resource requirement should not exceed the anticipated maximum permitted time-dependent quantities $Qi_{t,r}^{\max}$, the following resource limitations can be set in the model:

$$Qi_{t,r} \leqslant Qi_{t,r}^{\max} \quad t \in T \quad r \in R(i,p). \tag{17}$$

Even though the maximum allowed time-dependent resource quantities $Qi_{t,r}^{\max}$ are frequently considered as constant during the project, they can also be included in the model by a variety of (non)linear terms $Qi_{t,r}^{\max} = f(aw_t)$. The applicability of such constraints may particularly come to the fore in cases when the availability of resources (e.g. a labor force) is expected to vary over the time of project execution, for example, owing to simultaneous coverage of multiple active projects in a company with the same type of limited resource.

Restrictions can also be established on the cumulative quantity of resources. At this point, resource limitation can be determined for each working time unit of the project by the following inequality:

$$Qc_{t,r} \leqslant Qc_{t,r}^{\max} \quad t \in T \quad r \in R(i,p), \tag{18}$$

where $Qc_{t,r}^{\max}$ represents the maximum allowed cumulative quantity of the resource r, $r \in R(i,p)$, for the working time unit t, $t \in T$.

The limitation above can generally be determined as $Qc_{t,r}^{\max} = f(aw_t)$. Such a formulation allows the possibility to set upper bounds on cumulative quantities of resources in linear or nonlinear form, if required. An example that often occurs in practice, where such restrictions can be applied, involves scheduling a project that needs to be implemented under limited cumulative man-hours, e.g. per month or per the project as whole. To conclude this section, note that lists of symbols and abbreviations are given in "Nomenclature" and "Glossary," respectively. The next section is devoted to the presentation of the model's applicability.

Application examples

Input data

An actual case of road reconstruction is addressed here to demonstrate the advantages to cost optimization of project schedules under constrained resources and APPs using the proposed model. For that purpose, three separate, well-known production processes that are commonly used in our construction practice were selected as relevant alternatives $p \in P$ available for successful completion of a given road reconstruction project: ($p = 1$) conventional, ($p = 2$) cold-in-place-recycling and ($p = 3$) cold-in-plant-recycling production processes. The activities found in the APPs are presented in Table I.

Road reconstruction is planned to be executed under traffic and partial road closures. The symbols 1/2 and 2/2 thus denote partial road closures. Note here that the activities 16 and 20 appear only in case of total failure of a bottleneck resource, i.e. a recycler during cold-in-place-recycling or a tractor-towed stabilizer during cold-in-plant-recycling. Regardless of the production process selection, some common tasks need to be consistently performed and most occur during the initial and final phases of the given road reconstruction, for example activities from 1 to 12 at the beginning of the project and activities from 45 to 54 in its concluding phase. However, activities common to different APPs also occur during project implementation, for instance activities 37 and 38, in this particular case.

A conventional production process is widely established and technologically less demanding for reconstructing pavement structures. The replacement is carried out by excavating the existing pavement, where the asphalt milling machines first remove the damaged asphalt base and the wearing course. The milling process is followed by excavation of the sub-base material, i.e. the blanket layer and substructure, to its full depth. The excavated material is then transported to an authorized landfill. Later, the delivery of virgin aggregate from a gravel pit is needed for the unbounded substructure and blanket

Labels and description of activities, $i \in I(p)$			
1	Construction site organization with road closure	28	Asphalt milling incl. permanent deposit 2/2
2	Stake out process	29	Cold recycling with foamed bitumen 2/2
3	Asphalt cutting	30	Excavation of blanket layer and substructure 2/2
4	Removal of asphalt and worn-out street curbs	31	Cold recycling without foamed bitumen 2/2
5	Installation of new street curbs	32	Leveling of recycled material by compactor 2/2
6	Excavation of drainage ditches	33	Delivery of unbounded substructure layer 2/2
7	Production of culverts, inlet and outlet heads	34	Leveling of recycled material by grader 2/2
8	Disassembly of signs and signalization	35	Delivery of blanket layer 2/2
9	Verge removal 1/2	36	Leveling and consolidation of sub-base layer 2/2
10	Guard rail removal	37	Superelevation of bridge fascia beams
11	Verge removal 1/2	38	Paving of roadside ditch
12	Demolition of bridge fascia beams	39	Asphalting of base course using virgin material
13	Asphalt milling incl. permanent deposit 1/2	40	Asphalting of base course using recycled material
14	Asphalt milling incl. temporary deposit 1/2	41	Asphalt and blanket layer milling incl. permanent deposit
15	Spreading of cement 1/2	42	Asphalt and blanket layer milling incl. temporary deposit
16	Readjusting works: failure of a bottleneck resource 1/2	43	Execution of base course layer on transitions
17	Excavation of blanket layer and substructure 1/2	44	Execution of recycled base course layer on transitions
18	Cold recycling with foamed bitumen 1/2	45	Spreading of bitumen emulsion
19	Cold recycling without foamed bitumen 1/2	46	Execution of wearing course
20	Readjusting works: failure of a bottleneck resource 1/2	47	Execution of driveways
21	Delivery of unbounded substructure layer 1/2	48	Execution of verges
22	Leveling of recycled material by compactor 1/2	49	Signalization installation
23	Delivery of blanket layer 1/2	50	Guard rails installation
24	Leveling of recycled material by grader 1/2	51	Restoration of road markings
25	Leveling and consolidation of sub-base layer 1/2	52	Renewal of road slopes
26	Asphalt milling incl. temporary deposit 1/2	53	Grass planting on slopes and clean-up work
27	Spreading of cement 1/2	54	Handover procedure
Sets of GPRs[a]	Set members		
$(i, j, p) \in FS = \{$	(2, 3, 1); (2, 6, 1); (4, 5, 1); (5, 13, 1); (6, 7, 1); (7, 38, 1); (8, 9, 1); (9, 10, 1); (9, 17, 1); (11, 13, 1); (12, 37, 1); (13, 17, 1); (21, 23, 1); (25, 28, 1); (28, 30, 1); (33, 35, 1); (37, 50, 1); (38, 52, 1); (39, 41, 1); (41, 43, 1); (43, 45, 1); (46, 47, 1); (48, 49, 1); (48, 52, 1); (50, 51, 1); (2, 3, 2); (2, 6, 2); (4, 5, 2); (5, 15, 2); (6, 7, 2); (7, 38, 2); (8, 9, 2); (9, 10, 2); (9, 18, 2); (11, 15, 2); (12, 37, 2); (24, 27, 2); (37, 50, 2); (38, 52, 2); (39, 41, 2); (41, 43, 2); (43, 45, 2); (46, 47, 2); (48, 49, 2); (48, 52, 2); (50, 51, 2); (2, 3, 3); (2, 6, 3); (4, 5, 3); (5, 14, 3); (6, 7, 3); (7, 38, 3); (8, 9, 3); (9, 10, 3); (9, 15, 3); (11, 14, 3); (12, 37, 3); (14, 15, 3); (24, 26, 3); (26, 27, 3); (37, 50, 3); (38, 52, 3); (40, 42, 3); (42, 44, 3); (44, 45, 3); (46, 47, 3); (48, 49, 3); (48, 52, 3); (50, 51, 3)}		

(*continued*)

Table I.
Project activities and GPRs in APPs

Labels and description of activities, $i \in I(p)$	
$(i, j, p) \in SS = \{$	(1, 2, 1); (1, 8, 1); (3, 4, 1); (10, 11, 1); (11, 12, 1); (17, 21, 1); (23, 25, 1); (30, 33, 1); (35, 36, 1); (36, 39, 1); (45, 46, 1); (47, 48, 1); (49, 50, 1);
	(1, 2, 2); (1, 8, 2); (3, 4, 2); (10, 11, 2); (11, 12, 2); (15, 18, 2); (18, 22, 2); (22, 24, 2); (27, 29, 2); (29, 32, 2); (32, 34, 2); (34, 39, 2); (45, 46, 2); (47, 48, 2); (49, 50, 2);
	(1, 2, 3); (1, 8, 3); (3, 4, 3); (10, 11, 3); (11, 12, 3); (15, 19, 3); (19, 22, 3); (22, 24, 3); (27, 31, 3); (31, 32, 3); (32, 34, 3); (34, 40, 3); (45, 46, 3); (47, 48, 3); (49, 50, 3)}
$(i, j, p) \in FF = \{$	(1, 54, 1); (51, 54, 1); (52, 53, 1); (53, 54, 1);
	(1, 54, 2); (51, 54, 2); (52, 53, 2); (53, 54, 2);
	(1, 54, 3); (51, 54, 3); (52, 53, 3); (53, 54, 3)}

Table I.

Note: [a]Horizontal lines separate precedence relations in different production processes

layers, which are leveled by grader and compacted by rollers. Finally, the base and wearing course asphalts are transported from the asphalt batching plant to the construction site and then embedded into the road structure. In this way, the activities specific to the conventional production process are 13, 17, 21, 23, 25, 28, 30, 33, 35, 36, 39, 41 and 43.

The cold-in-place-recycling production process is performed with a recycling machine, also known as a recycler. This production process is implemented by milling the existing asphalt and lower layers by adding water, cement and bitumen. At this point, the cement is spread before recycling, using the binder spreader; mixing the materials is then done by employing the recycler, while the transportation of bitumen and water is done by tanker trucks. In the recycler's mixing chamber, the ground material is stirred with water, cement and bitumen. The recycled product forms a stabilized layer, which is prepared for leveling by grader and compacting by rollers. The production process is concluded by two-layer paving from virgin material. Accordingly, typical activities for the cold-in-place-recycling production process are 15, 18, 22, 24, 27, 29, 32, 34, 39, 41 and 43.

The cold-in-plant-recycling production process starts with the separate milling and transportation of the asphalt base and the wearing course to a stockpile at the cold-mixing plant situated close to the construction site. A new homogenous asphalt mixture for the base layer is produced by mixing the reclaimed asphalt pavement, cement and foamed bitumen. Meanwhile, the stabilization of pre-cemented existing sub-base layers is carried out by a tractor-towed stabilizer. The stabilized layer is then leveled by grader, compacted by rollers and thus prepared for delivery of the recycled asphalt material from a nearby mobile plant. Afterwards, base course of recycled material and a wearing course of virgin material are put in place. Characteristic activities for the cold-in-plant-recycling production process are 14, 15, 19, 22, 24, 26, 27, 31, 32, 34, 40, 42 and 44.

Technological and organizational particularities of each APP for the given road reconstruction are reflected in the characteristic set of precedence relations that need to be fulfilled among associated activities to achieve successful project completion. In this way, a specific combination of activities and GPRs was suitably allocated to each considered APP as shown in Table I and also in project network diagram of Figure 1.

Execution of a road reconstruction project applying any of these APPs also requires inclusion of lag/lead times between some interdependent activities. From viewpoint of GPRs in Table I, lag/lead time options between connected activities are determined in dependence of production process and EM. In this particular case, there are five instances of non-zero lag/lead times that appear in all APPs, i.e. $dl_{i,j,k,p} = \{[dl_{1,8,1,1} = 2;\ dl_{1,8,2,1} = 2;\ dl_{2,3,1,1} = -2;\ dl_{2,3,2,1} = -1;\ dl_{2,6,1,1} = -1];\ [dl_{1,8,1,2} = 2;\ dl_{1,8,2,2} = 2;\ dl_{2,3,1,2} = -2;\ dl_{2,3,2,2} = -1;\ dl_{2,6,1,2} = -1];\ [dl_{1,8,1,3} = 2;\ dl_{1,8,2,3} = 2;\ dl_{2,3,1,3} = -2;\ dl_{2,3,2,3} = -1;\ dl_{2,6,1,3} = -1]\}$.

The first example occurs between activities 1 and 8, where the SS precedence relation contains two days of lag time required for preliminary preparation of construction site organization and road closure, so that disassembly of signs and signalization can be performed. The same two-day lag time is considered in all cases irrespective of the selected EM. Furthermore, there is an FS precedence relation between Activity 2 and 3, with two days of lead time due to an organizational issue, where asphalt cutting can be performed two days before the completion of the staking out process on the construction site. However, lead time is allowed to change here in accordance with the chosen EM of the previous activity. More precisely, this lead time can be two-, one- or zero-days, depending on the duration of the preceding activity, in that accelerating the predecessor also reduces the necessary lead time for the successor. The final instance appears between activities 2 and 6, with an FS precedence relation including a one-day lead time, where excavation of drainage ditches can begin one day before the completion of the staking out process. If the preceding activity is accelerated from the normal mode, the lead time to the succeeding one is accordingly reduced to zero days.

Moreover, the set of EM constraints is determined for road reconstruction scheduling to fulfill specific technological demands of APPs, which in some instances require that the successor activity's EM be consistent with the predecessor's. In this particular case, there are nine EM constraints set in three APPs, six of which are used to cover the production technology of the recycler, and the other three for inclusion of technological demands during asphalt laying, i.e. $(i,j,p) \in \text{EM} = \{(45,46,1); (45,46,2); (15,19,3); (19,22,3); (22,24,3); (27,31,3); (31,32,3); (32,34,3); (45,46,3)\}$. There is one common EM relation valid for all APPs, namely between activities 45 and 46, where the spreading of bitumen emulsion depends on completion of the base and wearing courses to provide adhesion between the two layers. In cold-in-plant-recycling production processes, a tractor-towed stabilizer is allocated for work completion together with simultaneous use of a cement spreader, grader and roller. This machinery has to be operated efficiently so that the recycling process can be successfully completed. With the use of EM constraints, parallel implementation of connected activities (15, 19), (19, 22), (22, 24) during the first part of the road closure and (27, 31), (31, 32), (32, 34) during the second is covered for cold-in-plant-recycling production process, owing to cold recycling without bitumen.

Discrete duration options for project activities in APPs are shown in Table II. The conventional production process includes 11 one-option activities (i.e. activities 3, 10, 13, 25, 28, 36, 41, 43, 50, 51 and 54), 17 two-option activities (i.e. activities 4, 5, 6, 8, 9, 11, 21, 23, 33, 35, 37, 38, 39, 45, 46, 47 and 49), 4 three-option activities (i.e. activities 12, 48, 52 and 53) and 4 four-option activities (i.e. activities 2, 7, 17 and 30). The cold-in-place-recycling production process contains 15 one-option activities, 13 two-option activities, 4 three-option activities and 2 four-option activities, while the cold-in-plant-recycling production process comprises 9 one-option activities, 13 two-option activities, 12 three-option activities and 2 four-option activities.

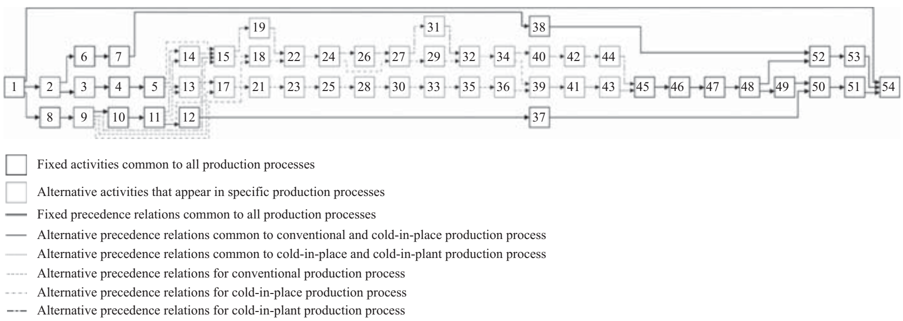

Figure 1.
Project network diagram with APPs consisting of fixed and alternative activities as well as associated GPRs

Table II. Discrete duration options for project activities in APPs

Activity $i \in I(p)$ ↓ Mod. →	Description of activity	Proc. → $p=1$ $k=1$	2	3	4	$p=2$ $k=1$	2	3	4	$p=3$ $k=1$	2	3	4
1	Construction site organization with road closure	*	*	*	*	*	*	*	*	*	*	*	*
2	Stake out process	4	3	2	1	4	3	2	1	4	3	2	1
3	Asphalt cutting	1	–	–	–	1	–	–	–	1	–	–	–
4	Removal of asphalt and worn-out street curbs	2	1	–	–	2	1	–	–	2	1	–	–
5	Installation of new street curbs	2	1	–	–	2	1	–	–	2	1	–	–
6	Excavation of drainage ditches	2	1	–	–	2	1	–	–	2	1	–	–
7	Production of culverts, inlet and outlet heads	4	3	2	1	4	3	2	1	4	3	2	1
8	Disassembly of signs and signalization	2	1	–	–	2	1	–	–	2	1	–	–
9	Verge removal 1/2	2	1	–	–	2	1	–	–	2	1	–	–
10	Guard rail removal	1	–	–	–	1	–	–	–	1	–	–	–
11	Verge removal 2/2	2	1	–	–	2	1	–	–	2	1	–	–
12	Demolition of bridge fascia beams	3	2	1	–	3	2	1	–	3	2	1	–
13	Asphalt milling incl. permanent deposit 1/2	1	–	–	–	–	–	–	–	–	–	–	–
14	Asphalt milling incl. temporary deposit 1/2	–	–	–	–	–	–	–	–	1	–	–	–
15	Spreading of cement 1/2	–	–	–	–	3	–	–	–	3	2	1	–
16	Readjusting works: failure of a bottleneck resource 1/2	–	–	–	–	–	–	–	–	–	–	–	–
17	Excavation of blanket layer and substructure 1/2	4	3	2	1	–	–	–	–	–	–	–	–
18	Cold recycling with foamed bitumen 1/2	–	–	–	–	3	–	–	–	–	–	–	–
19	Cold recycling without foamed bitumen 1/2	–	–	–	–	–	–	–	–	3	2	1	–
20	Readjusting works: failure of a bottleneck resource 2/2	–	–	–	–	–	–	–	–	–	–	–	–
21	Delivery of unbounded substructure layer 1/2	4	3	–	–	–	–	–	–	–	–	–	–
22	Leveling of recycled material by compactor 1/2	–	–	–	–	3	–	–	–	3	2	1	–
23	Delivery of blanket layer 1/2	2	1	–	–	–	–	–	–	–	–	–	–
24	Leveling of recycled material by grader 1/2	–	–	–	–	3	–	–	–	3	2	1	–
25	Leveling and consolidation of sub-base layer 1/2	1	–	–	–	–	–	–	–	–	–	–	–
26	Asphalt milling incl. temporary deposit 2/2	–	–	–	–	–	–	–	–	1	–	–	–
27	Spreading of cement 2/2	–	–	–	–	3	–	–	–	3	2	1	–
28	Asphalt milling incl. permanent deposit 2/2	1	–	–	–	–	–	–	–	–	–	–	–
29	Cold recycling with foamed bitumen 2/2	–	–	–	–	3	–	–	–	–	–	–	–
30	Excavation of blanket layer and substructure 2/2	4	3	2	1	–	–	–	–	–	–	–	–
31	Cold recycling without foamed bitumen 2/2	–	–	–	–	–	–	–	–	3	2	1	–

(continued)

Table II.

Activity $i \in I(p)$ ↓ Mod. →	Description of activity	Discrete duration options for activities $dd_{i,k,p}$ (days) Proc. → $p=1$, $k=1$	2	3	4	$p=2$, $k=1$	2	3	4	$p=3$, $k=1$	2	3	4
32	Leveling of recycled material by compactor 2/2	–	–	–	–	3	–	–	–	3	2	1	–
33	Delivery of unbounded substructure layer 2/2	4	3	–	–	–	–	–	–	–	–	–	–
34	Leveling of recycled material by grader 2/2	–	–	–	–	3	–	–	–	3	2	1	–
35	Delivery of blanket layer 2/2	2	1	–	–	–	–	–	–	–	–	–	–
36	Leveling and consolidation of sub-base layer 2/2	1	–	–	–	–	–	–	–	–	–	–	–
37	Superelevation of bridge fascia beams	2	1	–	–	2	1	–	–	2	1	–	–
38	Paving of roadside ditch	2	1	–	–	2	1	–	–	2	1	–	–
39	Asphalting of base course using virgin material	2	1	–	–	2	1	–	–	–	–	–	–
40	Asphalting of base course using recycled material	–	–	–	–	–	–	–	–	2	1	–	–
41	Asphalt and blanket layer milling incl. permanent deposit	1	–	–	–	1	–	–	–	–	–	–	–
42	Asphalt and blanket layer milling incl. temporary deposit	–	–	–	–	–	–	–	–	1	–	–	–
43	Execution of base course layer on transitions	1	–	–	–	1	–	–	–	–	–	–	–
44	Execution of recycled base course layer on transitions	–	–	–	–	–	–	–	–	1	–	–	–
45	Spreading of bitumen emulsion	2	1	–	–	2	1	–	–	2	1	–	–
46	Execution of wearing course	2	1	–	–	2	1	–	–	2	1	–	–
47	Execution of driveways	2	1	–	–	2	1	–	–	2	1	–	–
48	Execution of verges	3	2	1	–	3	2	1	–	3	2	1	–
49	Signalization installation	2	1	–	–	2	1	–	–	2	1	–	–
50	Guard rails installation	1	–	–	–	1	–	–	–	1	–	–	–
51	Restoration of road markings	1	–	–	–	1	–	–	–	1	–	–	–
52	Renewal of road slopes	3	2	1	–	3	2	1	–	3	2	1	–
53	Grass planting on slopes and clean-up work	3	2	1	–	3	2	1	–	3	2	1	–
54	Handover procedure	1	–	–	–	1	–	–	–	1	–	–	–

Note: *First activity duration is defined as a continuous variable.

As can be seen from Table II, for conventional and cold-in-plant-recycling production processes, one-option activities are scheduled to be completed in the smallest discrete time unit, i.e. in one day, so these were assumed not be accelerated in any case. On the other hand, some of the one-option activities found in the cold-in-place-recycling production process (i.e. activities 15, 18, 22, 24, 27, 29, 32 and 34) are designed to be unalterably implemented in three days on account of their technological dependence on the work of the bottleneck resource, i.e. the recycler. All other activities can be accelerated from their normal durations. Note here that the lowest-value non-zero option for activity duration in an APP also represents its lower bound, i.e. $D_{i,p}^{\mathrm{LO}}$, while the highest one denotes its upper bound, i.e. $D_{i,p}^{\mathrm{UP}}$.

The lower and upper bounds on the start times for activities (i.e. $S_{i,p}^{\mathrm{LO}}$ and $S_{i,p}^{\mathrm{UP}}$) found in APPs are given in Table III. For each APP, the upper bounds $S_{i,p}^{\mathrm{UP}}$ were set by the latest start times of activities for fixed normal project duration, while the lower bounds $S_{i,p}^{\mathrm{LO}}$ were determined at the earliest start times of activities for fixed crashed project duration.

The engagement of resources in different EMs of road reconstruction activities is reflected through various direct cost options. The direct cost options for activities found in these particular APPs are introduced in Table IV. It can be observed that most direct cost-duration dependences follow a non-decreasing pattern, i.e. direct costs are amplified as the activity is accelerated. However, there are some exceptions. The first can be found in Activity 1 in all production processes, where implementation requires a direct cost of 435 currency units per day, mostly due to the influence of road closure duration. The second can be identified in supportive activities 15 and 27 in the cold-in-plant-recycling production process, where performance depends on the EMs of the leading activities 19 and 31, see $(i, j, p) \in \mathrm{EM}$. Thus, the supportive activities of cement spreading as well as the leading activities of cold recycling without foamed bitumen should be performed with the same duration in order to fulfill the technological requirements of the production chain, see Table II. Hence, acceleration of activities 15 and 27 results in lower cost on account of the reduction in man-hours needed to complete the supportive tasks. Note here that the indirect cost of the project amounted to 150 currency units per day, while there were no agreed penalties and bonuses.

Production workers represent the most important resource in a labor-intensive industry such as the construction sector. Accordingly, resource-constrained scheduling is here focused on the allocation of production workers ($r = 1$) and the consumption of working time ($r = 2$) required for the given project. At this point, production workers were considered universally qualified for assignment to any activity, which is often characteristic of road reconstruction projects, with minor deviations that can sometimes appear in practice. Optional numbers of workers for activities of APPs are provided in Table V.

Time–cost–resource options were determined to enable potential acceleration of project activities by increasing the number of workers and/or machinery and/or working longer hours. These options were supported by estimated working time consumption, which took into account expected productivity losses related to the acceleration measures (Hanna *et al.*, 2008) and therefore yield different values, see Table VI.

Optimization setup and results

The proposed MINLP model was employed to optimally solve the sample tasks addressed in the upcoming sections. A high-level language called a general algebraic modeling system was used for the optimization modeling, data entry and acquirement of outputs; see GAMS (2018). A global solver BARON supported by GAMS (2018), which is based on a branch and reduce (BR) method proposed by Ryoo and Sahinidis (1996), was selected to perform the MINLP optimization. In this connection, the mixed-integer linear programming sub-problems were handled by CPLEX software (based on a branch and bound algorithm), while the nonlinear programming ones were managed by the computational program CONOPT (based on a generalized reduced gradient method). Default termination

Activity $i \in I(p)$ ↓ Proc. →	Description of activity	Lower and upper bounds on start times of activities (day) $S^{LO}_{i,p}$ $p=1$	2	3	$S^{UP}_{i,p}$ $p=1$	2	3
1	Construction site organization with road closure	1	1	1	1	1	1
2	Stake out process	1	1	1	1	1	1
3	Asphalt cutting	2	2	2	3	3	3
4	Removal of asphalt and worn-out street curbs	2	2	2	3	3	3
5	Installation of new street curbs	3	3	3	5	5	5
6	Excavation of drainage ditches	2	2	2	4	4	4
7	Production of culverts, inlet and outlet heads	3	3	3	6	6	6
8	Disassembly of signs and signalization	3	3	3	3	3	3
9	Verge removal 1/2	4	4	4	5	5	5
10	Guard rail removal	5	5	5	7	7	7
11	Verge removal 2/2	5	5	5	7	7	7
12	Demolition of bridge fascia beams	5	5	5	7	7	7
13	Asphalt milling incl. permanent deposit 1/2	6	–	–	9	–	–
14	Asphalt milling incl. temporary deposit 1/2	–	–	6	–	–	9
15	Spreading of cement 1/2	–	6	7	–	9	10
16	Readjusting works: failure of a bottleneck resource 1/2	–	–	–	–	–	–
17	Excavation of blanket layer and substructure 1/2	7	–	–	10	–	–
18	Cold recycling with foamed bitumen 1/2	–	6	–	–	9	–
19	Cold recycling without foamed bitumen 1/2	–	–	7	–	–	10
20	Readjusting works: failure of a bottleneck resource 2/2	–	–	–	–	–	–
21	Delivery of unbounded substructure layer 1/2	7	–	–	10	–	–
22	Leveling of recycled material by compactor 1/2	–	6	7	–	9	10
23	Delivery of blanket layer 1/2	10	–	–	14	–	–
24	Leveling of recycled material by grader 1/2	–	6	7	–	9	10
25	Leveling and consolidation of sub-base layer 1/2	10	–	–	14	–	–
26	Asphalt milling incl. temporary deposit 2/2	–	–	8	–	–	13
27	Spreading of cement 2/2	–	9	9	–	12	14
28	Asphalt milling incl. permanent deposit 2/2	11	–	–	15	–	–
29	Cold recycling with foamed bitumen 2/2	–	9	–	–	12	–
30	Excavation of blanket layer and substructure 2/2	12	–	–	16	–	–
31	Cold recycling without foamed bitumen 2/2	–	–	9	–	–	14
32	Leveling of recycled material by compactor 2/2	–	9	9	–	12	14
33	Delivery of unbounded substructure layer 2/2	12	–	–	16	–	–
34	Leveling of recycled material by grader 2/2	–	9	9	–	12	14
35	Delivery of blanket layer 2/2	15	–	–	20	–	–
36	Leveling and consolidation of sub-base layer 2/2	15	–	–	20	–	–
37	Superelevation of bridge fascia beams	6	6	6	10	10	10
38	Paving of roadside ditch	4	4	4	10	10	10
39	Asphalting of base course using virgin material	15	9	–	20	12	–
40	Asphalting of base course using recycled material	–	–	9	–	–	14
41	Asphalt and blanket layer milling incl. permanent deposit	16	10	–	22	14	–
42	Asphalt and blanket layer milling incl. temporary deposit	–	–	10	–	–	16
43	Execution of base course layer on transitions	17	11	–	23	15	–
44	Execution of recycled base course layer on transitions	–	–	11	–	–	17
45	Spreading of bitumen emulsion	18	12	12	24	16	18
46	Execution of wearing course	18	12	12	24	16	18
47	Execution of driveways	19	13	13	26	18	20
48	Execution of verges	19	13	13	26	18	20

(*continued*)

Table III. Lower and upper bounds on start times of project activities in APPs

Activity $i \in I(p) \downarrow$ Proc. →	Description of activity	Lower and upper bounds on start times of activities (day) $S^{LO}_{i,p}$ $p=1$	2	3	$S^{UP}_{i,p}$ $p=1$	2	3
49	Signalization installation	20	14	14	29	21	23
50	Guard rails installation	20	14	14	29	21	23
51	Restoration of road markings	21	15	15	30	22	24
52	Renewal of road slopes	20	14	14	29	21	23
53	Grass planting on slopes and clean-up work	20	14	14	29	21	23
54	Handover procedure	21	15	15	31	23	25

Table III.

tolerances were fixed for the BR search algorithm to carry out the optimization. The set initial solutions and the obtained optimal results for sample tasks that will be discussed in continuation of this text are presented in Figures 2 and 3.

As far as the feasible starting point for MINLP optimization is concerned, the conventional production process, with activities placed at their normal EMs, was initially set for the given reconstruction project. The corresponding initial project network diagram with critical paths, a Gantt chart of activities, a histogram of workers and an S-curve of cumulative man-hours is shown in Figure 2(a). In reviewing the initial network diagram and related time–cost–resource data, it emerges that when the road reconstruction is implemented by a conventional production process through the normal EMs of activities, the resulting project duration is 31 days and the associated total cost is 232,785 currency units, see the Gantt chart in Figure 2(a). Project duration is determined by two critical paths comprising 25 activities, which are indicated in the network diagram in Figure 2(a) with light gray, connected with thicker arrows. As regards minimum and maximum needs for labor, it can be observed that such a project execution requires between 7 workers (at the beginning of the project) and 31 workers (on the 20th working day), see the histogram in Figure 2(a). The total consumption of working time necessary for project completion in this manner is 5,118 man-hours, see the S-curve in Figure 2(a). Subsequently, the introduced feasible starting point for MINLP optimization was consistently applied in Examples from 1 to 3, while a specific initial solution, based on updated schedule information (which will be explained later), was used in Example 4:

Example 1. Project scheduling under APPs for initial input data.

The purpose of the first example is to present the capacity of the proposed model to select the optimal production process and provide associated exact optimal output data for project management in the form of a network diagram, Gantt chart, histogram and S-curve. The objective of MINLP optimization was to obtain an optimal time schedule for the relevant road reconstruction, with minimum total project cost, subject to a set of GPR constraints, activity duration and start time constraints, lag/lead time constraints, EM constraints, project duration constraints and working time unit assignment constraints. In this particular example, resource constraints, given by Equations (17) and (18), were not activated for either workers or man-hours.

The BR method was applied to solve the stated project scheduling problem to an optimal level taking into account the set input data as presented in section "Input data" and the feasible starting point as given in section "Optimization setup and results." Figure 2(b) shows the results of optimal project scheduling under APPs in the form of a network diagram and an accompanying Gantt chart of activities, a histogram of workers and an S-curve of cumulative man-hours. In this particular example, the

Table IV. Discrete direct cost options for project activities in APPs

Activity $i \in I(p)$ ↓	Discrete direct cost options for activities $dc_{i,k,p}$ (currency unit)											
	Proc. → $p=1$				$p=2$				$p=3$			
Mod. →	$k=1$	2	3	4	$k=1$	2	3	4	$k=1$	2	3	4
1	*	*	*	*	*	*	*	*	*	*	*	*
2	1,700	1,970	2,300	2,670	1,700	1,970	2,300	2,670	1,700	1,970	2,300	2,670
3	120	–	–	–	120	–	–	–	120	–	–	–
4	1,990	2,410	–	–	1,990	2,410	–	–	1,990	2,410	–	–
5	2,200	2,270	–	–	2,200	2,270	–	–	2,200	2,270	–	–
6	2,180	2,450	–	–	2,180	2,450	–	–	2,180	2,450	–	–
7	3,350	3,690	3,910	4,930	3,350	3,690	3,910	4,930	3,350	3,690	3,910	4,930
8	600	780	–	–	600	780	–	–	600	780	–	–
9	1,020	1,220	–	–	1,020	1,220	–	–	1,020	1,220	–	–
10	580	–	–	–	580	–	–	–	580	–	–	–
11	1,020	1,220	–	–	1,020	1,220	–	–	1,020	1,220	–	–
12	1,420	1,530	1,740	–	1,420	1,530	1,740	–	1,420	1,530	1,740	–
13	7,060	–	–	–	–	–	–	–	–	–	–	–
14	–	–	–	–	–	–	–	–	12,160	–	–	–
15	–	–	–	–	4,420	–	–	–	4,420	4,000	3,600	–
16	–	–	–	–	–	–	–	–	–	–	–	–
17	5,400	6,220	6,320	6,870	–	–	–	–	–	–	–	–
18	–	–	–	–	20,680	–	–	–	–	–	–	–
19	–	–	–	–	–	–	–	–	9,180	13,760	19,480	–
20	–	–	–	–	–	–	–	–	–	–	–	–
21	16,310	19,820	–	–	–	–	–	–	–	–	–	–
22	–	–	–	–	1,080	–	–	–	1,080	1,720	2,220	–
23	8,050	8,500	–	–	–	–	–	–	–	–	–	–
24	–	–	–	–	2,070	–	–	–	2,070	2,500	2,600	–
25	780	–	–	–	–	–	–	–	–	–	–	–
26	–	–	–	–	–	–	–	–	12,160	–	–	–
27	–	–	–	–	4,420	–	–	–	4,420	4,000	3,600	–
28	7,060	–	–	–	–	–	–	–	–	–	–	–
29	–	–	–	–	20,680	–	–	–	–	–	–	–
30	5,400	6,220	6,320	6,870	–	–	–	–	–	–	–	–
31	–	–	–	–	–	–	–	–	9,180	13,760	19,480	–

(*continued*)

Table IV.

Activity $i \in I(p)$ ↓	Discrete direct cost options for activities $dc_{i,k,p}$ (currency unit)											
	Proc. → $p=1$				$p=2$				$p=3$			
Mod. →	$k=1$	2	3	4	$k=1$	2	3	4	$k=1$	2	3	4
32	–	–	–	–	1,080	–	–	–	1,080	1,720	2,220	–
33	16,310	19,820	–	–	–	–	–	–	–	–	–	–
34	–	–	–	–	2,070	–	–	–	2,070	2,500	2,600	–
35	8,050	8,500	–	–	–	–	–	–	–	–	–	–
36	720	–	–	–	–	–	–	–	–	–	–	–
37	1,140	1,320	–	–	1,140	1,320	–	–	1,140	1,320	–	–
38	610	710	–	–	610	710	–	–	610	710	–	–
39	65,760	73,630	–	–	65,760	73,630	–	–	–	–	–	–
40	–	–	–	–	–	–	–	–	89,100	92,350	–	–
41	2,320	–	–	–	2,320	–	–	–	–	–	–	–
42	–	–	–	–	–	–	–	–	2,320	–	–	–
43	2,340	–	–	–	2,340	–	–	–	–	–	–	–
44	–	–	–	–	–	–	–	–	2,430	–	–	–
45	3,640	3,670	–	–	3,640	3,670	–	–	3,640	3,670	–	–
46	32,900	40,770	–	–	32,900	40,770	–	–	32,900	40,770	–	–
47	5,930	8,800	–	–	5,930	8,800	–	–	5,930	8,800	–	–
48	3,950	4,510	4,920	–	3,950	4,510	4,920	–	3,950	4,510	4,920	–
49	500	600	–	–	500	600	–	–	500	600	–	–
50	580	–	–	–	580	–	–	–	580	–	–	–
51	1,010	–	–	–	1,010	–	–	–	1,010	–	–	–
52	1,530	1,830	2,030	–	1,530	1,830	2,030	–	1,530	1,830	2,030	–
53	720	770	920	–	720	770	920	–	720	770	920	–
54	400	–	–	–	400	–	–	–	400	–	–	–

Note: *First activity continuously generates a direct cost of 435 currency units per day

Activity $i \in I(p)$ ↓ Mod. →	Description of activity	Proc. → $p=1$ $k=1$	2	3	4	$p=2$ $k=1$	2	3	4	$p=3$ $k=1$	2	3	4
		Optional numbers of workers $q_{i,k,p,1}$ (no.)											
1	Construction site organization with road closure	*	*	*	*	*	*	*	*	*	*	*	*
2	Stake out process	4	5	8	13	4	5	8	13	4	5	8	13
3	Asphalt cutting	1	–	–	–	1	–	–	–	1	–	–	–
4	Removal of asphalt and worn-out street curbs	4	10	–	–	4	10	–	–	4	10	–	–
5	Installation of new street curbs	5	10	–	–	5	10	–	–	5	10	–	–
6	Excavation of drainage ditches	6	12	–	–	6	12	–	–	6	12	–	–
7	Production of culverts, inlet and outlet heads	6	8	12	20	6	8	12	20	6	8	12	20
8	Disassembly of signs and signalization	3	5	–	–	3	5	–	–	3	5	–	–
9	Verge removal 1/2	3	6	–	–	3	6	–	–	3	6	–	–
10	Guard rail removal	3	–	–	–	3	–	–	–	3	–	–	–
11	Verge removal 2/2	3	6	–	–	3	6	–	–	3	6	–	–
12	Demolition of bridge fascia beams	5	7	11	–	5	7	11	–	5	7	11	–
13	Asphalt milling incl. permanent deposit 1/2	8	–	–	–	–	–	–	–	–	–	–	–
14	Asphalt milling incl. temporary deposit 1/2	–	–	–	–	–	–	–	–	8	–	–	–
15	Spreading of cement 1/2	–	–	–	–	2	–	–	–	2	2	2	–
16	Readjusting works: failure of a bottleneck resource 1/2	–	–	–	–	–	–	–	–	–	–	–	–
17	Excavation of blanket layer and substructure 1/2	6	7	12	24	–	–	–	–	–	–	–	–
18	Cold recycling with foamed bitumen 1/2	–	–	–	–	4	–	–	–	–	–	–	–
19	Cold recycling without foamed bitumen 1/2	–	–	–	–	–	–	–	–	4	8	12	–
20	Readjusting works: failure of a bottleneck resource 2/2	–	–	–	–	–	–	–	–	–	–	–	–
21	Delivery of unbounded substructure layer 1/2	10	16	–	–	–	–	–	–	–	–	–	–
22	Leveling of recycled material by compactor 1/2	–	–	–	–	3	–	–	–	3	4	7	–
23	Delivery of blanket layer 1/2	9	17	–	–	–	–	–	–	–	–	–	–
24	Leveling of recycled material by grader 1/2	–	–	–	–	3	–	–	–	3	5	9	–
25	Leveling and consolidation of sub-base layer 1/2	5	–	–	–	–	–	–	–	–	–	–	–
26	Asphalt milling incl. temporary deposit 2/2	–	–	–	–	–	–	–	–	8	–	–	–
27	Spreading of cement 2/2	–	–	–	–	2	–	–	–	2	2	2	–
28	Asphalt milling incl. permanent deposit 2/2	8	–	–	–	–	–	–	–	–	–	–	–
29	Cold recycling with foamed bitumen 2/2	–	–	–	–	4	–	–	–	–	–	–	–
30	Excavation of blanket layer and substructure 2/2	6	7	12	24	–	–	–	–	–	–	–	–
31	Cold recycling without foamed bitumen 2/2	–	–	–	–	–	–	–	–	4	8	12	–

(*continued*)

Table V.
Optional numbers of workers for project activities in APPs

Table V.

Activity $i \in I(p)$ ↓ Mod. →	Description of activity	Proc. → $p=1$ $k=1$	2	3	4	$p=2$ $k=1$	2	3	4	$p=3$ $k=1$	2	3	4
		Optional numbers of workers $q_{i,k,p,1}$ (no.)											
32	Leveling of recycled material by compactor 2/2	–	–	–	–	3	–	–	–	3	4	7	–
33	Delivery of unbounded substructure layer 2/2	10	16	–	–	–	–	–	–	–	–	–	–
34	Leveling of recycled material by grader 2/2	–	–	–	–	3	–	–	–	3	5	9	–
35	Delivery of blanket layer 2/2	9	17	–	–	–	–	–	–	–	–	–	–
36	Leveling and consolidation of sub-base layer 2/2	5	–	–	–	–	–	–	–	–	–	–	–
37	Superelevation of bridge fascia beams	3	6	–	–	3	6	–	–	3	6	–	–
38	Paving of roadside ditch	3	5	–	–	3	5	–	–	3	5	–	–
39	Asphalting of base course using virgin material	14	26	–	–	14	26	–	–	–	–	–	–
40	Asphalting of base course using recycled material	–	–	–	–	–	–	–	–	14	28	–	–
41	Asphalt and blanket layer milling incl. permanent deposit	7	–	–	–	7	–	–	–	–	–	–	–
42	Asphalt and blanket layer milling incl. temporary deposit	–	–	–	–	–	–	–	–	7	–	–	–
43	Execution of base course layer on transitions	9	–	–	–	9	–	–	–	–	–	–	–
44	Execution of recycled base course layer on transitions	–	–	–	–	–	–	–	–	9	–	–	–
45	Spreading of bitumen emulsion	1	1	–	–	1	1	–	–	1	1	–	–
46	Execution of wearing course	14	28	–	–	14	28	–	–	14	28	–	–
47	Execution of driveways	9	18	–	–	9	18	–	–	9	18	–	–
48	Execution of verges	8	13	20	–	8	13	20	–	8	13	20	–
49	Signalization installation	2	4	–	–	2	4	–	–	2	4	–	–
50	Guard rails installation	3	–	–	–	3	–	–	–	3	–	–	–
51	Restoration of road markings	6	–	–	–	6	–	–	–	6	–	–	–
52	Renewal of road slopes	3	6	9	–	3	6	9	–	3	6	9	–
53	Grass planting on slopes and clean-up work	2	3	6	–	2	3	6	–	2	3	6	–
54	Handover procedure	3	–	–	–	3	–	–	–	3	–	–	–

Note: *First activity requires continuous engagement of three workers

Table VI. Optional quantities of working time consumption for project activities in APPs

Activity $i \in I(p)$ ↓ Mod. →	Optional quantities of working time consumption $q_{i,k,p,2}$ (man-hours) Proc. → $p=1$ $k=1$	2	3	4	$p=2$ $k=1$	2	3	4	$p=3$ $k=1$	2	3	4
1	*	*	*	*	*	*	*	*	*	*	*	*
2	128	135	144	156	128	135	144	156	128	135	144	156
3	10	–	–	–	10	–	–	–	10	–	–	–
4	80	100	–	–	80	100	–	–	80	100	–	–
5	100	110	–	–	100	110	–	–	100	110	–	–
6	120	132	–	–	120	132	–	–	120	132	–	–
7	192	216	240	260	192	216	240	260	192	216	240	260
8	54	60	–	–	54	60	–	–	54	60	–	–
9	60	66	–	–	60	66	–	–	60	66	–	–
10	30	–	–	–	30	–	–	–	30	–	–	–
11	60	66	–	–	60	66	–	–	60	66	–	–
12	120	126	132	–	120	126	132	–	120	126	132	–
13	80	–	–	–	–	–	–	–	–	–	–	–
14	–	–	–	–	–	–	–	–	80	–	–	–
15	–	–	–	–	60	–	–	–	60	32	22	–
16	–	–	–	–	–	–	–	–	–	–	–	–
17	192	210	216	240	–	–	–	–	–	–	–	–
18	–	–	–	–	120	–	–	–	–	–	–	–
19	–	–	–	–	–	–	–	–	120	128	132	–
20	–	–	–	–	–	–	–	–	–	–	–	–
21	400	480	–	–	–	–	–	–	–	–	–	–
22	–	–	–	–	72	–	–	–	72	80	84	–
23	144	170	–	–	–	–	–	–	–	–	–	–
24	–	–	–	–	72	–	–	–	72	80	90	–
25	50	–	–	–	–	–	–	–	–	–	–	–
26	–	–	–	–	–	–	–	–	80	–	–	–
27	–	–	–	–	60	–	–	–	60	32	22	–
28	80	–	–	–	–	–	–	–	–	–	–	–
29	–	–	–	–	120	–	–	–	–	–	–	–
30	192	210	216	240	–	–	–	–	–	–	–	–
31	–	–	–	–	–	–	–	–	120	128	132	–

(*continued*)

Table VI.

Activity	Optional quantities of working time consumption $q_{i,k,p,2}$ (man-hours)											
$i \in I(p)$ ↓	Proc. → $p=1$				$p=2$				$p=3$			
Mod. →	$k=1$	2	3	4	$k=1$	2	3	4	$k=1$	2	3	4
32	–	–	–	–	90	–	–	–	72	80	84	–
33	400	480	–	–	–	–	–	–	–	–	–	–
34	–	–	–	–	90	–	–	–	72	80	90	–
35	144	170	–	–	–	–	–	–	–	–	–	–
36	50	–	–	–	–	–	–	–	–	–	–	–
37	60	66	–	–	60	66	–	–	60	66	–	–
38	48	55	–	–	48	55	–	–	48	55	–	–
39	280	312	–	–	280	312	–	–	–	–	–	–
40	–	–	–	–	–	–	–	–	280	308	–	–
41	70	–	–	–	70	–	–	–	–	–	–	–
42	–	–	–	–	–	–	–	–	70	–	–	–
43	90	–	–	–	90	–	–	–	–	–	–	–
44	–	–	–	–	–	–	–	–	90	–	–	–
45	10	11	–	–	10	11	–	–	10	11	–	–
46	280	308	–	–	280	308	–	–	280	308	–	–
47	180	198	–	–	180	198	–	–	180	198	–	–
48	192	208	240	–	192	208	240	–	192	208	240	–
49	40	44	–	–	40	44	–	–	40	44	–	–
50	30	–	–	–	30	–	–	–	30	–	–	–
51	60	–	–	–	60	–	–	–	60	–	–	–
52	72	84	90	–	72	84	90	–	72	84	90	–
53	60	66	72	–	60	66	72	–	60	66	72	–
54	30	–	–	–	30	–	–	–	30	–	–	–

Note: *First activity requires a continuous working time consumption of 30 man-hours per day

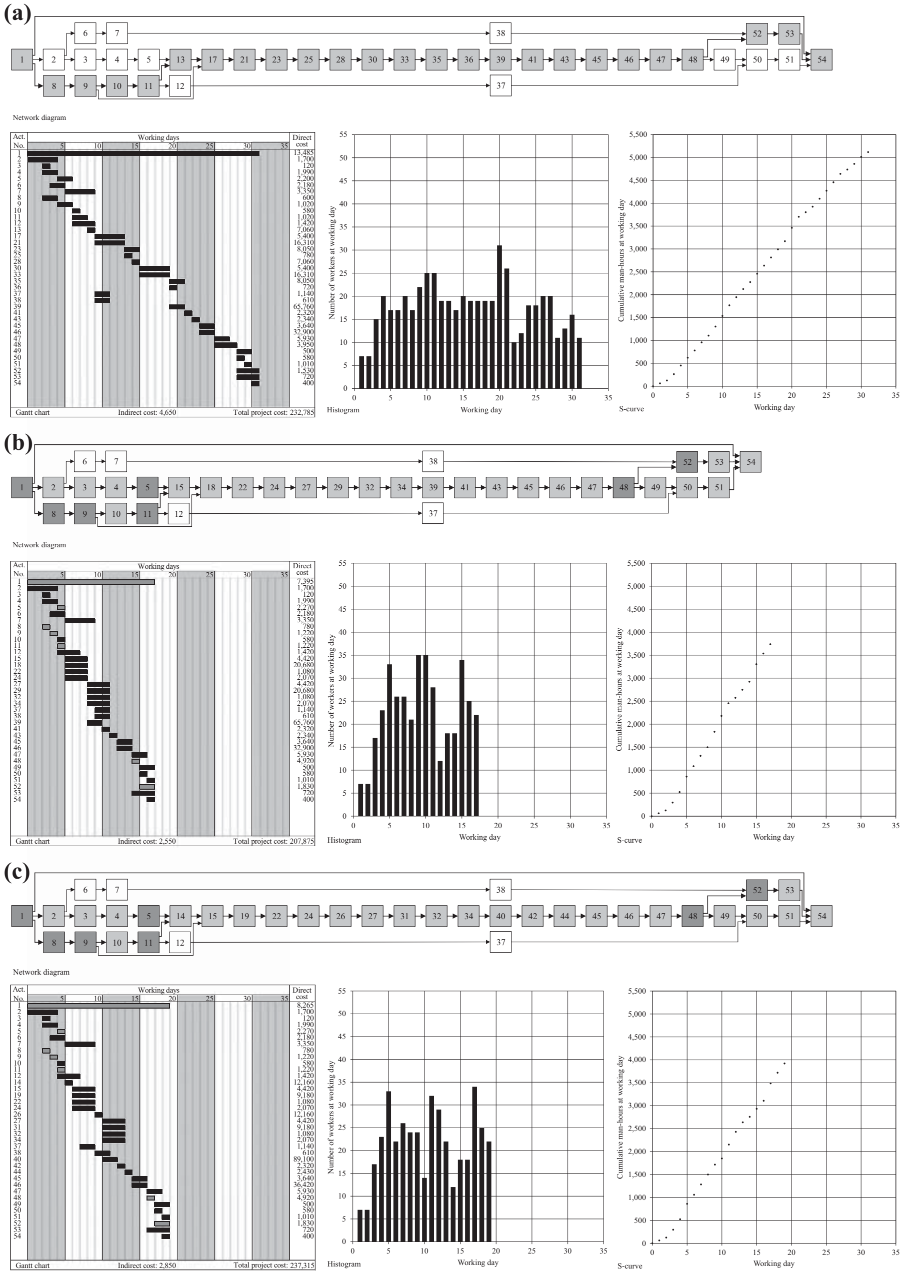

Figure 2.
Project network diagram, Gantt chart, histogram of workers and S-curve of cumulative man-hours related to: (a) initial solution; (b) optimal solution found under APPs for initial input data; (c) optimal solution found under APPs for updated cost data

cold-in-place-recycling production process emerged as the optimal one under which the given road reconstruction project should be implemented. The MINLP optimization of the project schedule under APPs achieved a minimum total project cost of 207,875 currency units and an optimal project duration of 17 days. The optimal project schedule for the

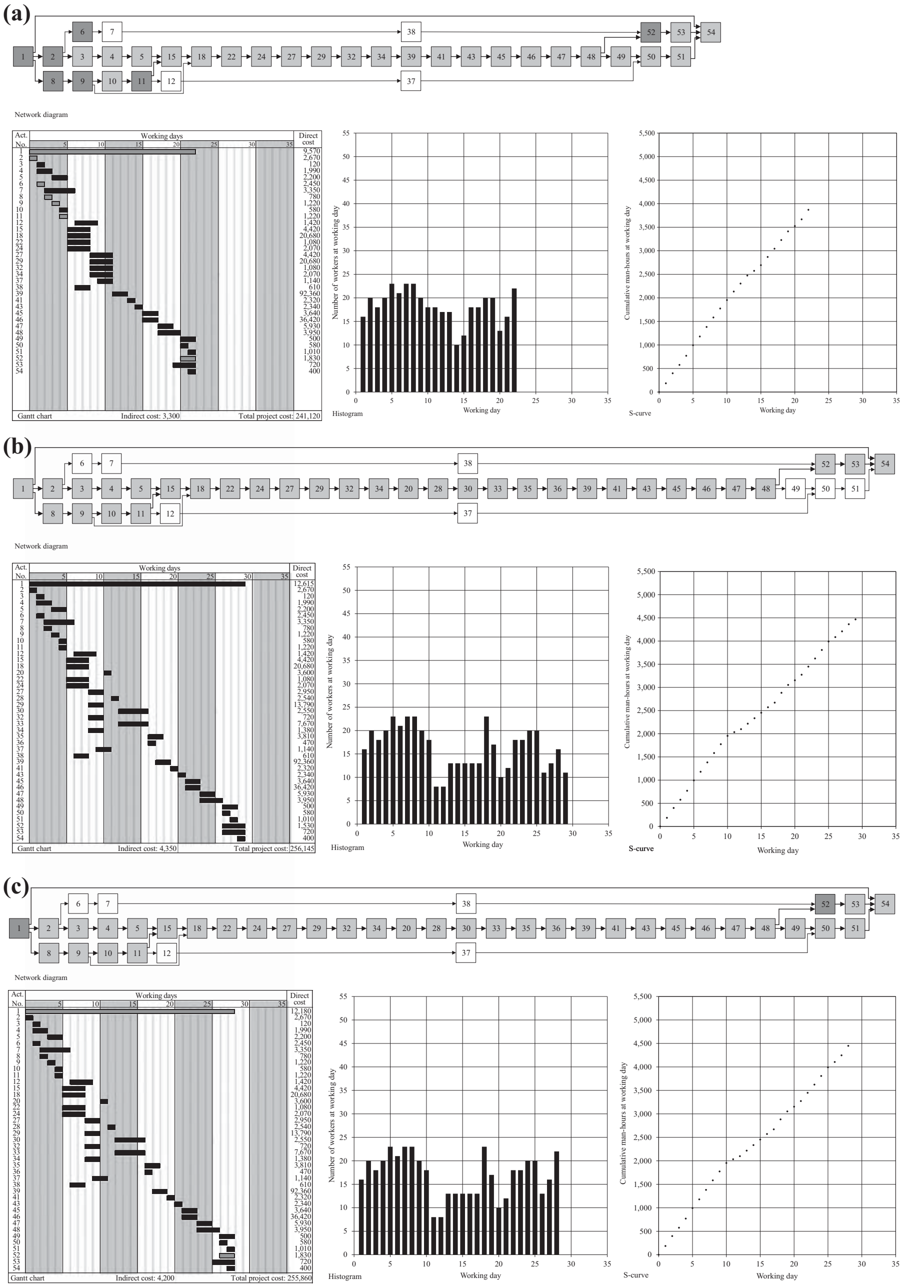

Figure 3.
Project network diagram, Gantt chart, histogram of workers and S-curve of cumulative man-hours related to: (a) optimal solution found under APPs for constrained number of workers and cumulative man-hours; (b) initial solution determined after total failure of a bottleneck resource; (c) optimal solution identified after total failure of a bottleneck resource

selected cold-in-place-recycling production process was attained by accelerating critical activities 5 (–1 day), 8 (–1 day), 9 (–1 day), 11 (–1 day), 48 (–2 days) and 52 (–1 day), as indicated by the dark gray in the network diagram and Gantt chart in Figure 2(b). In this context, project acceleration caused the formation of five critical paths determined by

30 activities, which are highlighted in the network diagram in gray and connected by thick arrows. Note here that the first activity, i.e. construction site organization with road closure, formally represents a critical activity, which always starts at the beginning of the project and ends with its completion. However, this activity, as a supportive task, does not determine the project duration but adapts to it.

Given the parallel implementation of activities, the increment in the labor requirement reached the maximum value of 35 workers between the 9th and 10th working days, see the histogram in Figure 2(b). Because optimization shortened the project duration from 31 to 17 days, the slope of the S-curve of cumulative man-hours consequently increased in comparison with the initial project scheduling solution. However, the total consumption of working time was reduced from 5,118 to 3,738 man-hours, owing to the selection of a more labor-efficient production process, see the S-curve in Figure 2(b):

Example 2. Project scheduling under APPs for updated cost data.

The aim of Example 2 is to show the model's capacity for updating with the latest cost data, and then to perform optimal project scheduling under different production process if such a solution appears to be more favorable. In this context, the present example will take into account a situation where the price of a certain key material has unexpectedly increased and needs to be considered in scheduling. More precisely, the increase in the price of asphalt virgin material raised the expected direct cost of activities 39 and 46 in the conventional and cold-in-place-recycling production processes. However, in the cold-in-plant-recycling process, the expected direct cost is amplified only for Activity 46, since Activity 40, i.e. an equivalent substitute for Activity 39, needs to be implemented with recycled asphalt material, which is not influenced by the virgin material price increase.

Accordingly, the initial direct cost options for Activity 39 given in Table IV, i.e. 65,760 and 73,630 currency units, were updated to new values of 92,360 and 100,230 currency units, respectively. In the same manner, the initial direct cost options of Activity 46, i.e. 32,900 and 40,770 currency units, were also raised to values of 36,420 and 44,290 currency units, respectively. All other input data were kept the same as in the previous example. After setting the optimization problem, the BR algorithm was run, resulting in the optimal scheduling solution demonstrated in Figure 2(c).

The cold-in-plant-recycling production process was found to be the optimal one for project execution with the updated cost data. Here, the optimal solution was attained at a project duration of 19 days and a minimum total cost of 237,315 currency units, see the Gantt chart in Figure 2(c). Since optimization was performed at amplified prices for virgin material, the selection of a production process relying on recycled material was expected, to a certain degree. However, the augmented cost data also resulted in a higher minimum total cost and a longer optimal project duration than in the prior example. Compared with the solution obtained in the previous example, the optimal project schedule was reached by accelerating the same common activities for an equal amount of time, though the five critical paths in this case included more activities, i.e. 32, see the network diagram in Figure 2(c). In such a project implementation, the maximum number of 34 workers occurs on the 17th working day, while the total consumption of working time is 3,922 man-hours, see the histogram and the S-curve in Figure 2(c), respectively:

Example 3. Project scheduling under APPs and constrained resources.

The third example is intended to demonstrate the model's potential to select the optimal production process as well as provide the project scheduling solution that requires the minimum total cost under constrained increment and cumulative quantities of resources. Taking into account the pronounced labor intensity of road reconstruction, the restrictions on available workers and man-hours were made the focus of this specific example. Compared to the preceding example, the input data applied here were only upgraded with the information that a maximum of 25 workers can be present at the same time on the construction site each working day and at most 4,000 man-hours can be spent on project completion. In terms of the formulation proposed by section "MINLP model formulation," the resource constraints determined by Equations (17) and (18) were accordingly activated within the MINLP model. After activation of these conditions, the search with the BR algorithm was started, and the optimal results presented in Figure 3(a) were acquired.

Unlike the optimal solution from the previous case, the results obtained here suggest that the cold-in-place-recycling production process should be used in executing the road reconstruction project under set restrictions of workers and man-hours. Activation of additional constraints in the model caused a raise in the minimum total project cost, i.e. 241,120 currency units. On the other hand, selection of a different production process resulted in project completion that was scheduled three days later than in the prior example, i.e. the optimal project duration of 22 working days. Here, the optimal project schedule was achieved by accelerating activities 2(−3 days), 6(−1 day), 8(−1 day), 9(−1 day), 11(−1 day) and 52(−1 day), see the Gantt chart in Figure 3(a).

Moreover, it can be noticed that the optimal project duration is determined by 30 critical activities, which are designated in the network diagram in Figure 3(a), in gray and mutually connected by thicker arrows. The highest increments to required workers occur on the 5th, 7th and 8th working days and show a number of 23, see the histogram in Figure 3(a). The anticipated cumulative consumption of working time at scheduled project completion is below the limit value and amounts to 3,870 man-hours, see the S-curve in Figure 3(a). Output results show that resource restrictions proved decisive for the optimal solution, although neither extreme increments nor maximum cumulative values were found on the boundary of feasible space. The main reason for such results lies in the fact that resource data was set with discrete alternatives and not with continuous parameters; see Tables V–VI:

Example 4. Project scheduling under APPs and constrained resources after total failure of a bottleneck resource.

The main goal of the final example is to present the model's capacity to determine the optimal production process with an associated network diagram and accompanying schedules in the case of total failure of a difficult-to-replace bottleneck resource during the project execution. At this point, this example takes the implementation of road reconstruction into account in accordance with optimal scheduling solution found in previous case, i.e. section "Project scheduling under APPs and constrained resources," and the total recycler failure that occurred the 10th day in Activity 29, on two-thirds of the project, under 2/2 of road closure and after the second day of operation. In the event of recycler breakdown, it was ascertained that the remaining non-implemented work could be completed by the conventional production process, while cold-in-plant-recycling was not a relevant alternative in this project phase, given the technological issues. Furthermore, it was found that the project should be finished under the limited number of 25 workers, while the restriction of 4,000 man-hours needs to be relaxed because of more labor-intensive APP.

The project input data were updated accordingly. Completed activities from 2 to 24 were fixed as shown in the optimal project scheduling solution of section "Project scheduling under APPs and constrained resources," see the Gantt chart in Figure 3(a). Activities 27, 29, 32 and 34 were determined to be two-thirds complete, while activity 37 was set to be finished. The remaining non-implemented work among these activities was set to be performed by substitute activities 28, 30, 33, 35 and 36, taken from the conventional production process, however with suitably updated options for direct costs and resource quantities, i.e. number of workers and man-hours. Activity 20, which comprises necessary readjustment work (i.e. removal of the recycler, cement spreader, bitumen and water tankers, etc.), was declared in the model and connected with its preceding task 34 and succeeding activity 28 by FS relationships without lag/lead times. Start times for the finished and partially completed activities were set as in the previously obtained solution, while the rest of them were allowed to be optimally selected between their lower and upper bounds. Updated project input data are given in Table VII.

All unfinished project activities subjected to optimal scheduling were initially set to their normal EMs. Accordingly, the initial project scheduling solution generated after total failure of a bottleneck resource was included in the model, as presented in Figure 3(b). The resulting updated normal project duration was 29 days, and the associated total cost was 256,145 currency units. Moreover, the corresponding network diagram shows that the project duration was determined by three critical paths composed of 33 activities. The minimum number of workers required was 8 and occurred between the 11th and 12th working days, while the maximum increment of 23 workers appeared on the 5th, 7th, 8th and 18th working days. The anticipated total consumption of working time for such a project implementation was 4,467 man-hours.

After setting the starting point, the BR algorithm was launched, and the optimal scheduling solution was identified, as shown in Figure 3(c). The optimal project schedule was attained at a minimum total cost of 255,860 currency units and an optimal project duration of 28 days, which was achieved by accelerating activity 52 by one day. This also led to the creation of three additional critical activities: 49, 50 and 51. Values and positions of the minimum and maximum worker increments remained the same as in the updated normal project scheduling solution, while the total consumption of working time was reduced to 4,449 man-hours.

Discussion

This section is devoted to addressing the advantages and limitations of the proposed model. Initially, it is necessary to emphasize that the model relies on a deterministic estimation of time–cost–resource data. In project-oriented industries like construction, it is commonly known that precise deterministic assessment of such data can be a challenging task that needs considerable experience and should be performed by qualified practitioners. However, the deterministic approach is widely used in construction practice on account of the recognized utility of its outputs in decision making. From that perspective, deterministic optimization of project schedules can frequently ensure valuable data for making efficient management decisions.

Problem complexity is another issue that should be discussed here. Project scheduling tasks as discussed herein may represent extensive nonlinear discrete optimization problems in many real-life circumstances, and globally optimal solutions in such instances can generally be difficult to attain through all existing methods. Although the area of nonlinear (discrete) optimization is demanding and not yet as mature as linear (continuous) optimization, high-quality exact solutions can still be expected from state-of-the-art MINLP methods, even for comprehensive project scheduling tasks.

To accelerate convergence of the optimal solution applying the proposed model, it is advisable to launch the MINLP algorithm from a feasible starting point. In this context, the MINLP optimization of the project schedule under APPs addressed in the first sample application was run on a conventional production process, with normal EMs for activities. This example demonstrated the model's potential to assure optimal scheduling results at a minimum total cost in forms relevant to project management, i.e. network diagrams, Gantt charts, histograms and S-curves. At this point, the capacity to provide the exact discrete optimal solution can be indicated as a beneficial characteristic of the proposed model. Namely, optimization software and project management tool (for example MS Project) can

Table VII. Updated project input data determined after total failure of a bottleneck resource

Activity $i \in I(p)$ ↓	Bounds on start times		Discrete direct cost options $dc_{i,k,p}$ (currency unit)				Optional number of workers $q_{i,k,p,1}$ (no.)				Optional working time consumptions $q_{i,k,p,2}$ (man-hours)			
Mod. →	$S_{i,p}^{LO}$ (day)	$S_{i,p}^{UP}$ (day)	$k=1$	2	3	4	$k=1$	2	3	4	$k=1$	2	3	4
1	1	1	*	*	*	*	*	*	*	*	*	*	*	*
2	1	1	1,700	1,970	2,300	*2,670*	4	5	8	*13*	128	135	144	*156*
3	2	2	*120*	–	–	–	*1*	–	–	–	*10*	–	–	–
4	2	2	*1,990*	2,410	–	–	*4*	10	–	–	*80*	100	–	–
5	4	4	*2,200*	2,270	–	–	*5*	10	–	–	*100*	110	–	–
6	2	2	2,180	*2,450*	–	–	6	*12*	–	–	120	*132*	–	–
7	3	3	*3,350*	3,690	3,910	4,930	*6*	8	12	20	*192*	216	240	260
8	3	3	600	*780*	–	–	3	*5*	–	–	54	*60*	–	–
9	4	4	1,020	*1,220*	–	–	3	*6*	–	–	60	*66*	–	–
10	5	5	*580*	–	–	–	*3*	–	–	–	*30*	–	–	–
11	5	5	1,020	*1,220*	–	–	3	6	–	–	60	*66*	–	–
12	7	7	*1,420*	1,530	1,740	–	*5*	7	11	–	*120*	126	132	–
15	6	6	*4,420*	–	–	–	*2*	–	–	–	*60*	–	–	–
18	6	6	*20,680*	–	–	–	*4*	–	–	–	*120*	–	–	–
20	11	11	3,600	–	–	–	2	–	–	–	20	–	–	–
22	6	6	*1,080*	–	–	–	*2*	–	–	–	*60*	–	–	–
24	6	6	*2,070*	–	–	–	*3*	–	–	–	*72*	–	–	–
27	9	9	2,950	–	–	–	2	–	–	–	40	–	–	–
28	12	13	2,540	–	–	–	5	–	–	–	35	–	–	–
29	9	9	13,790	–	–	–	4	–	–	–	80	–	–	–
30	13	14	2,550	3,020	3,340	4,010	4	6	9	16	112	126	144	160
32	9	9	720	–	–	–	3	–	–	–	*60*	–	–	–
33	13	14	7,670	9,890	–	–	6	10	–	–	240	300	–	–
34	9	9	1,380	–	–	–	3	–	–	–	60	–	–	–
35	16	18	3,810	4,680	–	–	6	12	–	–	84	108	–	–
36	16	18	470	–	–	–	4	–	–	–	28	–	–	–
37	10	26	*1,140*	1,320	–	–	*3*	6	–	–	60	66	–	–
38	7	25	610	710	–	–	3	5	–	–	48	55	–	–
39	16	18	92,360	100,230	–	–	14	26	–	–	280	312	–	–
41	17	20	2,320	–	–	–	7	–	–	–	70	–	–	–

(*continued*)

Table VII.

Activity $i \in I(p)$ ↓	Bounds on start times		Discrete direct cost options $dc_{i,k,p}$ (currency unit)				Optional number of workers $q_{i,k,p,1}$ (no.)				Optional working time consumptions $q_{i,k,p,2}$ (man-hours)			
Mod. →	$S_{i,p}^{LO}$ (day)	$S_{i,p}^{UP}$ (day)	$k=1$	2	3	4	$k=1$	2	3	4	$k=1$	2	3	4
43	18	21	2,340	–	–	–	9	–	–	–	90	–	–	–
45	19	22	3,640	3,670	–	–	1	1	–	–	10	11	–	–
46	19	22	36,420	44,290	–	–	14	28	–	–	280	308	–	–
47	20	24	5,930	8,800	–	–	9	18	–	–	180	198	–	–
48	20	24	3,950	4,510	4,920	–	8	13	20	–	192	208	240	–
49	21	28	500	600	–	–	2	4	–	–	40	44	–	–
50	21	28	580	–	–	–	3	–	–	–	30	–	–	–
51	22	29	1,010	–	–	–	6	–	–	–	60	–	–	–
52	21	27	1,530	1,830	2,030	–	3	6	9	–	72	84	90	–
53	21	27	720	770	920	–	2	3	6	–	60	66	72	–
54	22	29	400	–	–	–	3	–	–	–	30	–	–	–

Notes: *First activity duration is defined as a continuous variable. Underlined values were reached before total recycler failure be integrated together into a uniform system (see e.g. Valenko and Klanšek, 2017) so the export of exact optimization data is needed for presentation of optimal network diagrams, Gantt charts, histograms and S-curves in the software packages that are used in practice.

Unlike classic project scheduling approaches, the proposed model incorporates an adaptive production process based on generated alternatives rather than a fixed one. In this sense, the first example demonstrated that the optimal project structure was identified by model-endogenous decisions and not by the planner, as is usually done in practice. As shown through these examples, such an approach certainly enlarges the feasible space but enables more efficient scheduling solutions than would be possible if the project structure was fixed before optimization.

From the viewpoint of cost optimization in project scheduling, the well-known fact that resource prices change over time should be highlighted here. The planner may be aware of the cost amount for APPs at a time when developing a model. However, the cost data generally changes over time, and thus the related baseline values can considerably differ from those identified at the time of model application, which influences the selection of the optimal solution. For instance, in the construction industry, the time lag between signing a contract with an agreed price and the beginning of its execution, i.e. the formal introduction of the contractor to the work when the project deadline starts running, can be significant and may influence the total cost faced by the contractor in terms of current resource prices. The second example therefore showed the model's capacity to be updated with recent cost data and its ability to execute optimal project scheduling simultaneously with the selection of different production processes when such an outcome proves to be better.

However, these examples also draw attention to the fact that some increment in the quantities of resources (e.g. number of required workers) may increase significantly on account of reduced project duration and more pronounced parallelism in the execution of activities. In this way, peak increments of resources can sometimes exceed the contractor's capacity to cover such extremes during project implementation. The third example demonstrated the model's ability simultaneously to choose a production process that offers minimum total cost and to ensure an optimal project schedule taking into account limited resources increments. Although resource-constrained project scheduling is well-covered in the contemporary literature, this model's ability to perform it under APPs and in a cost-optimal manner represents an innovation offered in this paper.

Managing cumulative resource consumption can also be important for project success in many actual situations. For example, in the construction business, interim payments to the contractor are frequently associated with cumulative quantities of work done on the project. Thus, it is often crucial to support scheduled work progress with related cumulative consumption of necessary resources, but still paying attention to the contractor's capacity. To this end, the third sample application emphasized the model's advantage in selecting the optimal production process and providing the project scheduling solution at the minimum total cost, considering constrained cumulative quantities of resources (e.g. the quantity of man-hours).

A total failure of expensive and difficult-to-replace bottleneck resources can seriously jeopardize a project's success. Besides, in cases of high-value projects, such an event can sometimes even endanger the contractor's existence on the market. Therefore, it is important that the planner also take into consideration the APPs that can be suitably applied if such circumstances occur during the project execution. The last application case pointed out the model's advantageous capacity to be suitably updated after such events and its ability to select the optimal production process for project continuation, as well as to support it with associated scheduling data. Here it is worth mentioning that addressing APPs with a single model, rather than by separate ones, enables more efficient updating of input data within one file and execution of parametric optimization if it proves necessary. This may prove convenient during the model usage, especially when project's APPs contain some common activities, relations or resources so that the update can be performed simultaneously for all of them. This feature of the model enables the contractor to make a faster response and more efficient adaptation to a new situation on site.

This paper presents a natural continuation of previous work done on cost-optimal project scheduling by MINLP approach reported in Klanšek and Pšunder (2012) and Klanšek (2016). From the perspective of novelties, it is necessary to expose not only the new MINLP model features that allow it to handle APPs but also those improvements that are beneficial even for cost-optimal resource-constrained project scheduling under user-predetermined and fixed network diagram (e.g. for production process selected by the planner), concretely: enabling the lag/lead time to be considered as a decision variable related to the production process and the activity EM rather than as a fixed parameter; inclusion of EM constraints for dealing with dependencies between project activities that may arise from some specific production process requirements; and introduction of (non)linear time-dependent constraints on increment and cumulative quantities of multiple resource types for which it is also possible to take into account unfavorable effects of activity acceleration, for example productivity losses.

In order to facilitate the model implementation into practice as well as to enable a transparent repetition of presented optimization calculations and verification of reported results, the application examples took into account the case of an actual road reconstruction project, which was allowed to be performed under technologically conditioned limited number of fairly rigidly determined alternatives for the production process. However, this should not be considered as a general fact for construction projects which can be highly complex and executable under a wide variety of alternatives for a production process.

Since the paper proposes a newly developed MINLP model with peculiar features, the direct comparison with the existing ones, from the viewpoint of required convergence time, was not possible. Nevertheless, it is important to expose that all computer work in this study was done on a 64-bit operating system using a personal computer Intel Core i7, 2.80 GHz, 16 GB random-access memory and 250 GB solid-state drive with which the presented optimization results were achieved within a couple of minutes. At the end of this section, it is also necessary to emphasize, alongside practical benefits presented in above discussion, that the proposed MINLP model gives the exact optimal result, which is an advantage over the heuristic models that calculate approximate optimal solutions.

Conclusion

This paper has presented the MINLP model for cost optimization of project schedules under constrained resources and APPs. The proposed model ensures the exact optimal output data in forms relevant to project management, i.e. network diagrams, Gantt charts, histograms and S-curves. The paper reveals that the optimal project structure is gained as model-endogenous decision. The new optimization model incorporates an adaptive production process based on generated alternatives rather than a fixed process. Therefore, the feasible space is enlarged to allow more efficient scheduling solutions than would be possible if the project structure had been fixed before the optimization. In this way, the project planner is enabled to perform optimization of the production process simultaneously with resource-constrained scheduling of activities in discrete time units and at a minimum total cost. An additional feature of the model is that it enables an operative determination of the optimal production process with an associated network diagram and accompanying updated schedules in case of total failure of a bottleneck resource, that is difficult to replace during the project's execution.

The unique value this paper contributes to the body of knowledge reflects through the proposed MINLP model, which is capable of performing the exact cost optimization of production process (where presence and number of activities including their mutual relations are dealt as feasible alternatives, meaning not as fixed parameters) simultaneously with the associated resource-constrained project scheduling, whereby that is achieved within a uniform procedure. At this point, the simultaneous synthesis of optimal network diagram topology and resource-constrained project schedules underscores the scientific value of the paper while model's usage on an actual road reconstruction project supports its applicability. To the best of our knowledge, no such optimization model has yet been developed and applied to an actual construction project as it was demonstrated in application examples. Furthermore, the presented model avoids the need for time-consuming trial-and-error analysis of identified APPs and harmonization with associated project schedules, which are necessary in conventional scheduling approaches.

The ability to identify the exact optimal discrete solution under nonlinear terms can be put in the fore as an advantage of the MINLP model. Namely, the application of nonlinear expressions aids in achieving more compact optimization problem formulation including accelerating some model management tasks such as transforming the gathered data into model parameters and performing the model updates. Wherever reasonable, the model's combinatorial size can be decreased with continuous variables included into nonlinear dependences instead of employing number of integer variables for discretization of nonlinearities. Aware that NP hard problems are being addressed, the generation of MINLP superstructure of alternatives still requires to being performed attentively, in order to take full benefits of proposed model.

The MINLP model introduced in this paper shows a considerable degree of flexibility, which allows its use in a variety of projects, especially those in an industry that incorporates production processes. However, the superstructure of discrete alternatives (like options for activity duration, cost, workers, etc.), which are linked together over the assigned binary decision variables, takes some effort to be suitably generated, especially in cases of projects with numerous combinations. Moreover, the model's features that have been demonstrated suggest that its application may be particularly advantageous in the following instances: high-value one-of-a-kind projects where even a single implementation of such a model could create profit; repetitive projects where benefits can be achieved through multiple use of the model with possible minor modifications; and projects that rely on expensive bottleneck resources, such as specifically designed heavy machinery, facilities or equipment, where the model's output results can assist with efficient change in the production process if breakdown accidentally occurs.

Glossary

APP	alternative production process
BR	branch and reduce method
EM	execution mode
FF	finish-to-finish precedence relation
FS	finish-to-start precedence relation
GAMS	general algebraic modeling system
GPR	generalized precedence relations
LO	lower bound
MINLP	mixed-integer nonlinear programming
SF	start-to-finish precedence relation
SS	start-to-start precedence relation
UP	upper bound

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