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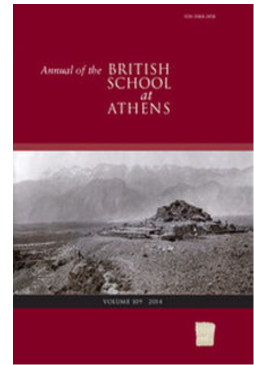

Towards Understanding Greek Temple Design: General Considerations

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TOWARDS UNDERSTANDING GREEK TEMPLE DESIGN: GENERAL CONSIDERATIONS

IN a recent paper on stylobate design in Doric temples[1] I stated, with the minimum of evidence, certain working hypotheses on some of the factors restricting ancient Greek architectural design procedures. The present paper is, in part, an attempt to examine more closely the evidence for those hypotheses, but it also has a wider aim; it tries to thin out the almost numberless possible theories concerning the way in which Greek temples were planned, by looking at some of the technical restrictions felt by ancient architects, by stating and examining the consequences of certain hypotheses which seem to be widely, if tacitly, accepted by students of Greek architecture, and by suggesting how many of the propositions put forward by modern investigators may be tested, more or less rigorously perhaps, but nevertheless objectively.

A first assumption is, of course, that Greek temples really were designed; that is, that the finished building was the realization of the architect's conscious intention. It is indeed possible that the sizes of the various elements of a temple were not predetermined by the architect, but decided by, for instance, the distance that a pebble could be thrown, the amount of stone that could be carried in one cartload, or by the size of the block that first came to hand. There is, however, no evidence that such procedures were ever adopted by the Greeks, and it is hard to see how they could produce the results we find in Doric temple architecture, where the ratios relating one part to another are fairly stable at any given time and place, but change quite definitely from one period or place to another.

Even if a building is the conscious product of the architect's mind, however, that does not necessarily mean that he could freely work out all the elements of his design with a view to the

Besides the abbreviations normal in *BSA*, the following are used:

(*a*) Bibliographical:

Aegina = A. Furtwängler, *Aegina, das Heiligtum von Aphaia* (1906).

Bouras = Ch. Bouras, *I Anastilosis tis Stoas tis Vravronos* (1967).

Corinth = American School of Classical Studies at Athens, *Corinth, Results of Excavations* . . . (1929–).

Didyma ii = A. Rehm (ed. T. Wiegand), *Didyma* ii (1958).

EAD = École française d'Athènes, *Exploration archéologique de Délos* (1909–).

FD = École française d'Athènes, *Fouilles de Delphes* (1902–).

Heath = T. L. Heath, *History of Greek Mathematics* (1921).

KP = R. Koldewey, O. Puchstein, *Griechische Tempel in Unteritalien und Sicilien* (1899).

Martin = R. Martin, *Manuel d'architecture grècque* i (1965).

Olympia = E. Curtius, F. Adler, *Olympia, Ergebnisse* . . . (1890–96).

Orlandos = A. Orlandos, *Ta ilika domis ton arkhaion Ellinon* (1955–8).

Penrose = F. C. Penrose, *Investigation of the Principles of Athenian Architecture* (2nd edn., 1888).

Roux = G. Roux, *L'Architecture de l'Argolide aux iv et iii siècles av. J.-C.* (1961).

(*b*) Elements of a columnar order:

A = Architrave height
Ab = Abacus height
AbW = Width of abacus
Cor = Cornice height
Cp = Capital height
D = Lower column diameter on arrises
F = Frieze height
H = Column height
I = Axial intercolumniation
L = Length over stylobate
mod. = module
OL = Over-all length
OW = Over-all width
S = Stylobate breadth from back to front
T = Triglyph width
W = Width over stylobate

Where the size of an element on the flank of a building differs systematically from that on the front, the letter W (or L), written subscript, indicates that the abbreviation refers to the front (or flank) of the building.

[1] *BSA* lxix (1974), 61–86.

best aesthetic effect. Factors of function, finance, structure, or site might well control the formulation of the design. Nevertheless, in the case of the Doric temple, it is probable that it will be safe, initially at least, to disregard these factors. Both the function and the structure of a Greek temple were very simple and conventional. Only in a very few extreme cases is there evidence of problems in these fields governing the form of a temple; the form of the Erechtheion is presumably governed largely by its function—the need to include and respect a large number of cults and holy places—as well as by its site, while the form of the Olympieion at Akragas is apparently governed to a large extent by structural problems arising from its colossal size.[2]

Finance also appears to have been of minor significance in the designing of a Greek temple. Certainly the very existence of a temple depended on the availability of money, but although the availability of money may have controlled the size and perhaps the material of a temple, there is no sign that it controlled the form. A shortage of funds was apparently more likely to result in an unfinished temple than in a temple of a different kind.[3]

The nature of the specific site rarely appears to have affected the over-all form of a Greek temple either. The Erechtheion is again an exception to the rule, but it is virtually unique, and normally a Greek temple was a freestanding building set on a level site—an artificially levelled platform if necessary. But although the site almost never controlled the general form of a Greek temple (apart from setting some limit to its size), it may have affected the design more subtly; the architect might modify his design to suit the character of the site. 'The look of a building when seen close at hand is one thing, on a height it is another, not the same in an enclosed place, still different in the open', says Vitruvius,[4] and he suggests that its design should be adjusted accordingly. Vitruvius is here talking of Roman houses, however, and it is not clear whether the character of the site was normally taken into consideration by Greek temple architects. In any case the modifications which Vitruvius has in mind are apparently to be small ones; their aim is to create the impression of a normal design by compensating for any optical illusions induced by the nature of the site, rather than to create an entirely different impression.

Thus, although we should obviously bear in mind the rather summary exclusion of these factors from immediate consideration, it may be reasonable, initially at least, to regard the basic design problem for a Greek temple architect as the evolution of an aesthetically satisfying building.

This assumption is of considerable importance to the argument. First of all, it will be much easier to analyse the design of Greek temples if only one factor is of primary importance. Secondly, any problems of finance and site at least would be special to a particular building, so that if such factors governed the design of a temple, each architect might be facing entirely different problems. But if the aesthetic form of the temple was the architect's major preoccupation, then all of them were facing a common problem, and we may reasonably go on to assume that they did not each invent a completely new system of design to deal with it. The uniformity of proportion in temples from a given area and period, and the strict adherence to convention both in general arrangement and in detail, provide the best support for this further hypothesis. It is unlikely that we should find such uniform results if each design was based on a different system. It is unlikely, therefore, that one architect worked only in ratios based on π, a second worked only in simple arithmetic ratios, a third based his design on equilateral triangles,

[2] M. Guido, *Sicily, An Archaeological Guide* (1968) 123–5.

[3] Thus almost all the colossal temples were either never finished or left unfinished for centuries. The desire for economy might lead to a non-peripteral temple, but the structural methods were unchanged, and in so far as there were columns at all, the forms were normally the same as in a peripteral one.

[4] vi. 2. 2.

a fourth took the triglyph width as a module, a fifth took the lower diameter of the column as a module, and so on. It would of course be unreasonable to suggest that every temple, no matter when or where built, was designed in one and the same way, but we should probably regard as suspect, if not necessarily invalid, a proposed rule which cannot be applied to more than one building.[5]

Even with the simplifications to the problem of analysing Greek design which have been suggested above, the problem we are left with is by no means an easy one. There are so many lines and rectangles that might perhaps be significant in the design of a Doric peripteral temple, that some of them are almost certain to coincide with the lines of almost any proposed scheme of design. For instance, it has recently been suggested that the temple of Zeus at Nemea was designed to have a length of 140 feet between the outer tangents of the east and west colonnades;[6] and another study suggests that the second temple of Hera at Paestum was designed so that the significant proportion of temple length to width was realized only at the level of the architrave taenia.[7] Can we then simply choose at will the level at which the architect may be supposed to have chosen to set out the length and breadth of the temple as simple dimensions or as significant ratios one of the other? Similarly there are various ways in which we may suppose him to have considered the diameter of the columns; he might have taken the diameter on the arrises, in the flutes or between the chords across opposite flutes; he might have taken the lower diameter, the average diameter,[8] or even the upper diameter measured in any of these three ways. In dealing with a single building there is no objective way of deciding between these possibilities, yet in each case it is virtually certain that only one of them was considered by the architect as of primary importance to his design.

Furthermore, even if we manage to make the right choice among each set of alternatives, the relationship linking any two of them can be expressed in a large number of ways. For instance, if two parts of a building are found to be related to each other as $1:2\frac{1}{2}$, the architect might have visualized this ratio simply as $1:2\frac{1}{2}$; but he might also have visualized it as $6:15$ or as $5\frac{1}{3}:13\frac{1}{3}$—or as an approximation to $1:\sqrt{6}$ or $4:3\pi$; but it is at least highly unlikely that he visualized it in all of these ways.[9] How is one to show that the design scheme proposed for a particular building is the one used by the original architect, and not just an alternative way of expressing the same design, perhaps by means of an entirely different procedure?

Previous scholars investigating the principles of Greek architectural design have generally based their work on an intensive study of a single building,[10] and they have, therefore, two criteria by which to assess or compare various possible schemes of design: first, the accuracy with which the measurements predicted by the proposed system agree with those actually measured on the building, and second, the simplicity with which the proposed rules can be expressed (for there is no reason to suppose that Greek architects liked unnecessary complexity). The first

[5] Any exceptional procedures will be best distinguished and understood if we know the methods of design normally in use, just as exceptional forms are distinguished and interpreted on the basis of a knowledge of the forms normal to Greek architecture. We should therefore concentrate initially on understanding the general procedures, rather than the exceptions and modifications to be found in individual buildings.

[6] *AJA* lxxii (1968) 188–9.

[7] F. Krauss, *Paestum, die griechischen Tempel* (1943) 62–3.

[8] Koldewey and Puchstein (KP *passim*) regarded the average diameter as the significant one in most Sicilian and South Italian temples.

[9] Compare the two different rules which would give a stylobate length $2\frac{1}{2}$ times its width to a temple with 6×14 columns (*BSA* lxix (1974) 61–86).

[10] Even the fundamental study of Greek design methods by Bundgaard (J. A. Bundgaard, *Mnesicles. A Greek Architect at Work* (1957)) is based primarily on the Propylaia. One exception is the attempt by Koldewey and Puchstein to work out the basic systems of the Sicilian and South Italian temples, but this is only one in quite a general way. Hambidge's attempt to demonstrate the use of dynamic symmetry in Greek architecture deals mainly with the Parthenon, with much less thorough studies of other buildings (J. Hambidge, *The Parthenon and other Greek Temples: their Dynamic Symmetry* (1924).

criterion is straightforward enough. Although the degree of accuracy achieved is not always clearly expressed, particularly when the system involves the superimposition of a geometrical scheme on a comparatively small-scale plan or elevation, it is nevertheless quite simple to compare objectively the accuracy with which two proposed systems fit the existing remains. The second criterion, although obviously important, is less objective. Some people may think that a ratio of $1:\sqrt{2}$ (easily constructed from the diagonal of a square) is more simple than one of $5:7$. To some extent we may look for guidance from Vitruvius in choosing which expression to prefer, but it will not be universally admitted that Vitruvius' testimony is a guide to the practice of architects working four or five hundred years earlier.[11] Since the measurement predicted by a rule of $1:\sqrt{2}$ ($= 1:1{\cdot}414$) may well be very close to that predicted by a rule of $5:7$ ($= 1:1{\cdot}4$), it will be clear that, even together, these criteria may not be sufficient to show convincingly that a certain scheme was in fact the one used for a particular building by its architect.

It is therefore of the greatest importance if we can reasonably assume that successive Greek temple architects were facing virtually the same problems, and were using conventional systems of design, just as they used conventional forms and proportions. For this will give us a third criterion by which to judge a proposed scheme of design. It is most unlikely that the architect of the second temple of Hera at Paestum, building a highly conventional temple, was the only man to design a temple so that there was a simple proportion of length to width at the level of the architrave taenia. If no other temple can be found to display the same characteristic, then it is likely that the existence of this proportion in the second temple of Hera is simply a matter of coincidence. If, however, simple proportions can be shown to exist at the same level in other temples, and particularly if these other temples belong to the same period and area, then the possibility of coincidence becomes much more remote. If in addition these other buildings are different in size and shape, the probability of a coincidence will be even smaller. A great advantage of this third criterion, the number of buildings to which a proposed system can be applied, is that it is objective. One can set out clearly to how many buildings a proposed rule applies, where and when they were built, and what difference there is between the predicted values and the measurements actually found for the relevant parts of the building. While it may be impossible to prove conclusively that a certain rule or system was used, it should at least be possible, using all three criteria, to show objectively which of any proposed rules or systems is more likely to have been used.

Let us now return to our first hypothesis, that Greek temples were in fact designed. If so, then there is a limit to the number of ways in which their architects could have worked. If the external factors of function, finance, structure, and site were rarely of major importance, then the architect must have worked on internal principles.[12] Of these we have clear written evidence of

[11] One restrictive factor which we have not considered is the occasional need to re-use material from another building, as in the Parthenon and the fourth-century temple of Apollo at Delphi. Such cases will normally be readily recognizable, however.

[12] Besides the general probabilities that will emerge in the course of this study (as for instance in the treatment of fractions), there is a positive argument for the relevance of Vitruvius to the study of classical Greek architecture in the discrepancy between the design method suggested by *De Arch.* i. 1. 4 and i. 2. 2 and that implied in *De Arch.* Books iii and iv. In i. 1. 4 and i. 2. 2, Vitruvius assumes that plans and elevations of a proposed building will be drawn, and that would appear to have been contemporary practice, for Roman scale plans of buildings have survived. However, plans and elevations are nowhere mentioned in Books iii and iv; they are not necessary to the application of the rules (cf. p. 63 below); and the absence of detailed prior planning is strongly suggested by phraseology such as 'The shafts of the columns having been erected, the rule for the capitals will be as follows' (iii. 5. 5). Some of Vitruvius' rules clearly go back only to the Hellenistic period (such as those for the pseudo-dipteral plan and for eustyle column-spacing), but it is unlikely that a new procedure of the sort implied by Books iii and iv would have been developed for Hellenistic temples, if earlier temples were already designed by means of plans and elevations drawn to scale; so this design procedure probably goes back at least to the fifth and fourth centuries, and perhaps to the beginning of Greek monumental architecture.

only two, proportion and dimension; any part of a building might be defined either as a simple proportion of another, or as a convenient number of feet or parts of a foot. For the use of convenient numbers of feet we have evidence in numerous building inscriptions going back to the fifth century B.C. It might be argued that many of these inscriptions are sub-contracts and inventories where dimensions might well be approximated for convenience; but Philo's specification for an Arsenal at Peiraieus (probably 347 B.C.)[13] is also expressed in simple dimensions. The main dimensions are in round numbers of feet, and all the dimensions given for the stonework are in feet and quarter-feet; dactyls are used only for the timber specifications. If the Arsenal was not designed in detail before construction began, of course some adjustments to these simple dimensions might have been found necessary, but the fact that measurements in dactyls *were* used for the woodwork makes it clear that the simpler figures given for the stonework were not just regarded as rough approximations, but were the sizes actually intended at this stage in the design process.

For rules of proportion used to define architectural elements, the written evidence is much later. Books iii to vi of Vitruvius' *De Architectura* are the most obvious evidence for the use of such rules, and through Vitruvius we can trace them, surely, back to Hermogenes in the second century B.C.[14] Vitruvius' extension of the practice right back to the archaic period[15] might seem less reliable, but the surviving remains bear him out. From as early as the first half of the sixth century B.C. Doric capitals of different sizes are closely similar in proportion.[16]

A third factor in defining the dimensions of the elements of a building might be the architect's eye. In the passage referred to above where Vitruvius discusses modifications to a design to suit the site, he implies that these will be devised by instinct rather than by rule—'haec autem etiam ingeniorum acuminibus non solum doctrinis efficiuntur.'[17] It is possible that a whole building could be designed by this means. The architect might draw out the plan and elevation of his building by eye so as to produce an attractive and fashionable appearance, and then measure up the various parts on the drawing, scale up the measurements to full size, and lay them out accurately on the building itself.

While this is a possible procedure, it seems a rather twisted way of designing, and it presupposes an improbably sophisticated technology, particularly for the sixth century B.C. For it would require precise architectural drawing and rulers with accurate and closely spaced divisions in order to reproduce exactly an effect created by eye at a much smaller scale. If such a method *was* used for the over-all design of a façade, one would expect that when the dimensions were scaled up from the drawing, there would be a considerable amount of approximation. If one imagines the façade of the temple of Zeus at Olympia, for instance, drawn out on a drawing-board 1·5 m. wide (as large as any a modern architect normally works with), then the scale could not be larger than 1:20, and 0·001 m. on the drawing would represent 0·020 m. on the ground. Unless the subdivisions on the architect's ruler were less than 0·001 m. apart, therefore, this method could not produce measurements in fractions of a dactyl, and the unavoidable rounding out of dimensions would be greater for any smaller drawing-board or for any coarser subdivision of the architect's ruler.[18]

If this sort of method was used at all in Greek architecture, it is perhaps more likely that it was

[13] *IG* ii². 1668; K. Jeppesen, *Paradeigmata* (1959) 69–101.

[14] Vitruvius iii. 3. 8.

[15] Vitruvius iv. 1. 6; Vitruvius iv. 1. 3 talks of an earlier stage before the adoption of rules of proportion.

[16] See, for example, in the proportional analysis of some early Doric capitals in *BCH* lxxxvii (1963) 639–52.

[17] vi. 2. 2.

[18] On the subdivision of Greek measuring instruments, see below, pp. 90 ff. Gudea's ruler has single dactyls divided into 2, 3, 4, 5, and 6 parts (i.e. down to 0·0028 m.: L. W. King, *History of Sumer and Akkad* (1923) figs. 64–5), and an Egyptian cubit rule has successive single dactyls divided into 2, 3, 4 . . . 14, 15, 16 parts (i.e. down to 0·0012 m.: A. E. Berriman, *Historical Metrology* (1953) 76).

used for the smaller elements of the building. For there is considerable evidence that a Greek architect provided the builders with full-scale models of certain parts of a building, such as capitals and triglyphs,[19] and from these models measurements could be taken directly with callipers, without any need for scaling up or for conversion of the measurements into feet and dactyls.

The result to be expected from the application of this method of design to a series of buildings is disorder. In so far as architects designed by eye, we should expect to find proportions, and even forms, varying considerably between contemporary buildings; few dimensions, and those just a random collection, would be in round numbers of feet, and there would be equally few parts related to each other by simple proportions. Such a system of design in fact does not tally with the existing remains of Greek architecture, and it is this that is the serious argument against it. For it could not account for the relative uniformity of Doric architecture over quite substantial areas and periods, and in buildings of different sizes. How could the contemporary ideal of the Doric order be transmitted with such consistency? Are we to imagine that Greek architects circulated accurate, large-scale drawings of their buildings throughout the Greek world, so that their colleagues might know the current fashion and design by eye?

In order to explain the state of affairs which we find in Greek architecture, it would be necessary to modify the system, and suppose that there were conventional rules of proportion which were followed within certain limits, and that the architect's eye served only to define the exact sizes and proportions within those limits. If rules of proportion governed the general forms, and some rounding out was unavoidable in executing the design, then this system would simply be an alternative to the assumption of substantial error on the part of the builders as an explanation of the fact that simple proportions seem to occur rarely. It would not affect the validity of a proportional analysis, for the significant field for investigation would clearly be the rules of proportion which restricted the workings of the architect's eye. The assumption that this is how Greek architects designed, besides being bizarre, does not therefore substantially affect the way in which our investigation is to be carried out.

At this point we may conveniently make a general distinction between a governing factor and a defining factor in fixing the size of a particular part of a building. One factor, the governing factor, may indicate the approximate size which that part should have, while another defines the precise size within the range allowed by the governing factor. Thus in the modified 'architect's eye' system discussed and rejected above, the governing factor would have been proportion, while the defining factor was the architect's eye. As mentioned above we have written evidence for proportion as an important factor in Greek architectural design, and the surviving remains overwhelmingly bear this out. Nevertheless, although it is possible that the governing system and the defining system were one and the same, and that a single series of simple proportional rules defined precisely and completely the size given to each part of a building, no convincing system of this kind has yet been proposed, and it is probable that more than one factor is required. We have seen that there is also substantial evidence for dimension as a factor in design. The choice of simple dimensions can only be a defining factor, however, not a governing factor, for unless all buildings are of a standard size, and the architect simply repeats the conventional dimensions (which is not the case with Greek temple design, of course), how is he to decide which out of all the round numbers, or whole numbers, or simple fractions, he is to use next? We can, however, suppose that the dimensions of each part of a building were governed by a system of proportion, but defined by being rounded out to the nearest convenient dimension. That might explain, theoretically at least, the general uniformity of Doric proportions and at the same time the infrequency of simple ratios.

[19] For references to *paradeigmata* see Orlandos ii. 268 n. 3.

Some scholars have suggested that instead of dimension, another system of proportion was used to define the precise dimensions of each part. Lloyd[20] proposed a system based on two series of ratios—6:1, 7:2, 8:3, etc., and 2:1, 3:2, 4:3, etc. Hambidge[21] proposed a system based on a series of ratios derived geometrically, and Tiberio[22] a system based on what he calls R rectangles. Their authors have not always recognized the fact, but all these are defining systems; the approximate sizes must already have been known to the architect before he began to apply such a system, for they all provide such a wide selection of suitable ratios, and such a degree of flexibility in applying them, that the basic uniformity of Greek architecture must be due to some more rigid underlying system, the governing system. In fact the wider the range of ratios which a proposed defining system offers, and the greater the flexibility with which they may be applied, the smaller is the significance that can be attached to it.[23] For the very obvious uniformity of Greek architecture must have resulted from a system which restricted the architect, not one which left him free play.

A further weakness in these systems is the absence of any written evidence for their use in Greek architecture. This is not a conclusive objection, for we cannot expect written evidence for everything; but it does mean that the real existence of such systems must be proved more rigorously from the monuments than the use of round numbers of feet and of a proportional system of the Vitruvian type.

There is little further we can say in general about the possible workings of the Greek architect's eye, but it may be helpful to offer some further consideration of the two factors which seem to lie at the bottom of Greek design procedure—proportion and dimension—and two matters which are closely associated with them—the treatment of fractions and standards of accuracy. Before doing so, however, it may be helpful to emphasize a last general point about the sort of explanation we should look for.

In so far as Greek temple design is indeed based on proportion and dimension, then within the limits of accuracy current for the building concerned, every member of a Greek building must have been defined *either* as a convenient number of feet or parts of a foot, *or* as a convenient and/or desirable proportion of a part of the building which had itself already been defined. We may not know the length of the foot or the nature of the proportional system, but from the architect's point of view there must have been a chain of rules, perhaps supplemented and modified by rounding out of dimensions, which would lead him from the first element whose size he decided on, whether it be intercolumniation, module, or over-all width, to the last element whose size needed to be defined. We should certainly remember the word 'convenient', for there is no reason why the architect should have wished to add unnecessary complexity to his task. Thus it is not an adequate explanation of design to say, for instance, that an architect made the architrave taenia $3\frac{19}{31}$ dactyls high so that it was $\frac{7}{48}$ of the architrave height of 1 foot $8\frac{24}{31}$ dactyls, for there is no reason why the architect should consciously choose such inconvenient fractions.[24] Any or all of these facts may be true, but they do not for that reason reflect the architect's intentions. In the case suggested here more reasonable explanations might be these: the architect intended the taenia height to be about $\frac{1}{7}$ of the architrave height, and the inaccuracy arose because he was not concerned with precision and laid out the proportion quite roughly; or because the execution did not coincide exactly with his design; or that the taenia

[20] Penrose 111–16.

[21] J. Hambidge, *The Parthenon and other Greek Temples: their Dynamic Symmetry* (1924).

[22] C. Tiberio, *Mnesicle. L'Architetto dei Propilei* (1964).

[23] Thus the flexibility of Lloyd's system, to which Penrose admiringly draws attention (Penrose 116), should be regarded as an argument not for, but against, its importance.

[24] This is, of course, an exaggerated example, but in some cases the explanation offered does seem to be little more than a conversion into other units and types of fraction, as, for instance, in *Hesperia* Suppl. v (1941) 93.

was designed in relation to something other than the architrave height—perhaps the abacus height; or that the architect was not using the foot standard we thought he was using, and that he in fact set out the architrave height as $1\frac{1}{2}$ feet, and the taenia height as $\frac{1}{7}$ of this, to the nearest half dactyl, i.e. $3\frac{1}{2}$ dactyls. Each of these possibilities could probably be tested in a real case either by reference to other buildings or to other parts of the same building. When offering an explanation of design, we must be sure that our explanation does not involve unnecessary work for an architect who was presumably more interested in architecture than computation.

PROPORTION

A first question which should be asked about the systems of proportion in Greek architecture is why they were used. Two reasons spring to mind, although they need not necessarily be exclusive: either it was because, as theoretically based systems with carefully calculated ratios in inner harmony with each other, they could create true beauty without previous reference to the physical form of the building; or it was because they were a practical aid to the architect, enabling him to produce a building which would stand up, and would conform to, or slightly modify, the current idea of what a temple should look like.[25] One might use the analogy of a musical score, the value of which lies not so much in the inherent beauty of the arrangement of lines and symbols as in its ability to record an elaborate composition so that a later artist may either repeat or modify it.

Explanations of the first type are usually offered, or assumed, in connection with the more complex defining systems mentioned above, and the same line of thought seems to be inherent in Le Corbusier's theory of the Modulor.[26] Some Greek philosophers, notably the Pythagoreans, attributed an almost mystical significance to certain numbers and numerical relationships, and the discovery that simple numerical ratios determined the principal intervals of the musical scale might well have led, as it did later, after the Renaissance, to doctrinaire applications of proportion to other fields of art. In Greece, however, even to philosophers aesthetic value, in so far as it was not derived from ethical or mimetic considerations, lay largely in such general concepts as *metriotes* and *symmetria*, without specific ratios or sorts of ratio being preferred.[27] More specifically Plato suggested that the most beautiful shapes were the plane and solid figures that could be constructed with ruler, compasses, and set-square,[28] but even that leaves a colossal range of possibilities; and since Plato regarded mathematical concepts as much superior to any embodiment of them,[29] his ideas would be neither helpful nor encouraging to an architect.

Indeed, although there is evidence that philosophers might be influenced by the ideas of artists,[30] there is no good evidence that artists were influenced by the mathematical theories of philosophers. Vitruvius certainly thought that an architect should know some mathematics,[31] but his knowledge would be on quite a different level from that of the pure mathematician, for it was to be limited to those parts indispensable to the architect; and when Vitruvius lists the famous philosophers,[32] it is to commend those of their results that had a practical use. In the fourth century B.C. Pytheos optimistically held that the architect should be better at each subject than the specialist in it,[33] but there is no suggestion that his mathematical expertise was to be more valuable than his knowledge of history, astronomy, or medicine. In any case, since Greek archi-

[25] The distinction is rightly made by P. H. Scholfield, *The Theory of Proportion in Architecture* (1958) 18.

[26] Le Corbusier, *The Modulor* (Eng. edn., 1954) 90, and note the procedure suggested ibid. 158–60.

[27] Plato, *Phlb.* 64 d–e, *Tim.* 87 c–d; Aristotle, *Metaph.* 1078a36, *Poet.* 1450b35, *Top.* 116b21.

[28] Plato, *Phlb.* 51 c–d.

[29] Plato, *Rep.* vii. 529 c–530 a.

[30] Demokritos and Anaxagoras were inspired by Agatharchos' scene painting (Vitruvius vii. praef. 11), and Chrysippos quoted Polykleitos' *canon* (Galen, *Placit. Hipp. et Plat.* 5).

[31] i. 1. 3–16.

[32] 9. praef.

[33] Vitruvius i. 1. 12.

tecture took on its characteristic form and proportional basis before the philosophical theories had been elaborated, it is clear that, whatever may have happened later, Greek architects cannot have adopted proportional systems in the first instance because they produced beauty by theory.

Vitruvius' approach to proportion is definitely non-doctrinaire. He does talk of perfect numbers—6, 10, and 16—but in the systems of proportion he recommends these numbers appear no more frequently than any others. In fact the theoretical value lies in the principle of *symmetria* in general, not in the specific rules of proportion;[34] and in giving his practical rules, Vitruvius never draws attention to the theoretical value of the specific proportions he advises. Indeed he gives only the minimum number of rules to define each part of the building, and never suggests that these will also create desirable relationships between parts not directly linked by one of his rules (e.g. cornice height to capital height, or entablature height to column height). Furthermore, the principle of varying the proportions to avoid undesirable optical effects produced by differences in scale or position[35] shows that he is interested in creating buildings which are beautiful because they look beautiful, not because they conform to some theory.

This is of course late evidence for the ideas of Greek architecture, but the fact that even in the fifth century B.C. we find sophisticated refinements which are based on what is seen to be satisfactory, not what is satisfactory in theory, shows that architects then too had a pragmatic rather than a theoretical outlook. Vitruvius attributes the same kind of thinking to the architects who built the first monumental temple in Ionia: finding that the height of a man was six times the length of his feet, they made their Doric columns six diameters high. For Vitruvius there was probably a theoretical value in the links between architectural proportions and human proportions, and he felt that these links were important also to those who originated the orders; nevertheless, it was not because of any inherent value in the number six, or the proportion 1 : 6, that this proportion was chosen; it was because it was structurally adequate and corresponded to the proportion actually found in a thing which was accepted as beautiful.[36] Similarly when Vitruvius tells us that the proportion was later changed to 1 : 7;[37] there is no suggestion of any theoretical reason for the change; more slender columns were simply more pleasing to contemporary taste.

There is therefore some evidence from an architectural context that the specific rules of proportion did not have a theoretical basis, but were a practical aid to the architect in producing the result he wanted. It has been persuasively argued that the Greek attitude to proportion in sculpture was similar; its aim was not to conform to a preconceived theory which would produce beauty automatically, but to serve as a basis upon which to create an appearance which would be accepted as beautiful in fact.[38] Here again, the theoretical content seems to lie in the value of *symmetria* in general, rather than in the specific rules adopted. Thus when we learn that Lysippos produced a new canon of human proportion with a smaller head and more slender body, it is clear that the ideas of a canon and of *symmetria* in general were important to him, but we are told of no theoretical basis for the specific changes in the rules; they simply produced the kind of effect that Lysippos wanted.[39] Furthermore the principle of subjective modifications to the rules of proportion was followed in the sculpture as well as the architecture of the fifth century.[40]

[34] Vitruvius iii. 1. 5–8; see also TABLE 2, p. 83. P. H. Scholfield (*The Theory of Proportion in Architecture* (1958) 16–32) has argued that a more sophisticated theory lies behind Vitruvius' words, but he admits (26) that the rules do not often follow the proposed theory.

[35] Vitruvius iii. 3. 12; iii. 5. 8–9; vi. 2. 1–4.

[36] Vitruvius vi. 1. 6. Six, as we have seen, is listed by Vitruvius as a perfect number, but the ratio chosen for the Ionic column (iv. 1. 7) was 1:8, later 1:9.

[37] Vitruvius iv. 1. 8.

[38] E. Panofsky, *Meaning in the Visual Arts* (1955) 62–72.

[39] Pliny, *NH* xxxiv. 61. Here again the idea of *symmetria* in general seems to have been important, but no theoretical basis is suggested for the specific rules adopted.

[40] Plato, *Soph.* 235e–236a; Tzetzes, *Chil.* viii. 358 (Overbeck, *Schriftquellen*, no. 772).

If we are right in concluding that systems of proportion were primarily aids to the architect, that suggests that they had little to do with higher mathematics, for the architect will have wanted the simplest system that would produce the desired result.[41]

The most commonly considered proportional system is the modular system, that is a system where all the elements making up the design are dimensioned as multiples or simple fractions of a single measurement which forms the basis of the system. An obvious example is the system set out by Vitruvius for the Doric order,[42] in which I = $7\frac{1}{2}$ modules, D = 2 mod., H = 14 mod., A = 1 mod., F = $1\frac{1}{2}$ mod., Cor = $\frac{1}{2}$ mod., T = 1 mod., etc. Since the module chosen will presumably be a convenient dimension, multiples of it will also be reasonably convenient dimensions, so that little rounding out of the sizes given by the rules would be required. If one can find the module of such a system, therefore, it will be easy to recognize the nature of the system, for all the other elements will turn out at once to be multiples or simple fractions of it. But even if the module is not immediately recognized, the nature of the system will still be made clear by a proportional analysis, for all the smaller elements of the design will form simple proportional relationships with each other. Thus if the system proposed by Vitruvius was used for an actual building unknown to us, we should get precisely the same results by analysing the proportions in terms of the triglyph width or the architrave height as those given above. Even if we calculated the proportions in terms of the frieze height, we should still get very simple results which would suggest a modular system: I = 5F, D = $1\frac{1}{3}$F, H = $9\frac{1}{3}$F, A = $\frac{2}{3}$F, F = 1F, Cor. = $\frac{1}{3}$F, T = $\frac{2}{3}$F, etc.

Another characteristic of the modular system is that it will be very stable. Since all the elements are tied directly to the module, a change in the rule for one element will not alter the interrelationships of the other parts. Thus even if we do not immediately recognize the module and use some other element as the basis of our calculations, we shall find the same proportions repeated in any other buildings which use the same modular system, and since an architect is more likely to change one rule than to change the whole system, most of these proportions will remain the same as the system evolves. Another reason for the probable stability of a modular system is that there are comparatively few values that can be given to the smaller elements of the system, for instance the capital height, triglyph width, and architrave height of the Vitruvian Doric, which are all one module. It is difficult to alter such values without either radically changing the appearance of the building (as by increasing any of these elements to $1\frac{1}{2}$ modules or reducing them to $\frac{1}{2}$ module), or spoiling the simplicity of the modular system (as by raising the capital height to $1\frac{1}{8}$ modules or reducing the triglyph width to $\frac{7}{8}$ module). In the larger elements of the design on the other hand, changes in proportion will be less of a problem; raising the column height from 14 modules to 15 modules (as Vitruvius recommends for stoas) neither spoils the simplicity of the system nor produces too abrupt a change in the appearance of the colonnade. Proportional relationships between the larger elements may therefore be less stable. They will also tend to be less simple; H/I according to Vitruvian rules will be 28/15 for a temple (but 2/1 for a stoa), H/W for a tetrastyle distyle façade will be 14/27 ($1:1\frac{13}{14}$), for a hexastyle systyle façade $14/29\frac{1}{2}$ ($1:2\frac{3}{28}$), but for a hexastyle distyle façade 1:3.

[41] The fourth-century Treasury of Kyrene at Delphi has been held to embody higher mathematical theory (*FD*, J. Bousquet, *Le Trésor de Cyrène* (1952), *passim*), but see Dinsmoor's review (*AJA* lxi (1957) 402–11). Dinsmoor shows that the dimensions of the building, as worked out by Bousquet, are suspect (and it is indeed doubtful whether the building is well enough preserved to give the required accuracy and certainty); that much of the mathematical theory depends on the conversion of these measurements into feet and dactyls, in terms of a foot standard which is not certain to have existed; and that many of the proportional relationships held to involve irrationals can be explained in simpler terms (thus the ratio $\sqrt{2}:\sqrt{3}$ is virtually the same as $4\frac{1}{2}:5\frac{1}{2}$ (0·816... as against 0·818...), so that, e.g., the upper diameter of the column was probably derived from the lower diameter by taking $4\frac{1}{2}$ parts out of $5\frac{1}{2}$ (cf. Vitruvius iii. 3. 12; $4{\cdot}5D/5{\cdot}5 = 0{\cdot}455$ m., $d = 0{\cdot}456$ m.), rather than by using the ratio $\sqrt{2}:\sqrt{3}$).

[42] Vitruvius iv. 3. 3–10.

Although a modular system is simple in operation, however, it is not necessarily a simple conception. There is, for example, no obvious reason why the whole design should be seen in terms of one of its parts, why the triglyph width should control the column height, or the column diameter the cornice height. The column height is much more naturally derived from the diameter, the intercolumniation, or the width of the stylobate, for these are the things to which it is visually related.

Not all the systems described by Vitruvius are modular systems, however. In his rules for Ionic,[43] for instance, he starts off by deriving a module from the stylobate width and this is taken as the lower diameter of the column; but this module is very soon forgotten, and the system proceeds basically by relating each element to the one defined previously; thus architrave height is derived from column height, frieze height and cornice height from architrave height, sima height from cornice height, etc. If we were to work out the sizes of these elements in modules, we should get, for a eustyle building with columns 15 to 20 feet high, the following results: $I = 3\frac{1}{4}$ modules, $D = 1$ mod., $H = 9\frac{1}{2}$ mod., $d = \frac{11}{13}$ mod., $A = \frac{19}{26}$ mod., $F = \frac{57}{104}$ mod., dentil ht. $= \frac{19}{91}$ mod., Cor. $= \frac{19}{91}$ mod., and sima ht. $= \frac{171}{728}$ mod. One could of course call this a modular system based on a module of D/728, with $H = 6{\cdot}926$ mod., $A = 532$ mod., $F = 399$ mod., dentil ht. $= 152$ mod., sima ht. $= 171$ mod. But at that rate the term modular loses all significance, and it seems obvious that a system like this would not have been applied in practice by calculating the size of each part directly from a module.

With a system of this sort, which we may perhaps call a successive system of proportion, the relationship between any two parts of the design taken at random is unlikely to be expressible simply. For instance, the proportion of sima height to lower column diameter in Vitruvian Ionic may be 171 : 728; not surprisingly, this relationship did not seem to the originator of the system a significant one. Furthermore, the relationship between two parts of the design taken at random is less likely to remain constant over a number of buildings than in a modular system. For the sima height is related to the lower column diameter in Vitruvian Ionic by five separate rules, whereas in a modular system there would be at the most two, linking each part directly to the module. While we might expect any one of these rules to remain constant over a number of buildings, the chances that all five will remain constant are obviously considerably less. Thus it should be possible to distinguish whether a group of buildings is built on a modular or a successive system, even without knowing fully how the particular system worked.

It is clear that a change in the relationship between two parts taken at random could result equally from a change in any of the series of rules relating them. Thus an increase in the sima height in terms of the lower column diameter might (in the Vitruvian system) be produced either by an increase in the column height in terms of the lower diameter or by an increase in the architrave height in terms of the column height—that is either by the adoption of more slender columns or by the adoption of a heavier entablature. It is doubtful whether these two changes could be regarded as of identical significance in the stylistic development of Greek architecture, so that it would be unsafe to use the proportions of sima height to lower diameter as a stylistic index. As far as I know, nobody has done so, but we can perhaps generalize from this example and say that it is safe to regard as stylistically significant only those relationships which were regarded as significant by the architect. Thus the proportion of architrave height to frieze height has been used as a stylistic index in Doric architecture of the fourth century B.C.[44] We know that in Vitruvian Ionic the frieze height was derived directly from the architrave height, but was it in fourth-century Doric? Or were the frieze height and architrave separately derived

[43] Vitruvius iii. 5. 1–15.

[44] e.g. by G. Roux in Roux 326–7; by the present author in *BSA* lxiii (1968) 171.

from other parts of the building, and related to each other only indirectly, so that a change in their relationship in one building might have been produced by a cause quite different from that producing the same change in another building?

This should not be an impossible question to answer, however. If, as we have seen is likely, the proportional relationship between two parts taken at random cannot be simply expressed and does not remain constant over a series of buildings, while the proportional relationships used by the architect in working out the design will be both simple and relatively constant, then it should be possible by analysing the proportions of a number of buildings to distinguish initially those proportions which are more likely to have been used from those which are less likely to have been used, it may eventually be possible to point to a set of rules which in all probability *was* used. Thus if we find that the ratio F/A is more constant than the ratio F/D or F/I, that would indicate that the frieze height is more likely to have been derived from the architrave height than from the lower diameter or the intercolumniation. It may of course be that the frieze height was in fact derived from some other element which was itself derived directly from the architrave height, but at least we may say that the more constant a ratio is over a series of related buildings, the more likely it is to have been the one envisaged by the architect, and the more reliable it will be as a stylistic index.

A full analysis of the Doric order on the lines suggested here would involve the comparison of all the possible proportional relationships to be found in each of a large number of specimens of the Doric order. Since there are, at a conservative estimate, some thirty-five different elements in a specimen of the Doric order, any one of which might, theoretically at least, have been defined in relation to any other, it would be necessary to calculate some 600 different proportions for each building, and to repeat the process for 50 or 100 buildings, making a total of perhaps 50,000 proportions. It would then be necessary to take the 35 or so proportions which relate the first element to be studied to all the other elements, and to compare the stability of each of the 35 over the whole field of 50 to 100 buildings. That would then be repeated for each of the other 34 elements. Obviously this is a job for a computer, and presumably it will sooner or later be done, but there still remains the task of telling the computer precisely what to look for, and how to present the results. As an aid to that task it would seem to be worth while to conduct some initial exploration to discover the general character of the use of proportion in Greek architecture, looking only at the more probable proportions; and as a preliminary to the preliminary, we may pursue somewhat further our examination of what can be regarded as probable in this field, by noting some of the characteristics of the Vitruvian systems.

The rules for Ionic are not the only successive system of proportion in Vitruvius; the rules for Tuscan temples and for circular temples form systems of the same type.[45] The difference between these systems on the one hand and the modular system of Doric can be clearly seen by representing the sequence of rules diagrammatically (FIG. 1). The Doric system produces a markedly radial pattern, while the others are more linear. There are, however, substantial differences between the successive systems. In both Ionic and Tuscan (the details of the circular temple follow those of Ionic) the size and design of the base and capital of the column are derived

[45] Vitruvius iv. 7–8. The rules of human proportion attributed to Chrysippos and through him to Polykleitos (Galen, *Placit. Hipp. et Plat.* 5 (ed. Müller 425)), appear also to form a successive system, relating finger to finger, all fingers to palm and wrist, palm and wrist to forearm, forearm to upper arm. The final words 'πάντων πρὸς πάντα', in fact 'everything to everything else', suggests, if taken literally, a modular system. The words, however, are probably not those of Polykleitos himself, but of a summarizer, who perhaps did not notice the difference, or was more interested in the concept of a commensurability than in the precise nature of the series of rules.

The rules of human proportion given in Vitruvius iii. 1. 2 are neither modular nor successive. They may perhaps be called fractional, since the dimensions of all the main elements of the figure are defined as submultiple fractions of the total figure height.

naturally enough from the lower diameter; but in the Ionic temple the lower diameter of the column is derived directly from the stylobate width, and from it are derived both the column height and the intercolumniation, while in Tuscan temples it is the column height which is derived directly from the stylobate width, and the lower diameter is derived from the height; the intercolumniation is also derived from the stylobate width, but indirectly via the cella width. Thus although the rules for the Ionic temple are very far from being modular, the lower diameter of the column controls far more of the design than in the Tuscan temple—column

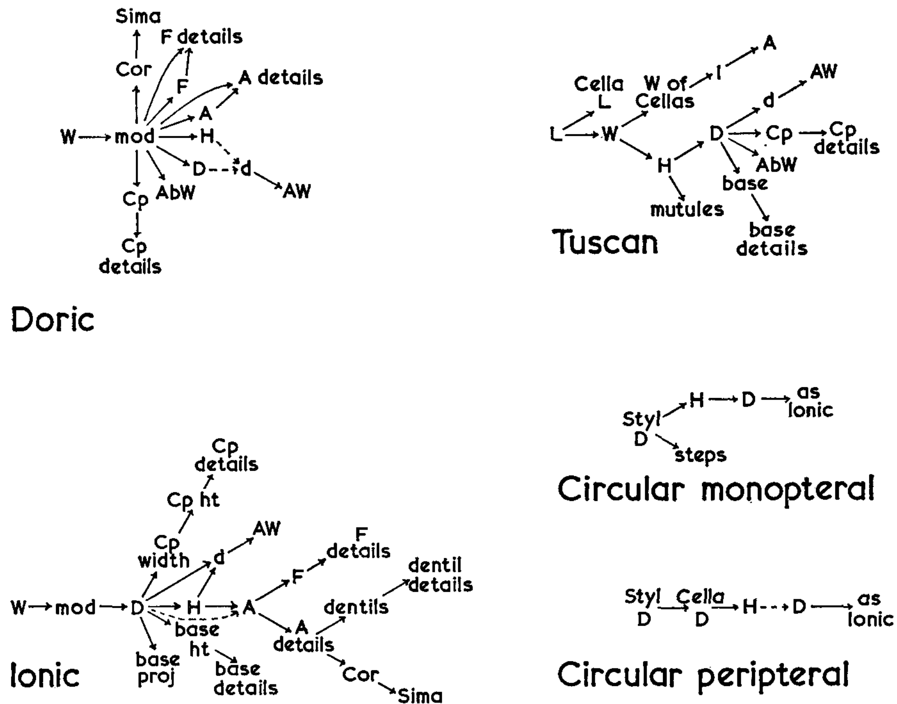

FIG. 1

height, intercolumniation, base, upper diameter and capital, and, for small columns, the architrave height as well.

This flexibility of successive systems in general is one reason for regarding them as more likely to have been used by Greek architects than any modular system. Vitruvius' rules for increasing the relative height of the Ionic architrave as the scale of the order increases[46] show how quite small variations can be introduced by the architect, and he could presumably do this not only to suit changes in scale, but also to suit changes in taste. Certainly Greek architects must have used some system which was capable of alteration in comparatively small steps, in order to account for the gradual development that we find. If, however, Greek architects did in fact follow rules of proportion, the stylistic development as envisaged by the architect must have been by discrete steps rather than as a continuous process. If we cannot see these steps, it may be because we are looking at the wrong proportional relationships, or it may be that rounding

[46] Vitruvius iii. 5. 8.

out of dimensions in the application of the rules has blurred the sharpness of the steps; but it will not mean that intermediate stages in fact existed.[47]

One general feature of the successive systems described by Vitruvius is the way in which the details of each element are derived from the main dimensions of that element. We have already seen how the various parts of the column are defined in terms of the lower diameter, but the details of capital and base are derived by subdivision of the height and/or width of those parts. Similarly the height of the Ionic architrave is derived from the column height or lower diameter, its width from the upper diameter, but the sizes of kymation and fasciae are derived by subdivision of the architrave height. It is also noticeable that each part is defined in terms of a part which, if the building were actually being built, must have existed physically before the part in question could be put in place. The four exceptions to this rule all relate to the column, the Ionic and Tuscan bases which are designed in terms of the lower column diameter, although the base must have been already in position before the lowest column drum could be placed on it, and the lower column diameters of the Tuscan and circular temples which are defined in terms of the column height, although the lowest drum embodying the lower diameter must have been set in place before the full column height could be realized. It appears, therefore, that the design of the column was considered as a whole.

The general rule, however, has one important result. It would have been possible to derive the size of most parts not only by calculation from a preliminary design of some sort, but also by direct measurement from the appropriate part, which might well already exist at full scale in the building itself. Both these characteristics may suggest how we may choose the more probable ratios from the 600 or so theoretically possible.

It is in some ways surprising that no reference is made to such compound entities as the entablature height or the total height of the façade, although such entities figure prominently in some modern studies of temple design. It seems likely that the aesthetic importance of such ratios as entablature height to column height and façade height to façade width was appreciated by ancient architects, too, but they could well be taken into account not directly through rules of proportion involving them, but indirectly, through the knowledge that by following a certain set of rules for the parts, a satisfactory over-all result would be achieved. The absence of the total façade height from the architectural rules contrasts strikingly with the importance of the total figure height in the rules given by Vitruvius for the human figure. The Greek sculptor would normally have the total height of his statue before him from the start, in the height of his block of stone, and so could easily use it as the basis of his design. So too an architect, if he designs on a drawing-board, can decide at once on the total height of his building, and subdivide it to define the sizes of the constituent parts. Otherwise, however, the total height will have no existence for him until the whole building is complete, and if he wishes to derive the design from an over-all measurement, it must be from the width, as we find in Vitruvius.

It is interesting to find that in the Tuscan temple and the circular temple the lower diameter of the column is derived from the column height, for we are inclined to think of the relationship the other way round—a column five diameters high, not a column one-fifth of a height thick. Nevertheless, even in the Ionic temple, where the height is apparently derived from the diameter, the relationship is still expressed not as a height of so many diameters, but as a thickness equal to such and such a part of the height. This is in fact a good example of another general feature of Vitruvius' successive systems; in most cases the various elements are derived not by multiplying a smaller part, but by dividing a larger part. Thus the column height is derived

[47] Hence, instead of trying to arrange the proportions of Doric capitals to form a continuous series, we should try to arrange them into a greater or smaller number of discrete groups.

from the stylobate width (except in Ionic), the mutule projection (in Tuscan) is derived from the column height, and, as we have already noted, the main elements are not built up by addition of the parts, but the parts are formed by subdivision of the main members. Thus whereas in a modular system the design proceeds mainly by multiplication, in the Vitruvian successive systems it proceeds mainly by division, so that unless the division is done geometrically at full scale on the actual parts of the building themselves, the architect will either have to round out his dimensions or deal in extremely complicated fractional measurements; thus following Vitruvius' rules for the sima height as given on p. 69 above for an order with columns two feet in diameter, we should theoretically have a sima $7\frac{47}{91}$ dactyls high; who could resist making it half a foot?

This sort of process provides us with another clue in the investigation of ancient design. We have already seen how it may be possible to establish which elements were related to each other in the architect's design, but not the direction in which the relationship operated. Thus if we find that the triglyph width is regularly one-fifth of the intercolumniation, but less simply and consistently related to the lower diameter, we may believe that triglyph width and intercolumniation were consciously related by architects, but we will not know whether the triglyph was designed as one-fifth of the intercolumniation or the intercolumniation as five times the triglyph. We may learn the answer, however, if it is possible to convert the measurements into ancient feet. Thus in the temple of Zeus at Olympia, the intercolumniation was built as 16 feet, and the triglyph width as $3\frac{1}{4}$ feet[48]. If the intercolumniation were designed as five times the triglyph width, it should have been $16\frac{1}{4}$ feet, and there is no reason why the architect should not have adopted that measurement; but if he derived the triglyph width from the intercolumniation, he would have to divide 16 feet by 5, which gives a theoretical answer of between 3 feet and 3 dactyls and 3 feet and 4 dactyls ($3\frac{1}{4}$ft.), and it would not be surprising that he should choose the simpler measurement, $3\frac{1}{4}$ feet. The dimensions we find in this temple are thus more readily explained on the assumption that the triglyph was derived from the intercolumniation, and not vice versa. We may perhaps apply that result, with some caution, to other buildings of the same stylistic group.

The assumption has been, throughout this section, that the proportional systems used in Greek architecture were based on commensurable ratios, in this respect at least similar to those set out by Vitruvius. Although there is little written evidence for architectural proportions outside Vitruvius, the little that exists is for rather than against this assumption,[49] and the scanty evidence for the ratios used in sculptural design argues the same.[50] Many of the general remarks above apply equally to systems based on incommensurable ratios, however. Such systems may be either modular or successive, and if we still accept the hypothesis that methods of design are

[48] That is if the commonly accepted equation of 1 ft. = 0·326 m. is valid for the temple.

[49] e.g. Pliny *NH* xxxvi. 23, 56, 178–9. Especially valuable is the record of the ratios used in the Old Temple of Artemis at Ephesos. This must go back to the fourth century B.C., when the temple was destroyed, and since it is unlikely that anybody in the fifth or fourth century took the trouble to re-measure the building, Pliny's information probably comes from the book on the temple written by its architects, Chersiphron and Metagenes (Vitruvius vii. praef. 12). It therefore represents not simply fact, but intention. The only literary evidence for incommensurable ratios is the occurrence twice of the ratio $1:\sqrt{2}$ in Vitruvius (iv. 1. 11, vi. 3. 3). It seems unlikely that these are the meagre remains of a much more widespread use of incommensurable ratios in earlier Greek architecture, for the rules in question deal with the Corinthian capital and the Roman *atrium*, neither of which can be regarded as basic to Classical Greek architecture.

[50] The rules of Polykleitos in the passage of Galen cited above (n. 45) involved *symmetria*, which would naturally mean normal commensurability, not commensurability in square. The rules for the human figure given by Vitruvius (iii. 1. 2) certainly involve commensurable ratios, and probably go back to the fifth or fourth century B.C. (on the date see n. 125 below). Similarly, when Plato speaks of geometrical designs drawn by artists (*Rep.* vii. 529 d–30 a), the ratios he imagines they might display (but not embody) are 1:1, 1:2, or some other commensurable ratio (ἄλλης τινὸς συμμετρίας). However, the argument here is that the

likely to be to a considerable extent conventional, then the relationships which are most likely to have been significant to the ancient architects will still be those which tend to be most constant over a number of related buildings. Thus a straightforward proportional analysis of the sort suggested above would not prejudge the question what types of ratio were used, provided that the proportional rules were still of the type: let x = ky.

There are, however, other types of proportional rule which will be less easy to detect through direct calculation and comparison of proportions. Dinsmoor, for instance, has noticed that in Periclean Athens the intercolumniation of Doric buildings is normally about 0·50 m., more than twice the lower diameter.[51] If Periclean architects in fact derived the intercolumniation from the lower diameter in this way, then we should have a rule of the type: let x = ky+l. Vitruvius, too, implies some rules which are not of the normal type. For instance the height, intercolumniation, and lower diameter of an Ionic column are all tied together by what amounts to a rule $H = 12\frac{1}{2}D - I$,[52] that is, of the type: let x = ky+z. The rule for the Ionic architrave carried by columns over 15 feet amounts to $A = \frac{2H}{29-H/5}$.[53] It is clear that all these rules may produce different ratios in different buildings, so that it may not be apparent that a single rule is being applied. If sufficient buildings obey rules such as these, however, it is likely that several of them will belong to the same category (e.g. of scale or column spacing). In each group the usual consistent pattern should then emerge, so that a whole series of separate rules may be recognizable, which will then be seen to be related. Another possibility is that it may already have become apparent that, for instance, simple arithmetic ratios are used in the buildings concerned, so that if it is found that the ratio of architrave height to column height, although not constant, consistently approximates to a simple fraction of the column height, we will have reason to believe that the architrave height was derived from the column height.

FRACTIONS

The problem of fractions is one which affects the modern student of Greek architecture as well as the ancient architects, for some acceptable way of expressing proportions is needed. The use of decimal fractions, allowing the expression of an unbroken series of fractional expressions to any desired degree of accuracy, forms an excellent basis both for the comparison of two or more individual proportions and for the comparison of the spread of values in two or more series of proportions. It is easy enough to compare vulgar fractions with the same numerator or with the same denominator, but it is hard to say at once in which order such terms as $\frac{5}{17}$, $\frac{12}{37}$, and $\frac{13}{42}$ should be placed, let alone whether the difference between the largest and the middle value is more or less than the difference between the middle and the smallest value; but if they are written as 0·294, 0·324, and 0·310, both these questions are easily answered.

For these obvious reasons proportional relationships must be expressed as decimal fractions initially in any investigation of their significance.[54] Just as obviously, however, no ancient

stars seen in the heavens, although very beautiful, can only serve as diagrams for the true science of astronomy, just as geometrical diagrams, even if beautifully drawn by professional artists, can never contain true mathematical entities; it is uncertain whether Plato meant 'geometrical diagrams such as *are* drawn by artists (as a basis for their works of art)' or 'geometrical diagrams, even if they *were to be* drawn by artists (for the use of geometers)'. Both are possible interpretations of the Greek, and both would suit the argument

[51] *Hesperia* ix (1940) 22.

[52] Vitruvius iii. 3. 2–5, iii. 3. 10; the eustyle system stands outside this rule.

[53] Vitruvius iii. 5. 8, provided that fractions of a foot are neglected in the term H/5.

[54] Note, for instance, the lack of clarity in Morgan's comparison of Vitruvius' rules for column taper with actual Greek practice (Vitruvius, *The Ten Books of Architecture,*

architect could have designed in decimal fractions. In so far as he conceived his design in terms of arithmetic proportions, he must have used expressions equivalent to vulgar fractions. Since the aim of our investigation is to discover the actual rules used by Greek architects (rather than simply the facts about Greek buildings), an important stage must be to suggest the most suitable ways in which the actual proportions we find may have been expressed as simple vulgar fractions. The difficulty is that some quite simple vulgar fractions are hard to recognize when expressed decimally; thus 0·426 may not be recognized at once as approximately $\frac{3}{7}$, nor 0·0775 as approximately $\frac{1}{13}$. Some of the advantages of both systems can be obtained, however, by expressing the larger values in terms of the smaller, rather than vice versa; thus the reciprocals of 0·426 and 0·0775, 2·345 and 12·9, are immediately recognizable as approximately $2\frac{1}{3}$ ($= \frac{7}{3}$) and 13.

Strictly speaking, expressions of fraction and of proportion are not the same, and we find Plato expressing the proportion 256 : 243 without reducing it to a fraction.[55] However, in so far as rules of architectural proportion are intended not just to describe a state of affairs, but to define the dimensions of one part of a building in terms of another already defined, the simplest way of applying them would be to reduce them to expressions involving fractions. If the lower diameter of the columns : axial intercolumniation = 6 : 13, then the architect, knowing the lower diameter, would define the intercolumniation as $2\frac{1}{6}$ diameters, or, knowing the intercolumniation, would define the lower diameter as $\frac{6}{13}$ of an intercolumniation. Otherwise a periphrasis would be required: divide the intercolumniation into thirteen parts and take six as the lower diameter (or vice versa).

Fractions would also play an important part in the application of the rules of proportion to a particular case. Thus we might have a general rule 'Let the architrave equal $\frac{3}{5}$ of the lower diameter' applied to a specific case where the lower diameter was $4\frac{1}{2}$ feet. The architect would have to calculate $4\frac{1}{2} \times \frac{3}{5} = 13\frac{1}{2}/5$ feet = 2 ft. $11\frac{1}{5}$ dact. $\simeq 2\frac{3}{4}$ feet. The way in which calculations of this kind were handled, and the stage at which, and the extent to which, rounding out might take place, will clearly affect the precise proportion which will result. Some discussion of the way in which fractions were written, expressed, and handled in classical Greece may therefore be useful.

Two systems of writing numerals were commonly used in Greece, the acrophonic and the alphabetic. The acrophonic system was the standard one in mainland Greece during the fifth and fourth centuries B.C., at least for official purposes, and its use goes back to the sixth century.[56] However, there was no recognized way of writing fractions in acrophonic numerals. The symbols themselves often represented things rather than pure numbers; for instance, no additional

trans. M. H. Morgan (1914) 85. The figures given by Morgan are set out below, with the simple decimal equivalents in brackets beside them:

	Vitruvian rule		Actual practice	
Col. ht.	Morgan	Decimal	Morgan	Decimal
Below 15′	5/6	(0·833)	4·9/6	(0·817)
15′–20′	$5\frac{1}{2}/6\frac{1}{2}$	(0·845)	$5·66/6\frac{1}{2}$	(0·87)
30′–40′	$6\frac{1}{2}/7\frac{1}{2}$	(0·866)	$6·38/7\frac{1}{2}$	(0·85)
50′–60′	$7\frac{1}{2}/8\frac{1}{2}$	(0·883)	$7·44/8\frac{1}{2}$	(0·875)

It is clear from the decimal equivalents that the second example completely breaks Vitruvius' rule, and that the others all taper more strongly than they should according to Vitruvius. These three approximate more closely, in fact, to other terms in the Vitruvian series: $4\frac{1}{2}/5\frac{1}{2}$ (0·818), $5\frac{1}{2}/6\frac{1}{2}$ (0·845), and 7/8 (0·875).

[55] *Timaeus* 36 b.

[56] Herodian, *Peri ton Arithmon* (see H. Stephanus, *Thesaurus Linguae Graecae* (Didot edn., Paris, 1865) viii. 345, says that the system was used in the record of Solon's legislation. Nothing as early as that has survived, but there are acrophonic numerals on late sixth-century sima sections from near Kaulonia (L. H. Jeffery, *The Local Scripts of Archaic Greece* (1961) 258). What appear to be acrophonic numerals occur on marble roof tiles from the Acropolis at Athens, and these may belong to the Pisistratean buildings there (N. T. Peppa-Delmazoglou in *Charisterion A. Orlandos* iv (1967–8) 369–850).

sign was needed in financial documents to show that a number represented drachmae, for a special sign (⊢), meaning not 1 but 1 drachma, would be used for the units. Subdivisions of the drachma were not written as fractions but as a number of obols, and the sign C meant a half obol, not a half. There was no way of expressing any fraction of the smallest existing coin denomination, and such an expression would have no meaning. Thus there would be no way of writing a fraction of a foot except as a number of dactyls, and one might expect that, since there was no recognized unit of length less than a dactyl, fractions of a dactyl would be disregarded.[57]

The alphabetic system of numerals can be traced back to the second half of the sixth century B.C. It occurs in the price graffiti on Attic pottery until *c.* 480 B.C., when it was replaced by the acrophonic system—perhaps because merchants of a different nationality now handled the trade.[58] The alphabetic system then almost disappears from sight until the Hellenistic period. It is the only system found in papyri, even those of the late fourth century B.C., and it was widely used in mathematical works from the third century on.[59] In official epigraphy it became increasingly common during the Hellenistic period, particularly in the eastern Greek world, until it replaced the acrophonic system almost entirely in the first century B.C.[60]

It is not so clear, however, what happened to the alphabetic numeral system in the fifth and fourth centuries B.C. It must have continued in use somewhere, in order to re-emerge in the late fourth century B.C., but between *c.* 480 and 320 B.C. it occurs only in one Attic inscription (a table of figures, where its use is not wholly understood),[61] and in a late fifth-century inscription from Halikarnassos (where it is used side by side with the acrophonic system).[62] It might be supposed that it was more widely used in unofficial contexts, but there is little evidence yet to support that view. The acrophonic system was not just an official one, for it was used, as we have seen, in the price graffiti on Attic vases, and also on Athenian mortgage stones, although those were not written by professional letterers.[63] Acrophonic numerals also occur on abaci and gaming tables, not only on large, official or semi-official ones, but also on smaller ones.[64] The only evidence for alphabetic numerals used in an unofficial context in mainland Greece during this intervening period is provided by the graffiti on some fifth-century amphorae from the Athenian agora. These have been interpreted as giving prices partially in alphabetic numerals, but although the interpretation is plausible, it is by no means certain.[65] If correct, it indicates a good deal of uncertainty in the use of numerals, with 14, for instance, not being represented by ⌗, as one might expect, but once by FH (6+8), and once by four vertical lines followed by ten horizontal ones linked by a single vertical.

The use of simple strokes to represent numbers even above 5 is also found sometimes on architectural elements in mainland Greece,[66] and does not suggest great familiarity with a more convenient system of numerals such as the alphabetic one. Indeed when letters of the alphabet

[57] For fractions of a dactyl see pp. 78 ff. below.

[58] R. Hackl, in *Münchener archäologische Studien* (1909) 79–82, 92–4; the explanation may well be correct if the acrophonic numerals were already used in Athens in the latter part of the sixth century (see n. 56 above).

[59] E. Mayser, *Grammatik der griechischen Papyri* i (1906) 51. ⊢A = 1,000 drachmae occurs in a papyrus of 311 B.C. (M. David, B. A. van Groningen, *Papyrological Primer* (3rd edn., 1952) 42 no. 25, ll. 4, 11, 12. For mathematical texts see Heath, i. 42–3.

[60] A. G. Woodhead, *The Study of Greek Inscriptions* (1967) 109–11.

[61] *IG* ii². 760.

[62] *Hermes* xxix (1894) 248–80; *SIG*³ no. 46.

[63] J. V. A. Fine, *Horoi* (*Hesperia*, Suppl. ix (1951)), esp. 48–9.

[64] Probably official: *IG* ii². 2777; *IG* xii⁵. (5) 99; *AM* xxiii (1898) 2–3, no. 1. Probably unofficial: *Corinth* i. 4 (1954) 64, pl. 15. 1 (the large letters at the bottom are probably X, H, and Δ); *Hesperia* xxxvii (1968) 243 no. 5; *IG* ii². 2779–80; cf. also the die, *AM* xxiii (1898) 17. 4.

[65] *Hesperia* iv (1935) 515–17; for the date, ibid. 497.

[66] On the Athenian Treasury at Delphi (*FD*, J. Audiat, *Le Trésor des Athéniens* (1933) 35) the numbers are 5 and 8 (not 5 and 9 as Orlandos 158), and on the Propylaia at Athens (Orlandos 159) the number is 10 not 2 (as Martin i. 225). The same system is used on parts of the Vix krater (the numbers are 1–7; *Rend. Linc.* ser. viii. 18 (1963) 1–19).

were used for identifying the order of architectural elements, they were used simply as letters, not as numerals; after ι′ comes κ′, not ια′.[67] The earliest example of true alphabetic numerals used for this purpose is perhaps in the Stoa of Philip at Delos, built in the late third century B.C.[68]

There is therefore no good reason to suppose that architects in the fifth and fourth centuries B.C. on the Greek mainland would have used anything but acrophonic numerals in calculating a design.[69] East of the Aegaean, the situation is less certain, chiefly for lack of evidence. The use of alphabetic numerals at Halikarnassos in the late fifth century B.C. suggests that if alphabetic numerals survived, it is more likely to have been in Asia Minor than in mainland Greece, and that would agree with the apparent origins of the system.[70] Even in Asia Minor, however, the acrophonic system was also used.[71]

The possibility that architects might have used alphabetic numerals in the fifth and fourth centuries is important not only because it is an easier system to use for calculations in general, but also because several ways of writing fractions were developed. Some of these were as flexible and convenient as the modern way of writing vulgar fractions in Arabic numerals,[72] but there was a strong tendency to follow the Egyptian practice of writing only submultiple fractions; thus $\frac{1}{3}$ was written as γ′, $\frac{1}{4}$ as δ′, and so on; other fractions were expressed as the sum of two or more submultiples, $\frac{3}{4} = \frac{1}{2}+\frac{1}{4}$ = ⟨δ′, $\frac{5}{7} = \frac{1}{2}+\frac{1}{7}+\frac{1}{14}$ = ⟨ʒ′ιδ′, and so on. It is this system which we find in the inscriptions relating to the temple of Apollo at Didyma, both in the official accounts and in the more practical specifications of the intended diameter on several drums of an unfluted column.[73] The existence of this method in the fifth or fourth centuries, however, is uncertain. There are no fractions among the early price graffiti on Attic pottery. Monetary fractions do occur with alphabetic numerals in the inscription from Halikarnassos, but the way of writing them is a primitive one, with $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$ of an obol represented by one, two, or three horizontal strokes; such a method works only when, as in monetary subdivisions, the denominator of the fractions can be assumed. The use of this method might be taken as evidence that alphabetic submultiple fractions had not yet been invented in the late fifth century B.C. Even in the Didyma inscriptions, however, where alphabetic submultiple fractions are used for parts of a foot, a set of various conventional signs was used for the subdivisions of the drachma.

Dealing with fractions is not only a matter of writing them in numerals, but also of expressing them in words. If fractions of all sorts were widely talked of and written in words, it would be a little surprising if there was no convenient way of writing them in numerals; and certainly if

[67] The non-numeral character of the first ten letters of the alphabet cannot, of course, be demonstrated, but the Ψ on a drum of the fourth-century temple of Apollo at Delphi (*FD*, F. Courby, *La Terrasse du Temple* i (1915) 88) and the X on the Old Temple at Sounion (Orlandos 160) can hardly be numerals, since they would mean 700 and 600, numbers which are much too high. The letters on the Sikyonian Treasury at Olympia are probably masons' marks, not position indicators: pairs of letters as well as single letters are used, and although IE could be 15 and ΠA perhaps 81 (although too large a number for so small a building), AΔ and FΧ cannot be numerals (*Olympia* (1896) no. 668).

[68] *EAD* vii. 1, R. Vallois, *Le Portique de Philippe* (1923) 63–6; for the date, ibid. 154–63.

[69] Acrophonic numerals occur on the Treasury of Kyrene at Delphi (*FD*, J. Bousquet, *Le Trésor de Cyrène* (1952) 40, pll. 15. 1, 17. 6).

[70] *Hermes* xxix (1894) 248–80; L. H. Jeffery, *The Local Scripts of Archaic Greece* (1961) 327.

[71] *SIG*³, no. 46 (the Halikarnassos inscription already mentioned); *OGIS*, no. 46. 6 (Halikarnassos, 3rd cent. B.C.); Collitz–Bechtel, *Sammlung der griechischen Dialektinschriften* iii. 2 (1900) no. 5616 (Smyrna, *c.* 300 B.C.); E. Schwyzer, *Dialectorum Graecorum Exempla Epigraphica* (1923), no. 709a (Klazomenai, 3rd cent. B.C.).

[72] Heath i. 41–4. The briefest and clearest notation was with the denominator above and the numerator below, reversing our current system.

[73] *Didyma* ii. nos. 39–43 and 48. Not surprisingly, the Egyptian system is the only one found in papyri (E. Mayser, *Grammatik der griechischen Papyri* i (1906) 52–3). It seems also to have been used by builders working at Alexandria. On an unfluted column-drum was engraved: Π AꓕO′H′I′Ꮯ′, Y ΔꓕI′Ꮯ′, which is presumably to be interpreted as: diameter (πάχος) 1 $\frac{1}{2}$ $\frac{1}{4}$ (?) $\frac{1}{8}$ $\frac{1}{16}$ ft. (= 0·678 m., using the Ptolemaic foot of *c.* 0·35 m.), height (ὕψος) 4 $\frac{1}{2}$ $\frac{1}{16}$ ft. (= 1·497 m.). The actual diameter and height of the drum are 0·68 m., and 1·51 m. See W. Hoepfner, *Zwei Ptolemaierbauten* (*AM*, 1971) 69–70, where a different interpretation is offered which does not explain all the symbols recorded. The date is probably *c.* 230–220 B.C.

there was a convenient way of writing them in numerals, we might expect to find simple and convenient verbal expressions for fractions. There are two main types of verbal expression for fractions in classical Greek, one based on the ordinals, as τὸ τέταρτον, the other more specifically fractional, of the type τὸ τεταρτημόριον.[74] The latter type appears to have been more common in the fifth and fourth centuries B.C. for most purposes, but there were two limitations to its range. Apart from the general expressions ὁποστημόριον, τὸ πολλοστημόριον, and τὸ μυριοστημόριον, there were terms in this series for $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{8}$, $\frac{1}{10}$, $\frac{1}{12}$, $\frac{1}{13}$, $\frac{1}{15}$, $\frac{1}{30}$, and $\frac{1}{40}$.[75] Only two of these, $\frac{1}{13}$ and $\frac{1}{15}$, are compounds of compound numbers, and their ungainliness—τὸ τρεισκαιδεκατημόριον and τὸ πεντεκαιδεκατημόριον[76]—shows why expressions of this type were not freely used for fractions such as $\frac{1}{48}$. The other limitation is that these terms seem only to have been used in the singular, as simple submultiples: I have found no example earlier than Archimedes of such phrases as τὰ τρία πεμπτημόρια to mean $\frac{3}{5}$.[77]

Fractional terms of the ordinal type could be rather more flexible, and terms such as $\frac{1}{71}$ (τὸ ἑβδομηκοστόμονον)[78] occur in Hellenistic mathematical texts. In the fifth and fourth centuries, however, much the same limitations apply as to the other series. There were the general expressions τὸ ὁπόστον μέρος, τὸ πολλοστὸν μέρος, and τὸ μυριοστὸν μέρος, and a series of more specific fractions almost all based on a simple rather than a compound number.[79] In many cases the expression refers to a levy or tax, ranging in severity from $\frac{1}{5}$ to $\frac{1}{500}$; in such cases the noun is omitted and the ordinal is in the feminine;[80] the sense may in fact be more ordinal than fractional—'every tenth item goes to the state' rather than 'the state takes one-tenth of the total'. Two of the other pre-Hellenistic examples of ordinal fractions quoted by *LS*[9] refer to coin denominations (again in the feminine with the noun omitted),[81] while the remainder all have μέρος or μοῖρα expressed, not just understood as was normal later.[82] As with the -ημόριον fractions, the ordinal fractions are used in the fifth and fourth centuries only in the singular, so as to express a submultiple, and this limitation often applies even in the Hellenistic period, too.

In addition to these two main series, there are a few other types of expression which may be relevant. A series of the type ἐπίτριτος was used to represent fractions of the type $(n+1)/n$ or proportions of the type $(n+1):n$. Ἐπίτριτος and ἐπόγδοος are used by Plato of proportions,[83] but ἐπίπεμπτος, ἔφεκτος, ἐπόγδοος, and ἐπιδέκατος are used in the fourth century to mean (capital plus) 20, $16\frac{2}{3}$, $12\frac{1}{2}$, and 10 per cent interest respectively.[84] The full range of such expressions does not appear, however, until Ptolemy, who uses ἐπιτεσσαρακοστόπεμπτος to mean 46:45, for instance.[85] Slightly later a series of the type ὑπεπίτριτος was developed for the reciprocal proportion, $n:(n+1)$,[86] and two rather more commonplace expressions for fractions of the type $n/(n+1)$ are given by Vitruvius; δίμοιρος ($\frac{2}{3}$) occurs also in Aeschylus, but πεντάμοιρος

[74] The discussion which follows is based primarily on the references given in *LS*[9]. Although all occurrences of commonly used expressions are not noted there, the absence of a word from *LS*[9] is reasonable evidence that it does not occur in surviving ancient Greek. The following pages have benefited from the help of my colleague D. B. Robinson, who has pointed out several relevant passages which I had missed; he does not, however, share all the views expressed here.

[75] See *LS*[9] under the appropriate words ($\frac{1}{40}$ = πετρωκοστομόριον: Archim., *Aren.* ii. 4, iv. 1).

[76] Eudox., *Ars* 16. 11: Hippoc. *Oct.* 13.

[77] Archim., *Aequil.*

[78] Archim., *Circ.* 3.

[79] Note τὸ δωδέκατον μέρος (Plato, *Leg.* 956 c and Alexios in Athenaios iii. 117 e). Cf. also *IG* 1675. 18.

[80] *$\frac{1}{5}$, *$\frac{1}{6}$, $\frac{1}{10}$, $\frac{1}{20}$, *$\frac{1}{24}$ (3rd cent.), $\frac{1}{30}$, $\frac{1}{40}$, $\frac{1}{50}$, *$\frac{1}{60}$, $\frac{1}{100}$, $\frac{1}{500}$. See LS[9] under the appropriate words. The fractions marked with an asterisk are not used in this sense before *c.* 300 B.C.

[81] ἕκτη (*IG* 310 al.), τεσσαρακοστή (Thuc. viii. 10). Note also δωδεκάτη (*IG* ii². 1675. 18) of a special type of bronze.

[82] Hippoc., *Inst.* 26, Isoc., xii. 177, Plato, *Leg.* vi. 784a, Xen., *Cyr.* ii. 1. 6, vi. 3. 2; $\frac{1}{4}$: Plato, *Rep.* ii. 369 e, 370 a; $\frac{1}{5}$: Plato, *Apol.* 36 b (etc.); $\frac{1}{12}$: Plato, *Leg.* xii. 956 c; $\frac{1}{10,000}$: Aristoph., *Thesm.* 555.

[83] Plato, *Rep.* viii. 546 c, *Tim.* 36 a, b.

[84] Xen., *Vect.* iii. 9; Dem. xxxiv. 23, l. 17; *IG* i². 377.

[85] Ptol., *Harm.* i. 15.

[86] There are expressions for 2:3 (ὑφημιόλιος), 3:4, 4:5, 8:9 (LS[9] s.v. ὑπεπιμόριος); the terms first occur in Aristotle.

($\frac{5}{6}$) is not listed by *LS*⁹.[87] Finally, when a contrast is being made between a certain fraction of the whole and the remainder, we get expressions such as τὰ δύο μέρη . . . τὸ τρίτον . . . , the denominator for the first phrase being understood from the subsequent mention of the third part.[88]

All the expressions we have looked at so far, however, are variations involving submultiples—$1/n$, $1+1/n$ and $1-1/n$. Other fractions are very rare; in surviving Greek of the fifth and fourth centuries I know of only two examples, and both involve a paraphrase such as 'two out of five parts'.[89] While this would be quite satisfactory if such fractions were not often used, it would not be convenient if architects were continually specifying parts of a foot as, for example, 'a block two feet and seven out of twelve parts of a foot long, one foot and five out of twelve parts of a foot high, and one foot and five out of six parts of a foot wide'.[90] Nevertheless, simple expressions such as τὰ τρία τέταρτα ($\frac{3}{4}$) do not seem to occur before the Hellenistic period.

Although it might be objected that the verbal evidence at our disposal is inadequate because no mathematical or architectural treatises have survived from the fifth or fourth century, the fact that the verbal evidence fits in with the evidence of the use of numerals much increases its value. Thus we have found that in the fifth and fourth centuries verbal expressions for fractions are rather restricted in range and flexibility, and there was probably no generally applicable numeral notation for fractions. In the Hellenistic period, the development of a much fuller range of verbal expressions is associated with the spread of the alphabetic system of numerals and the development of various ways of writing fractions in such numerals. The tendency to deal only in submultiple fractions is strong, both in the verbal expressions and in the numeral notation, and continues into the Hellenistic period. It is natural to suppose that the use of such a system would in itself lead to approximation in dealing with fractions, for although it is not difficult to write 5 as $5\frac{1}{3}\,\frac{1}{15}$, it is less easy to multiply $5\frac{1}{3}\,\frac{1}{15}$ by $3\frac{1}{2}\,\frac{1}{6}$ and get $19\frac{1}{2}\,\frac{1}{5}\,\frac{1}{10}$.

There is one other factor which might affect the way Greek architects dealt with fractions—the use of the abacus. Heath concludes that the Greeks 'had little need of the abacus for calculations',[91] but he is thinking of those who would use the alphabetic numerals. The acrophonic numerals were not so convenient; and in any case there is good literary evidence for the use of pebbles as an aid to calculating,[92] so presumably the Greeks did find the abacus helpful. The Salamis Table has often been taken as the best surviving example of a Greek abacus,[93] but Pritchett has shown that it is in all probability a gaming-board, possibly for the game of *pente grammai*.[94] That would mean that all surviving remains of Greek abaci consist simply of a row of

[87] Vitruvius iii. 1. 6; Aesch., *Suppl.* 1070.

[88] Thuc. i. 104, Aeschin. iii. 143; Plato, *Rep.* ii. 370 a, *IG* ii². 1675. 19–20; cf. *IG* ii². 1670. 29–32. In Dem. lix. 101, τὰ δύο μέρη = $\frac{2}{3}$ is used without τὸ τρίτον being expressed.

[89] Thuc. i. 10, Empedocles in H. Diels, W. Kranz, *Fragmente der Vorsokratiker* (6th edn., 1951–2), fr. 31B96.

[90] e.g. Archim., *Aequil.* 2. 10 (ed. Heiberg, p. 210).

[91] Heath i. 51.

[92] Hdt. ii. 36. 4, Aristoph., *Vesp.* 656–64, Lysias, fr. 50, Dem. xviii. 227, 229, Alexis in Athenaeus iii. 117e, Polyb. v. 26. 13, Diog. Laert. i. 59, Plutarch, *Mor.* 812e.

[93] *IG* ii². 2777; Heath i. 50–1; *RE* Suppl. iii (1918) 10–11; *Hesperia* xxvi (1957) 271–87; xxxiii (1964) 146–67.

[94] *Hesperia* xxxiv (1965) 131–47, esp. 138–40. Add to the references for large stone gaming-tables in sanctuaries: *AM* xxiii (1898) 1–14; note also two such tables at Goritsa (*AAA* v (1972) 492, fig. 9). For the large sums of money involved, compare the die from the Athenian Acropolis (*AM* xxiii (1898) 14), and the fortunes lost by Alkibiades, son of Alkibiades (Lysias xiv. 27) and by Hegesandros and Timarchos (Aeschin. i. 95).

Two further points in favour of the gaming-board interpretation may be added. On the abacus hypothesis, the central cross-line on the Salamis table is normally taken to separate the pebbles representing units, tens, hundreds, etc., from those representing fives, fifties, five hundreds, etc.; but in that case only one or two pebbles would ever have to be placed above it, as against four or five below it, so that the cross-line should not have been centrally placed. In our ignorance of the way Greek board-games were played, no full explanation for the cross-line can be offered on the gaming-board hypothesis either, but on general grounds a central cross-line would not be inappropriate to a competitive board-game for two players. The gaming-board hypothesis also allows the best explanation of the fact that the Salamis table has two sets of acrophonic numerals facing in opposite directions—the players sitting opposite each other each used a set of numerals to record

acrophonic numerals with space above or below each for placing pebbles or marks. In one case the columns are separated by engraved lines,[95] but that is not normally so. Such an abacus may be independent, or form part of a table of standard measures[96]—or, of course, a gaming-table. It would be most helpful in the addition of a large series of figures (a task which would be rare in architectural design), but could also be used for subtraction and, with practice, for multiplication and simple division.

Whenever the surviving remains of an abacus include one or other end of the series of numerals, the inclusion of symbols for talents and/or for drachmai, obols, etc., shows that the abacus was intended for financial calculations; no abacus has just numbers. It is therefore uncertain whether abaci were used in other contexts. Financial calculations would often be done in shops or offices, where a heavy, immobile counting-table would be an advantage; for other purposes a portable abacus, one made of wood rather than stone, might well be preferable. That might be sufficient to explain the absence of remains of non-financial abaci, but it is worth noting that where the literary references give any context for the use of the abacus, it is a financial context.[97]

If an abacus was used in architectural calculations, however, its chief effect would be in the handling of fractions. It is certainly untrue, as the remains of Roman abaci show, to say that the abacus is by nature decimal, and can only handle decimal fractions.[98] Rather, it can only be used for a preselected system of fractions. Thus the Roman abacus is equipped for the division of the unit into 12 parts, while the Greek abacus was marked with columns representing the normal subdivisions of the drachma: obols, half-obols, and sometimes *chalkoi*. With 48 *chalkoi* = 6 obols = 1 drachma, one could divide 1 drachma by 2, 3, 4, 6, 8, 12, and so on, but not by 5 or 7. One-seventh of a drachma, even at an intermediate stage of a calculation, would have to be approximated as 7 *chalkoi* or possibly as 1 obol. For architectural calculations, the abacus would presumably have columns for hundreds, fifties, tens, fives, and units of feet, and then for palms and dactyls. Dimensions in 12ths or 10ths of a foot could therefore not be set out on the abacus, nor could it be used to divide a foot by 3, 5, or 7 without approximation; 1 ft./3 ($5\frac{1}{3}$ dact.) would have to be represented as 5 dactyls, 1 ft./5 ($3\frac{1}{5}$ dact.) as 3 dactyls, and 1 ft./7 ($2\frac{2}{7}$ dact.) as 2 dactyls. Further, ease of calculation might make $\frac{5}{16}$ (5 dactyls in the foot) more widely used as an approximation for $\frac{1}{3}$ than was strictly necessary. For instance, an accurate calculation of 2 ft./3 to the nearest dactyl would give 11 dactyls, but the approximation 2 ft./3 = 1 ft./3 × 2 ≏ 5 dact. × 2 = 10 dactyls might well be preferred. Here again, therefore, is a factor tending to introduce approximation when fractions are involved.

The extent of the approximation would probably depend a good deal on the preference of the architect, but it is worth noting that even a close approximation may have quite a considerable

their winnings (or losses), and the third set of numerals may have been used to record the stakes for the game currently in play.

The only abacus interpretation which takes both these difficulties into account is that of K. Menninger, *Zahlwort und Ziffer* (2nd edn., 1958) 109, who suggests that two men sat opposite each other to check each other's calculations, as with some medieval accounting tables (ibid. 153–6); the central cross-line would then be to separate the two calculating areas, while the two opposite-facing rows of numerals are self-explanatory. If this is how the Salamis table was used, however, it is surprising that the two calculating areas were not kept more separate, as in the medieval examples; the second calculation would be more likely to create confusion than correction. Also the third set of numerals at one end is unexplained; Meninger uses it to set out the multiplier in his example, but the numerals here, too, are for a sum of money, not a number, and in any case one would expect this row to be duplicated for the benefit of the second calculator.

[95] *IG* xii (7). 282.

[96] *IG* xii (5). 99; *Hesperia* xxxvii (1968) 241–3.

[97] So all the references in n. 92 above, except Hdt. ii. 36. 4. The context is different in Aesch., *Agam.* 570 and Eur., *Rhes.* 309–10, but the image of accounting may well be intended.

[98] So *Hesperia* xxxiii (1964) 147, 150; but see the descriptions of the Roman abacus in Daremberg–Saglio s.v. 'abacus', *RE* Suppl. iii (1918) 5–10 and Heath i. 47.

effect if it is introduced at an early stage in the calculation, particularly if it comes before a multiplication. For instance, 8 ft. $\times \frac{1}{3} \times 16$ (= 42 ft. 11 dact. to the nearest dactyl) may be worked out as 40 ft., 44 ft., 42 ft., or 43 ft., depending on whether the step 8 ft. $\times \frac{1}{3}$ is calculated to the nearest half foot ($2\frac{1}{2}$ ft.), the nearest palm (2 ft. 3 palms), the nearest half palm (2 ft. $2\frac{1}{2}$ palms), or the nearest dactyl (2 ft. 11 dact.). This sort of approximation will only take place, of course, if the dimensions of the various parts are calculated, not if they are found by physically dividing an already existing element into the required number of parts.

So far our investigation of the treatment of fractions by architects of the fifth and fourth centuries has been based on contemporary evidence from non-architectural contexts. It may be helpful to complete it by looking at the evidence from architectural treatises, even though they are not contemporary. Vitruvius' *De Architectura* is one obvious choice in spite of the difference in date between him and the greatest works of Greek temple design; but we do not find in Vitruvius the kind of calculations that will arise when his general rules are applied to a particular case. In order to decide what sort of work may be appropriate, we must examine the relation of architecture to mathematics.

The Greeks made a distinction between ἀριθμητική, the theory of numbers, and λογιστική, the art of calculation.[99] This distinction was normally reinforced by the fact that ἀριθμητική was normally concerned with pure number, while λογιστική dealt with numbers of things. On both grounds, the architect was more concerned with λογιστική. We have already seen reason to believe that the use of proportion in Greek architecture was to a large extent practical rather than theoretical, and it is likely that calculation in architecture was primarily a practical matter, too. Thus Plato, in the *Philebos*, holds up τεκτονική as a purer art than music or medicine, because it uses measurements and instruments to produce accurate results, instead of relying on the fallible judgement of a practised ear or eye.[100] Nevertheless, at the end of the passage it is agreed that measuring and calculation in τεκτονική are to be classed with measuring and calculation in commerce, because they are applied; they are therefore quite distinct from the pure geometry and calculations of philosophy. The procedures used by architects are therefore likely to have been related to the procedures of applied calculation (λογιστική) rather than those of Pure Mathematics. The procedures to be found in the *Stereometrica* and *De Mensuris* attributed to Hero of Alexandria are therefore likely to be more relevant than those of Archimedes or any other of the great Greek mathematicians, and so we may look at the treatment of fractions in them.

These works were probably compiled in the Byzantine era, but in spite of their late date, they contain much earlier material.[101] Because, naturally, they use the alphabetic not the acrophonic numerals, they probably do not follow the practice of the fifth or fourth century B.C., but they may well give an idea of Hellenistic practice. At least they are dealing with problems of a sort which must have occupied ancient architects, and the examples are set out with measurements, not just numbers.[102] The problems are more concerned with quantity surveying than with architectural design (e.g. how many tiles are required for the floor of a building of given size), but this, too, must have formed part of the architect's job.

The Egyptian system of fractions is normally used throughout these works, and the handling

[99] Heath i. 13–16. Note also [Hero], *Definit.* 135. 6 (ed. Heiberg, p. 98): '*Logistike* treats one as the least in a group of things of the same type; it takes . . . one drachma in a number of drachmae as indivisible, even though as a coin it can be divided.' This does not coincide with the practice in either the genuine or the spurious works of Hero, so presumably refers back to earlier ideas.

[100] *Phlb.* 55 d–57 a. Τεκτονική here includes not only building but also shipbuilding and other carpentry.

[101] *Heronis Alexandrini Opera* v (ed. Heiberg, Leipzig, 1914) praef. 21–35.

[102] In both respects they contrast with the *Metrica*, which is probably a genuine work (*Heronis Alexandrini Opera* iii (ed. Schöne, Leipzig, 1903) praef. 11–12).

of this cumbersome method is most impressive to a novice in the art (e.g. $104\ \frac{1}{2}\ \frac{1}{7}\ \frac{1}{14}\ \frac{1}{21} - 21\ \frac{1}{2}\ \frac{1}{3}\ \frac{1}{12} = 82\ \frac{1}{2}\ \frac{1}{3}\ \frac{1}{84}$).[103] Nevertheless, there are numerous approximations, not only in the use of approximate formulae and approximate square roots but also in normal calculations involving fractions. For instance $21 \times 7\ \frac{1}{15}\ \frac{1}{93} = 147\frac{1}{2}$, rather than $148\ \frac{1}{2}\ \frac{1}{31}\ \frac{1}{62}\ \frac{1}{93}\ \frac{1}{465}$;[104] presumably this was obtained by $21 \times 7 = 127$, with $\frac{1}{2}$ added as a bad guess for $21 \times \frac{1}{15}\ \frac{1}{93}$. The effect of several approximations in the same calculation can be seen in TABLE I,[105] where the result given in [Hero] for each stage (A) is followed by the accurate result for that stage (B) and then the accurate result which would have been reached at that stage if the calculation had been precise throughout (C).

TABLE I

	A	B	C	Error (A–C)
1	$13\frac{1}{2} \times 13\frac{1}{2} = 182$	$182\frac{1}{4}$	$182\frac{1}{4}$	$-\frac{1}{4}$
2	$182 \times 11 = 2002$	2002	$2004\frac{1}{2}\ \frac{1}{4}$	$-2\frac{3}{4}$
3	$\frac{1}{28} \times 2002 = 71\frac{1}{2}$	$71\frac{1}{2}$	$71\ \frac{1}{2}\ \frac{1}{16}\ \frac{1}{28}$	$-\frac{11}{112}$
4	$71\frac{1}{2} \times 2\frac{1}{2} = 178\frac{1}{2}\ \frac{1}{4}$	$178\frac{1}{2}\ \frac{1}{4}$	$178\ \frac{1}{2}\ \frac{1}{4}\ \frac{1}{8}\ \frac{1}{14}\ \frac{1}{32}\ \frac{1}{56}$	$-\frac{55}{224}$
5	$\frac{1}{2} \times 13\frac{1}{2} \times 2\frac{1}{2} = 17$	$16\ \frac{1}{2}\ \frac{1}{4}\ \frac{1}{8}$	$16\ \frac{1}{2}\ \frac{1}{4}\ \frac{1}{8}$	$+\frac{1}{8}$
6	$178\frac{1}{2}\ \frac{1}{4} + 17 = 195$	$195\frac{1}{2}\ \frac{1}{4}$	$195\ \frac{1}{2}\ \frac{1}{4}\ \frac{1}{14}\ \frac{1}{32}\ \frac{1}{56}$	$-\frac{195}{224}$

To some extent the errors cancel each other out, but it will be noticed that the approximation in stage 1 produces quite a large error after the multiplication in stage 2, while the large error in stage 2 is substantially reduced by the division in stage three. The final approximation in the simple addition of $178\frac{3}{4} + 17$ was presumably aimed at leaving a nice round figure for the answer; without it the total error would have been less than one-eighth of a foot.

Approximations of the sort found in this calculation are quite common in the works we are considering. Of the fifty-two approximations and errors[106] apparently resulting from the handling of fractions, seven involve rounding up to the next unit, although in some cases the next unit below is closer;[107] eleven involve rounding down to the next unit (i.e. ignoring the fractions in the answer), although in some cases the next unit above is closer;[108] there are seven cases of rounding up or down to $\frac{1}{2}$,[109] one of rounding up to $\frac{2}{3}$,[110] and ten of ignoring more or less of the tail of the fraction.[111] In twelve cases the denominators of some fractions in the chain are changed;[112] in five of these cases there is no obvious reason for the change,[113] but in the other seven, fractions in the series $\frac{1}{2}, \frac{1}{4}, \frac{1}{8} \ldots$ are preferred, corresponding to the normal division of the Greek foot into 16 dactyls,[114] In addition there are four cases where the error is greater than 1, but can still be reasonably attributed to difficulties with fractions.[115]

The evidence from these works of [Hero] thus on the whole agrees with what we had argued on general grounds; approximations do indeed occur when calculations involving fractions

[103] *Geometrica* 20. 12.

[104] *Ster.* i. 21. 1. Similar mistakes could occur in the Hellenistic period (e.g. *Didyma* ii. nos. 39, 54–5). Of the thirteen multiplications in *Didyma* ii. nos. 39–41 where all three terms are preserved, ten are correct (in six of these no fractions are involved). The other two errors result from writing down the wrong numeral.

[105] *De Mens.* 28. 1.

[106] The list may not be absolutely complete. It is based on the footnotes to Heiberg's edition of the works (n. 101).

[107] *Ster.* i. 31. 2; ii. 22; ii. 57. 2; ii. 59. 4; *De Mens.* 27; 28. 1; 48.

[108] *Ster.* i. 41. 1; i. 87. 2; ii. 1. 2; ii. 59. 3; ii. 60. 4; ii. 60. 5; ii. 63. 4; ii. 64. 3; *De Mens.* 28. 1 (twice); 38.

[109] *Ster.* i. 67. 2; ii. 11; ii. 66. 4 (twice); *De Mens.* 28. 2 (twice); 52.

[110] *Ster.* ii. 66. 5.

[111] *Ster.* i. 51. 2; i. 76; i. 80; i. 81; i. 91. 1; ii. 10; ii. 60. 5; *De Mens.* 28. 2 (twice); 48.

[112] *Ster.* i. 36. 2; i. 60; i. 66; i. 68; i. 69; i. 82; i. 86. 2; ii. 1. 2; ii. 10; ii. 16; ii. 66. 3; ii. 60. 4.

[113] *Ster.* i. 36. 2; i. 68; ii. 1. 2; ii. 60. 4; ii. 66. 3.

[114] *Ster.* i. 60; i. 66; i. 69; i. 82; i. 86. 2; ii. 1. 2; ii. 10.

[115] *Ster.* i. 21. 11 (twice); ii. 11; *De Mens.* 38.

have to be made. If the resulting errors seem small, it must be remembered that many of the fractions used by [Hero] probably could not have been written as symbols by fifth- and fourth-century architects, and could not have been set out on an abacus. Archaic and classical Greek architects, in so far as they applied their proportional rules by calculation, would necessarily have used rougher approximations.

The information that we get from Vitruvius is complementary to that from [Hero]. TABLE 2 shows the frequency of occurrence of different numerators and denominators in Books iii to vi of

TABLE 2. *Frequency of fractions in Vitruvius*

(*a*) Integral values	Frequency of numbers as:		(*b*) Fractional values	Frequency of numbers as:	
	Numerators	Denominators		Numerators	Denominators
1	144	0	$1\frac{1}{2}$	2	0
2	6	32	$2\frac{1}{2}$	1	1
3	10	27	$3\frac{1}{2}$	1	2
4	3	20	$4\frac{1}{2}$	1	1
5	2	9	$5\frac{1}{2}$	2	0
6	1	14	$6\frac{1}{2}$	1	2
7	1	9	$7\frac{1}{2}$	0	2
8	0	13	$8\frac{1}{2}$	0	2
9	0	3	$9\frac{1}{2}$	0	2
10	0	7	$10\frac{1}{2}$	0	0
11	0	0	$11\frac{1}{2}$	0	1
12	0	9	$12\frac{1}{2}$	0	1
13	0	2	$13\frac{1}{2}$	0	0
14	0	2	$14\frac{1}{2}$	0	0
15	0	1	$15\frac{1}{2}$	0	0
16	0	1	$16\frac{1}{2}$	0	0
17	0	0	$17\frac{1}{2}$	0	0
18	0	3	$18\frac{1}{2}$	0	0
19	0	0	$19\frac{1}{2}$	0	1
20	0	0	$20\frac{1}{2}$	0	0
27	0	1	$24\frac{1}{2}$	0	1
42	0	1	$29\frac{1}{2}$	0	1
50	0	1	$\sqrt{2}$	0	2

the *De Architectura*, which contain Vitruvius' rules for architectural design. This table is based on the way the fractions are expressed, not on what they amount to. Thus $\frac{1}{9\frac{1}{2}}$ is not counted as $\frac{2}{19}$; $\frac{1}{2}\,\frac{1}{6}$ $(= \frac{2}{3})$ is counted as $\frac{1}{2}$ and $\frac{1}{6}$ not as $\frac{2}{3}$; and *dempta tertia* is treated as $\frac{1}{3}$ not as $\frac{2}{3}$. Expressions such as 'Divide X into 7 parts and take 4 of them', are, however, taken as equivalent to $\frac{4}{7}$, etc.

A number of points emerge. First, there are only two proportions involving irrational quantities, and in both cases these rules could be applied geometrically, without calculation.[116] Second, all the first ten numbers are used as denominators. There is some preference for the even numbers, but more noticeable is the preference for smaller numbers, with the frequency decreasing fairly regularly as the numbers get bigger. Above 10, denominators of 12, 14, and 18 are preferred; these are all even numbers so that an initial division by 2 would leave the architect

[116] iv. 1. 11; vi. 3. 3.

once more with a division by a number less than 10. Except where a module is to be derived from the stylobate width,[117] there is only one denominator over 20.[118] Third, when a series of quite closely related proportions is required, it almost always takes a form such as $\frac{1}{8}, \frac{1}{8\frac{1}{2}}, \frac{1}{9}, \frac{1}{9\frac{1}{2}}, \frac{1}{10}, \frac{1}{10\frac{1}{2}}$, etc., and not, for example, $\frac{8}{64}, \frac{7}{64}, \frac{6}{64}, \frac{5}{64}$, etc.; that is by variation of the denominator not the numerator. A possible exception is in the relation of the tablinum to the atrium width, where the series is $\frac{2}{3}, \frac{1}{2}, \frac{2}{5}$;[120] but this may perhaps have been seen as $\frac{2}{3}, \frac{2}{4}, \frac{2}{5}$. The overwhelming frequency of 1 as a numerator is readily apparent from the table; what is not apparent is the rarity of any other number in a truly fractional expression. Only in three instances is there a fraction with a numerator other than 1 expressed directly and simply; in two of those three the fraction is $\frac{2}{3}$;[121] but even such a simple fraction as $\frac{2}{3}$ is normally expressed either as $\frac{1}{2}, \frac{1}{6}$; as *dempta tertia* $(1-\frac{1}{3})$; or by a paraphrase.[122] Otherwise proportions which involve a numerator greater than 1 are expressed in more practical terms: 'Divide the remainder into 12 parts and take 3.'

It is remarkable how often expressions of this kind occur, even when just one part is to be taken, so that there could be no difficulty in expressing the fraction. We have already noticed how the successive systems of proportion in Vitruvius are arranged so that each element could usually have been derived directly, without calculation, from a part which would already exist physically before the element in question could be put in place.[123] The method of expressing proportional relationships in practical terms suggests that some parts of the building were in fact designed in that way. Even if this were not so, however, this method of expressing proportional relationships suggests that when the element being defined was related to another part in the proportion (e.g.) 4:7, the other part was first divided by seven and the result then multiplied by four to give the new element, rather than first multiplying by four and then dividing by seven. That would be important in so far as the design was worked out by calculation, because we have seen that approximation is more likely to occur in division than in multiplication, and its effect is likely to be increased if the approximate answer is then multiplied.[124]

The treatment of fractions in Vitruvius corresponds, if anything, more closely than that of [Hero] to what we had suggested on general grounds for Greek architecture of the fifth and fourth century. There is a general difficulty in expressing fractions overcome by the practical method of expression; and there is the strong predominance of unit numerators and low denominators. Since the evidence on which we based our general arguments was derived as far as possible from sources of fifth- and fourth-century date, this correspondence suggests that Vitruvian practice may not be very far removed from classical practice.[125]

The exclusive use of submultiples in Vitruvius' rules for human proportion is particularly telling, since they almost certainly go back to the fifth or fourth century B.C., the period when Greek sculptors and painters were most interested in systems of proportion for the human figure. It is also worth noticing that the treatment of fractions in Vitruvius' rules for catapult design, which clearly cannot go back to the fifth century, at least, is quite different from that in

[117] iii. 3. 7; iv. 3. 3.

[118] iii. 3. 11—the diameter of an angle column to be increased by $\frac{1}{50}$.

[119] iii. 3. 10; iii. 5. 8; cf. iii. 3. 12.

[120] vi. 3. 5.

[121] $\frac{2}{3}$: iii. 5. 11, iv. 6. 4; $\frac{3}{7}$: iv. 6. 6.

[122] $\frac{1}{2}$ $\frac{1}{6}$: iv. 3. 6, iv. 6. 5; *dempta tertia*: vi. 3. 5, vi. 3. 6; 'divide X into 3 parts and take 2': v. 1. 2, vi. 3. 3.

[123] See above.

[124] See p. 64 above.

[125] Vitruvius iii. 1. 2–3. A. Kalkmann (*Berl. Winck. Prog.* xxxv (1893) 42–5) argued that these rules are likely to derive from Euphranor, whose work on symmetry was directly or indirectly known to Vitruvius (vii. praef. 14). J. E. Raven argues that they derive from Polykleitos (*CQ* N.S. i (1951) 147–52), but with less probability in view of their difference from the system attributed to Polykleitos by Galen (n. 45 above).

the architectural rules. The specifications for catapults are set out using a system of fractions based on sixteenths, which is presumably Hellenistic.

DIMENSION

We noted above that the only factor other than proportion for which we have written evidence as entering into the process of architectural design is dimension,[126] that is, the preference which an architect might show for sizes which could be expressed in round numbers of feet or simple fractions of feet.[127] We have seen in the preceding section how a certain amount of rounding out might be more or less forced on an architect if he calculated the sizes of the parts of his building, because of the difficulties he faced in handling fractions. The extent to which he approximated, and the strength of his preference for round numbers, however, were a matter for each individual architect to decide. It is interesting to see how in Vitruvius' description of his basilica at Fano simple dimension seems to override simple or consistent proportion.[128] Thus the length and width of the central space are given in tens of feet, although this means that intercolumniation of the long sides is considerably shorter than on the short sides. The upper pilasters have a height 9 times their width and a projection $\frac{1}{2}$ their width, while the lower pilasters have a height 8 times their width and a projection $\frac{3}{5}$ of their width. These variations in proportion, which are certainly far greater than what might be demanded by the exigencies of the fractional system, allow simple dimensions to be used, and that presumably was their aim.

This tendency to design in round numbers of feet, whole numbers of feet, and simple fractions of feet is not just a characteristic of Vitruvian design. Philo's specification for the Arsenal at Peiraieus[129] is expressed in exactly the same sort of terms, although in considerably greater detail, and the same tendency has been found in practice in such buildings as the temple of Zeus at Olympia and the temple of Athena at Priene.[130]

The rounding out of dimensions would clearly simplify the ancient architect's task, but it complicates that of the modern investigator of Greek design, for we do not initially know what elements would have been designed as a whole number of feet, and we do not know what length were the feet used for any particular building. In several cases it has proved possible to provide fairly certain answers to the second of these questions. Most scholars would accept that the temple of Zeus at Olympia was designed in feet *c.* 0·326 m. long, that the temple of Athena at Priene was designed in feet *c.* 0·294 m. long, that the temple of Apollo at Didyma was designed in feet *c.* 0·2943 m. long, and so on. That does not tell us, however, how many other foot standards were in use in the Greek world, nor how rigidly they were adhered to.

Dinsmoor maintains firmly that there were only two basic standards in architectural use,[131] a Doric foot of *c.* 0·3265 m. and an Ionic foot of *c.* 0·29395 m., and that these standards were adhered to over large areas of the Greek world through about three centuries, with a maximum variation from the standard of about 0·001 m. He also mentions a third Samian–Ptolemaic–Philetairic standard of 0·350 m. 'and various less important local feet', but does not discuss them. It is unfortunate that Dinsmoor did not publish the analysis of ninety Greek buildings on which his conclusions were based, so that the full evidence is not available. The equations of feet

[126] Vitruvius x. 10–11. For the system see E. Schramm in *SB. BerlAkad.* 1917, 719. The signs (except for ιϛ = $\frac{1}{16}$ and S = $\frac{1}{2}$) are the Greek alphabetic numerals, but they express not the denominator, but the numerator, with the constant denominator of 16 understood. The convenience of using a constant denominator was appreciated by Hellenistic astronomers (who used a sexagesimal system) and by the Romans (who used a standard division into 12ths).

[127] See p. 79 above.

[128] v. 1. 6–10.

[129] See n. 13 above.

[130] *Olympia* ii (1892) 19; T. Wiegand, H. Schrader, *Priene* (1904) 86–7, 101–5.

[131] W. B. Dinsmoor, *The Architecture of Ancient Greece* (2nd edn., 1950) 54 n. 3, 161 n. 1, 195 n. 1, 199 n. 1, 222 n. 2, 229 n. 2; id., in *Atti del VII Congresso Internazionale di Archeologia Classica* i (1961) 357–60; for a third foot standard in an architectural context see note 73 above.

and metres that he does give are in several cases open to other interpretations. Certainly his general proposition needs to be proved, not just accepted, for such external evidence as there is seems to argue against it.

There was in fact little to be gained by the architect using a widely accepted or accurately standardized foot unit. The builders were often paid according to a specified contrast, rather than at a standard rate per foot or per square- or cubic-foot, so that a short foot would not defraud the authorities.[132] Even if they were paid per foot, as at Didyma,[133] the use of a special local length standard would cause no difficulties providing it was accepted by both parties beforehand. The architect's foot would have to correspond within limits to that used by the quarrymen, but since the building-blocks were of limited size and were always delivered from the quarry considerably larger than the intended size, a difference of several millimetres between the architect's foot and the quarrymen's foot might not be significant.[134]

A fixed standard of length does not seem to have been included with the standard weights and measures provided by the market authorities of a Greek city. No standard of length has been found in the Athenian agora,[135] and none is mentioned in the inscription giving specifications for official standards;[136] comparatively few commodities would be sold by length, so the absence of a commercial length standard is not unreasonable. The only positive evidence for official standards of length is an early third-century inscription from Delos, which mentions an *orguia* (fathom) kept in the Prytaneion there.[137] It is significant that this is an inscription dealing with building contracts, not commercial matters, and that the length standard was not kept with the official standards of weight in the *agoranomion*.[138] Therefore while it is possible that such official standards of length may have existed in other cities and in earlier centuries, one cannot use as evidence the fact that official standards of weight and capacity were imposed for commerce at least as early as the fifth century B.C.

No actual remains of Greek length standards have been found. The tile standard from Assos, probably belonging to the Hellenistic period, has cuttings perhaps for bronze inserts, and these could have been standards of length.[139] If bronze was the material used for standards of length, that would explain the absence of remains; but the length standards from the *macellum* at Lepcis Magna, of Imperial date, are engraved on a slab of stone. This slab, with cubits of two different sizes and a foot corresponding to neither, shows that even in the Roman period standards of length had not been unified, and the inaccuracy in the calibration of the subdivisions, amounting to several millimetres from one palm to another in the same foot or cubit, suggests that the standards themselves may not have been very accurately set out.[140]

[132] At Epidauros the accounting suggests that the builders were paid per contract, rather than at a standard rate per foot or cubic foot (A. Burford, *Greek Temple Builders at Epidauros* (1969) 191–8).

[133] *Didyma* ii. nos. 20–44.

[134] [Hero], *Tab. Her.* 4. 4 8nd 5. 11 mentions a πῆχυς λιθικός and a πῆχυς τοῦ πριστικοῦ ξύλου; their length is unknown, but could be slightly larger than normal (like a 'baker's dozen' of 13) to allow for this problem. Since Greek architects seem to have worked in feet rather than cubits, however, these special cubits probably do not relate to classical Greek architectural practice.

[135] M. Lang, M. Crosby, *The Athenian Agora* X, *Weights, Measures and Tokens* (1964).

[136] *IG* ii^2. 1013.

[137] *ID* 502. 24. Some doubtful light on the length of the *orguia* is shed by an inscription ΟΡΓΥΑΙ ΜΖ on the central wall of the South Stoa at Delos (*BCH* xxvi (1962) 545). If this refers to the length of the building (72·47 m. over all; *EAD*, R. Vallois, *Le Portique de Philippe* (1923) pl. 1), it would imply an *orguia* of not more than 1·541 m., and so a foot of not more than 0·2501 m!

[138] *BCH* vi (1882) 139; xxxi (1907) 72.

[139] J. Bacon, F. Clark, R. Koldewey, *Investigations at Assos* (1902–21) 71, 73. The slot lengths are: AB *c.* 2·32 m., CD *c.* 0·518 m., EF 0·367 m., GH 0·485 m. *If* these slots did hold length standards and their lengths bear some relation to the standards they held, then EF could have held a Pergamene–Ptolemaic foot of *c.* 0·35 m., GH an 'Ionic' cubit of *c.* 0·44 m., and CD a 'Doric' cubit of *c.* 0·49 m. However, AB is too long to be an *orguia* unless derived from a foot of *c.* 0·38 ft. It is therefore perhaps more likely that the slots held not general length standards, but standard lengths for specific objects used, like the tiles, in building (the tile dimensions do not come out as round numbers of feet of any of the 'accepted' sizes, either).

[140] There is a good photograph of the slab in R. B.

We may also make some estimate of the standardization of length on the basis of other standardized measurements. In coinage there was a strong motive for adopting a widely acceptable standard of weight and adhering to it closely, yet the various Greek cities used many different standards of coin weight and there were often wide variations from the standard.[141] In commercial weights and measures there was also officially organized standardization in individual cities. Yet the Athenians seem to have changed their weight standards at least twice and possibly five times in the course of about five centuries, and the variations from standard may be over 5 per cent.[142] The need which the Athenians felt to impose their own standard measures on the cities of the Delian League[143] also suggests that there was a substantial lack of uniformity from city to city in this field too. The variation in standards of both weight and length in pre-metric Europe shows that such a regrettable state of affairs is by no means improbable, and Graham[144] has produced good evidence that two foot standards were in use at Olynthos in the late fifth and early fourth centuries B.C.—one of 0·295–6 m. and one of 0·327–8 m.

It is also remarkable, if the foot was so rigidly standardized, that the lengths of athletic stadia in the Greek world varied so much. For although we know that the course was theoretically a stade long (= 600 ft.), no stadium has been found with a course of 176·40 m. or 195·90 m. which are the lengths produced by 600 'Ionic' feet of 0·294 m. and 600 'Doric' feet of 0·3265 m. 'Knowing' that the course was 600 ft., we could deduce from the Olympic stadium a foot of about 0·3202 m., from the stadium at Epidauros a foot of *c.* 0·3022 m., and so on. In no case is there an obvious reason for cutting short the course. Extending it to the full length of 600 'Doric' feet would naturally involve a considerable amount of earth-moving, but no more than for a comparable length of the course that was actually built. The failure to provide the full length is particularly striking at Olympia, where in spite of the great effort that was obviously expended on the stadium, the last 10 ft. or so of the track was omitted—that is, if we assume that its builders used a 'Doric' foot of *c.* 0·3265 m. Even if these various stadium lengths do not prove the existence of foot standards equal to $\frac{1}{600}$ of each course, at least they show that the Greeks did not attach great importance to the standardization of length measurements.

Thus while there is every reason to accept the existence of foot standards of 0·326–7 m. and 0·294 m., it would seem wrong to *assume* that these must be the only ones. It may appear dangerous to suggest that Greek architects may have used a foot of almost any size. Does not this leave the modern theorist with too free a field for uncontrolled speculation on the design of ancient buildings? It does indeed remove a 'fixed point', but if that point is only fixed artificially, then it is better out of the way. Besides, the modern speculation need not be ucontrolled, for though it is difficult to show that a foot of one particular size is to be preferred for a building being studied to a foot of any other size, it is possible to show which, out of a *limited number* of proposed foot sizes, is most likely to have been used in that building, and so to show that a foot of unusual size is (or is not) more likely to have been used than a foot of one of the conventionally accepted sizes.

I have argued in another paper[145] that this is not simply a matter of the accuracy with which the measurements in terms of a proposed foot standard coincide with those actually found in a building. For Greek buildings were not built with mathematical accuracy, and *any* actual measurements can be expressed in feet and dactyls of *any* reasonable size with a maximum error of half a dactyl, i.e. about a centimetre; and of course if fractions of a dactyl are used, the

Bandinelli *et al.*, *The Buried City: Excavations at Lepcis Magna* (1966) pl. 69. A much more accurately set out standard cubit, probably of Ptolemaic or Roman date, is described by F. Petrie, *Ancient Weights and Measures* (1926) 40.

[141] C. M. Kraay, M. Hirmer, *Greek Coins* (1966) 17.

[142] M. Lang, M. Crosby, op. cit. (n. 128) 18–21.

[143] R. Meiggs, D. Lewis, *Greek Historical Inscriptions* (1969) no. 45.

[144] D. M. Robinson, J. W. Graham, *Olynthus* viii (1938) 45–51.

[145] *BSA* lxviii (1973) 81–3.

maximum error will be less. Thus while accuracy of fit is obviously important, it is the simplicity of the dimensions when expressed in feet which matters in the last resort. It is precisely because, as well as fitting the known dimensions of what appear to be important elements of the buildings, they express those dimensions in simple terms, that the foot of 0·326 m. is accepted for the temple of Zeus at Olympia and the foot of 0·294 m. for the temple of Athena at Priene. Since both the accuracy of fit and the simplicity of expression achieved by any proposed foot can be tabulated, it is possible for anybody who disagrees with the foot standard that has been proposed for a particular building to show clearly and objectively that another foot performs better.

There remains, however, the difficulty that we do not know initially which dimensions an architect might consider important, and which dimensions he might wish to have simply expressed in feet. We saw how the same difficulty arose in trying to decide what parts of a building might have been intentionally related to each other in proportion, and it was suggested that the question might be decided on the basis of the simplicity with which a particular relationship could be expressed and the constancy with which it recurred in buildings of the same stylistic group. In some cases this may tell us also which of a group of alternative dimensions an architect is likely to have designed as a simple number of feet. If it is the lower diameter of a column on the arrises which is important in the proportional relationships of a group of buildings, then an architect belonging to the same school is likely to have chosen to make the lower diameter on the arrises a simple number of feet, rather than the lower diameter in the flutes or the average diameter, and so on. But there are other cases where the distinction may not be so easy. Thus both the axial intercolumniation and the stylobate size might reasonably be regarded as important elements of a Greek temple, which an architect is likely to have designed in simple numbers of feet. Nevertheless, if the stylobate is first dimensioned, and the intercolumniation is derived by subdivision of the stylobate size, then the architect has little control over the intercolumniation, so that if the stylobate width is a round number of feet, the chances are small that the intercolumniation will even be a whole number of feet. On the other hand if the intercolumniation is decided on first, and the stylobate size then derived from it, either by multiplying by a suitable factor or by adding up all the normal intercolumniations, the two contracted intercolumniations, and the stylobate width, then the intercolumniation would tend to be a whole number of feet, but the stylobate size might well not be; it is certainly unlikely to be a round number of feet.

The existence of proportional relationships may even be positively misleading in some circumstances. This is particularly so in the case of the modular system. Let us suppose that an architect following Vitruvius' rules for the Doric order[146] used the 'Ionic' foot of 0·294 m., and decided to adopt a module of $1\frac{1}{8}$ ft. (1 ft. being too small, and $1\frac{1}{4}$ ft. being unnecessarily large for the requirements of the building). This would produce the following dimensions in the metric system: D = 0·66 m., I = 2·48 m., H = 4·63 m., Cap. = 0·33 m., A = 0·33 m., F = 0·495 m., Cor. = 0·165 m., etc. A modern investigator might well suppose that he had used a foot of 0·33 m., since this would give the most important dimensions of the design in round numbers of feet: D = 2 ft., I = $7\frac{1}{2}$ ft., H = 14 ft., Cap. = 1 ft., A = 1 ft., F = $1\frac{1}{2}$ ft., Cor. = $\frac{1}{2}$ ft. The foot of 0·33 m. would perform much better by our objective tests than the foot of 0·294 actually used. The clue that something is wrong, however, is given by the fact that the proportions are so simple. In the modular system, once the architect has decided on his module, he simply follows the rules, and will probably not need to do any rounding out of dimensions. It is only where it appears that he has not followed the rules of the system that we may assume that he has altered some dimension for the sake of a simple expression in feet. In the present instance a clue might

[146] Vitruvius iv. 3.

also be given by the abacus height which should be $\frac{1}{3}$ module. If the architect had calculated the abacus height in terms of a capital height equal to 1 ft. of 0·33 m., then 16 dactyls/3 $\simeq$ 5 dactyls = 0·103 m., while if he calculated it in terms of a capital height equal to $1\frac{1}{8}$ ft. of 0·294 m., then 18 dactyls/3 = 6 dactyls = 0·11 m. Unfortunately, he might also have found the abacus height by geometrically dividing into three parts the height of an actual capital block, which would give 0·11 m. in either case.

Even when a successive system is used, the same sort of danger may exist. Thus we find that the triglyph width is normally one fifth of the intercolumniation. Since the intercolumniation is necessarily established at a very early stage in a classical temple, it would be quite possible for the architect to make the triglyph width exactly one fifth of that length without any reference to his foot-rule. Hence, if he chose an intercolumniation of 7 'Doric' feet of 0·3265 m. (= 2·285 m.), and made the triglyph width exactly one fifth of that (0·4571 m.) by a direct division from the lowest step course jointing, this would not be expressible in terms of the foot he used, and a modern investigator might well prefer to think that he used a foot of 0·3047 m., so that I = $7\frac{1}{2}$ ft. and T = $1\frac{1}{2}$ ft. This is of course particularly likely to happen when only a few dimensions are considered by the modern investigator.

These examples should show that a thorough study of the proportional relationships actually used in Greek temples is necessary before one can with confidence decide what foot standard was used in a particular building. For the study of proportions will not only tell us which parts of the building are likely to have been significant to an ancient architect, but it will also tell us whether a particular element might have been defined by the direct application of a proportional rule or whether its size varies slightly from what the normal rule would produce. It is in the latter case, of course, that we may presume that some rounding out has taken place. If we know which way the rule in question normally worked, then we may suppose that the rounding out took place in the element being defined by that rule, which should therefore be simply expressible in feet; the element from which it was defined may or may not also be simply expressible in feet. Even where we do not know in which way the rule normally worked, however, it will be of some value to know that one or other, or both, of two parts of the building should be simply expressible in feet. The amount by which the dimension found differs from that predicted by the rule should in addition give us some idea of the roundness of the number of feet (or parts of feet). If the difference is small, the dimension may simply have been rounded out to the nearest palm or half-palm, but where the difference amounts to a palm or more (say 0·08 m. or more), the dimension should have been rounded out to the nearest foot, for if the rule would have made the element, say, 1 ft. $3\frac{1}{2}$ dact. long, there is no reason why the architect should round that figure to $1\frac{1}{2}$ ft., rather than to the nearer and simpler figure of 1 ft.

ACCURACY

Whether we are trying to find the foot unit used in a building or the rules of proportion which apply to it, a question which continually recurs is what degree of accuracy we are to expect in the execution of the design. Is it reasonable to say that the stylobate was designed as 200 ft. long if, using the proposed foot standard, its length falls short of 200 ft. by 0·10 m.? Is it reasonable to say that the frieze height was designed as one third of the intercolumniation, if it falls short of that amount by 0·01 m.? At what stage do we have to call in some other factor than inaccuracy to account for the discrepancies between predicted and actual measurements? These are questions to which the literary and epigraphic sources provide no direct answer; we must therefore base our answers on the existing remains of buildings, and on some general considerations of the way in which measurements would have been executed.

The first impression left by Greek architecture, and particularly Doric architecture, is of extreme accuracy. That is partly because the standard of accuracy is indeed high, but also because the clear articulation of the parts and the thoughtful refinement in detail automatically suggest accuracy. Nevertheless, it is important to realize that mathematical accuracy in architecture is an impossibility. Neither an ancient architect, nor a modern archaeologist, can measure a stylobate with unqualified accuracy, for no matter how precise the instrument used, and no matter how much care is taken, there will always be an element of imprecision, even if it is only a matter of microns or fractions of a micron. We are dealing, therefore, not with outright accuracy as against inaccuracy, but with various degrees of accuracy, which can, in theory at least, be assessed.

Even when that is admitted, however, it may not be easy to say to what standard of accuracy an architect and his team were working. The difference between the stylobate length of the north and south sides of the Parthenon is about 0·0025 m., rather less than 1 part in 20,000. That might suggest that Iktinos and his masons were normally working to tolerances of 1 : 20,000, so that any larger discrepancies between theoretical and actual measurements cannot simply be due to inaccuracy. Such a conclusion may seem logical, but in fact it is not. Precision in measurement depends on the equipment used, the methods according to which it is used, and the care taken. It is most unlikely that the equipment and procedures were the same for all the measurements in the Parthenon, and it is quite possible that the same care was not taken with each separate measurement.[147] We cannot therefore simply take the extreme accuracy of the stylobate lengths as indicative of the general standard of accuracy in other parts of the building. It may be helpful, first, to consider the evidence for the instruments and procedures used in measuring by the Greeks.

Nothing is known in detail about the measuring instruments used by the Greeks, for none have survived. In inscriptions relating to building projects, we find frequent mention of κανόνες,[148] but both in inscriptions and in literature, the κανών is normally an instrument for checking the truth of lines and surfaces rather than their length; it is a straight-edge rather than a measuring-rod. Only once, in Apollonios Rhodios, is the word κανών used of a measuring instrument.[149] Aristophanes once uses the word πῆχυς to mean a cubit-rule,[150] an instrument not a unit of measurement, but it is unlikely that a cubit-rule was the only measuring instrument used in laying out temples which might be 200 cubits long.

In the later technical literature on surveying and mensuration we find both the measuring-rod (κάλαμος and ἄκαινα) and the measuring-cord (σχοινίον).[151] None of these words is found in that specific sense before the Hellenistic period, but both methods of measurement were probably used by the Pharaonic Egyptians, and so both could well have been known to the Greeks considerably earlier. Hero of Alexandria in the *Dioptra* twice talks of measuring with a chain (ἅλυσις),[152] but regards a cord (σχοινίον) as the normal thing. That word was also used as the name of a land measurement, equivalent to 60 or 72 Philetairic feet, which occurs frequently in tables of measurements, and presumably this indicates the length of the normal measuring-cord. Hero was clearly aware of the possible inaccuracy of a measuring-cord, for in three of the four problems in the *Dioptra* which involve the use of one he emphasizes that it must be tested beforehand, and that it must be one that will not stretch.[153] The verdict of a modern surveying handbook on linen tapes is that 'they should not be used where precise results are

[147] It has often been noticed, for instance, that the intercolumniations of the Parthenon are much less accurately set out, with variations of up to 0·048 m. (Penrose, pl. 4).

[148] e.g. *IG* i². 372E4; for other references see Orlandos ii. 140 n. 1.

[149] Ap. Rhod. i. 724.

[150] Aristoph., *Ran.* 799.

[151] See F. Hultsch, *Metrologicorum Scriptorum Reliquiae* (1894–6), index.

[152] Hero, *Dioptra* (ed. Schöne, 1903) 262. 12, 292. 18.

[153] Ibid. 254. 14, 262. 13, 270. 15.

required. . . . They stretch when pulled and may easily be permanently elongated. . . . Exposure to wet causes them to shrink.'[154] The stretching or shrinkage may amount to as much as one part in 200, and ancient measuring cords must have been subject to much the same defects. Nevertheless, it should be noted that these defects relate for the most part to the laying out (or the recording) of a specific number of feet. Two lines intended simply to be of the same length as each other may be set out with very great accuracy by the same people with the same tape on the same day, for any permanent alterations to the length of the cord will not affect the similarity of the two measurements (even if it affects their absolute length in feet), climatic variations will be eliminated, and since the same people are involved, the amount of stretch due to pulling should be roughly the same for both measurements.

The measuring-rod would provide a more accurate, though more time-consuming, means of setting out a given length. Wood expands and contracts along the grain comparatively little with changes in temperature and humidity, and brass-shod ends would prevent wear or irregular surfaces from affecting the accuracy of the measurement. The chief sources of error are likely to be imprecise standardization of the rod lengths and the occurrence of foreign bodies between the ends of the rods when placed end to end. Both the ἄκαινα and the κάλαμος meant specific units of measurement—10 and 7½ Philetairic feet respectively, according to the table in [Hero], *Geom.* iv. 11–12—and that should give an indication of the length of rod used. Roman surveyors also, typically, used a measuring-rod, the *pertica*; it was normally 10 ft. long, but might be 12 ft., 15 ft., or 17 ft. according to local conditions.[155]

So far we have considered the laying out of long measurements to an integral number of rod- or cord-lengths. Intermediate distances would certainly be needed in architectural work, and here there was opportunity for further error to arise. The calibration of the cord or rod might be less accurate than the over-all length was; the feet at one part might not be exactly of the same length as those at another. Further it is unlikely that either cord or rod would have been fully calibrated in dactyls, or even palms, so that if a long measurement of, say, 203¾ ft. was required, the fraction of a foot at the end would probably require a different instrument, a shorter rod with finer graduations. There is thus not only the increased possibility of error because of the process of changing from one instrument to another, but also the possibility that the second instrument, used for the smaller measurements, may not have been calibrated with precisely the same length of foot as that used for the longer measurements.

The calibrations of a group of Egyptian measuring-rods were studied by Petrie,[156] and the mean, minimum, and maximum lengths of the divisions of these rods are given in TABLE 3, together with the difference of the minimum and maximum lengths from the mean, expressed as a percentage of the mean length. Only two rods have no division differing by more than 1 per cent from the mean length, and one of these is an official standard not meant for use (no. 16). On the other hand in four cases there are at least two divisions varying more than 10 per cent, or, in absolute terms, more than 0·008 m. from the mean length. There is no evidence for the purpose for which these rods were made, but they were probably used by craftsmen of some sort. I have found no comparable study of the calibration of Roman measuring-rods. Petrie gives some supplementary information on a number of Roman foot measures in Rome and Naples, indicating a variation in foot length of 0·004 m.;[157] but his figures are not complete. One might expect standards of accuracy to be higher in Greece and Rome than in Egypt, but it

[154] D. Clark, *Plane and Geodetic Surveying* i (5th edn., 1958) 51.

[155] F. Hultsch, *Metrologicorum Scriptorum Reliquiae* ii (1896) 113. 12; cf. Hor., *Car.* ii. 15. 14–16. For the variant lengths, F. Hultsch, op. cit. 137. 6 (Isidorus).

[156] F. Petrie, *Ancient Weights and Measures* (1926) 38–40.

[157] Ibid. 48. Cf. A. Grenier, *Manuel d'archéologie gallo-romaine* iii. 1 (1958) 36.

would be wrong for our conception of the accuracy of calibration of the instruments actually used by builders to be based on the accuracy of calibration found in official standards.[158]

The calibration of measuring instruments will effect Greek design practice in another way. Architects will not have used, as such, fractions of a foot or of a dactyl that were not marked on measuring-rods.[159] Roman foot-rules are normally divided into palms and dactyls on one side, into palms and inches on another, so that their users could work in either the Greek or the Roman system of measurement. The dimensions specified in Greek building inscriptions show that measurements in feet, or feet and palms, were preferred, but that measurement in dactyls could readily be made if required.[160] Greek builders' rules must therefore also have been calibrated in

TABLE 3. *Calibration of Egyptian measuring-rods*

Rod no.	Length of divisions in inches			Percentage variation from mean of:	
	Mean	Min.	Max.	Min.	Max.
1	3·409	3·396	3·424	0·381	0·44
2	3·43	3·04	3·85	11·37	12·25
3	1·03	0·86	1·23	16·5	19·4
4	0·737	0·58	0·93	21·3	26·2
5					
6	2·97	2·916	3·034	1·82	2·16
7					
8	2·99	2·865	3·062	4·17	2·4
9	3·518	3·498	3·555	0·569	1·05
10	3·555	3·464	3·592	2·56	1·04
11	3·55	3·50	3·65	1·41	2·82
12	3·02	2·91	3·06	2·74	1·32
13	3·64	3·11	4·49	14·6	23·4
14	3·82	3·67	4·00	3·93	4·71
15					
16	3·829				

palms, and then partly or wholly in dactyls. The evidence for calibration in fractions of a dactyl is less satisfactory. Dinsmoor's interpretations of Greek design in terms of 48ths of a foot[161] imply that Greek measuring-rods were calibrated in thirds of a dactyl; but that suggestion seems to have been based on the belief that Greek foot-rules, like Roman ones, were calibrated in inches as well as dactyls,[162] so that the inch calibration would allow the measurement of $1\frac{1}{3}$ dactyls. In fact, however, there appears to be no evidence that the Greek foot was ever divided duodecimally, and no Roman foot-rule has even one inch or dactyl subdivided. The only exception is the official standard from the *macellum* at Lepcis Magna;[163] each of the two cubits and one foot set out on it is fully divided into palms, with the palm at one end of each divided into three inches, and the palm at the other end divided into four dactyls; the last dactyl on one of the cubits is then divided into halves. Turning to the building inscriptions, we find two instances of

[158] Thus Dinsmoor (*Atti del VII Congresso Internazionale di Archeologia Classica* i (1961) 357) noted that metre sticks sold in Athens varied by up to 0·0015 m.; yet modern official standards of length would not vary by a tenth of that amount.

[159] Some ancient architects' rules were very finely divided (see above, note 18); there is no evidence that such rules were used by Greek architects, but even if they were, it would not be possible to specify, as such, dimensions which could not be measured out with the builders' rules.

[160] Most inscriptions show a clear preference of measurements in feet and palms, but contrast *IG* ii². 1678.

[161] e.g. *Hesperia* ix (1940) 46; *Hesperia* Suppl. v (1941) 93.

[162] Cf. *AJA* xxvi (1922) 263.

[163] See note 140 above.

measurements involving half-dactyls before the Hellenistic period,[164] and these are both small measurements ($\frac{1}{2}$ and $1\frac{1}{2}$ dact.) where the effect of the half-dactyl, even if it were just estimated, would be significant, much more so than in a longer measurement. The only long measurement in an architectural context which involves a half-dactyl is that on one of the drums of an unfluted column at Didyma, dating from the Hellenistic period.[165] The absence of half-dactyls in *IG* ii². 1678, where measurements of three or four feet are consistently taken to the nearest dactyl, suggests that Greek builders normally measured only to the nearest dactyl.[166] There is no evidence for the use of any fraction of a dactyl less than a half.[167]

In setting out the various parts of a building, different procedures will have been used, as well as different instruments, The most important distinction here is between measurements which set out a given distance in cubits, feet, palms, dactyls, etc., and measurements which are related directly to some other measurement already laid out—either repeating it, subdividing it, or multiplying it. The first type of measurement is what we have so far considered, and we have seen that it is in setting out measurements in feet that the shortcomings of the measuring instruments, particularly inaccuracy and lack of fineness in calibration, are most likely to have significant effect. Even when the same instruments were used, direct repetition would involve less inaccuracy; but for some purposes it is likely that dividers or callipers would be used for measurements of this kind,[168] and in that case difficulties with calibration would not be involved. Thus if the intercolumniation was divided into five parts to obtain the triglyph width by stepping out the distance with dividers, the accuracy obtainable would depend only on the patience of the measurer. Of course any inaccuracy in the setting out of the initial dimension in feet would necessarily be repeated, multiplied, or divided in the course of such measurement by repetition, multiplication, or division, but the relationship of the two parts concerned in the process could be very accurately maintained.

Repetition is by far the most important of these processes, for no matter how the intended size of a part might be defined, there would often be difficulties in setting it out. Thus even if the height of the column was designed as 20 ft., it would be difficult to set out that measurement in practice on 36 columns or so with the accuracy necessary to prevent the entablature from undulating unpleasantly. It would not be easy to use measuring rods up the side of a column which first tapered and then projected again at the capital. A measuring-cord would probably not be accurate enough, and serious cumulative errors would tend to arise if the heights of successive drums were measured in feet and dactyls and added up. The simplest way to achieve the desired result would be to cut a piece of timber to a measured length of 20 ft., and then use a *diabetes* to test the height of each column against that of the timber.[169] In this way, even if the piece of timber had been very carelessly and inaccurately measured, so that the column height varied considerably from what the architect had intended, at least all the columns would be the same height, which was what mattered most.

[164] *IG* ii². 1671. 45, 1675. 15. The convenient compound form ἡμιδακτύλιον is not recorded before *c.* 200 B.C. (Philo, *Bel.* 63. 42, 65. 3, 65. 23 with three other later Hellenistic instances noted by *LS*⁹).

[165] *Didyma* ii. no. 48 ($6\ \frac{1}{4}\ \frac{1}{8}\ \frac{1}{16}\ \frac{1}{32}$ feet = $6\frac{15}{32}$ ft. = 6 ft. $7\frac{1}{2}$ dact.). Fractions of a dactyl would be almost unavoidable if the entasis of a column were set out by means of co-ordinates, as it apparently was at Didyma, but J. Bundgaard, *Mnesicles* (1957), 137–8, has suggested that a springy piece of wood was normally used to give the required curve. Such a method could well have been considered impracticable with the 19·7-m.-high columns of the temple at Didyma, so that the use of co-ordinates was perhaps exceptional.

[166] This does not imply, of course, that they *worked* only to the nearest dactyl; cf. pp. 95–7 below.

[167] The occurrence in *IG* vii. 3073. 115 of δακτυλίσκος (= 'about a dactyl' rather than 'a small dactyl'; cf. C. D. Buck, W. Petersen, *A Reverse Index of Greek Nouns and Adjectives* (1945) 637–8) suggests that small fractions of a dactyl were not normally measured, for one would otherwise expect a more precise expression.

[168] Both callipers and dividers are shown on Roman reliefs (Orlandos ii. figs. 73–4).

[169] For the *diabetes* as a levelling instrument see Orlandos ii. 135–7.

It is indeed generally true that regularity was more important to a Greek temple than absolute size. It was much more important that all the capitals, or all the triglyphs, should be of the same width than that they should be a specific number of feet in width—more important even than that they should be accurately related by a simple proportion to some other part of the building. The size as a specific number of feet was obviously of practical rather than aesthetic significance, while a slight variation from the theoretical proportion would probably be unnoticed—as is suggested by the willingness of architects to modify their proportions to obtain dimensions easily expressible in feet.

This uniformity between several examples of the same element was apparently achieved by means of the models, παραδείγματα, which are frequently mentioned in inscriptions concerned with building.[170] Full-size models in wood or stone were prepared for such elements as the capital and the triglyph, presumably under the close supervision of the architect, if not by him. The masons working on the carving of such elements then took their measurements directly from the models. For simple but important measurements such as the lower and upper diameters of the column, pieces of wood could have been measured and cut to the required length as suggested above for the column height. The transfer of dimensions from the model to the piece being worked on could be done with great accuracy and ease by means of callipers, without any reference to standard units of measurement.

Thus if we suppose that the architect's design was conceived in terms of feet or of simple proportions, the finished building was two stages away from that design. Differences between design and execution would occur, first, when the models were being made, and, second, when the masons were carving the actual elements of the building on the basis of the models. We can judge the degree of error in the second process by seeing how much variation there is within a single building in such elements as frieze height, abacus width, etc. If sufficient examples of any element survive, then their mean dimensions should give us an accurate notion of the dimensions of the model, and we can judge what was the allowable variation of size in that particular element. The only direct way, however, of assessing how far the model differed from the architect's theoretical design would be by working out that design and comparing it with the mean values found in the building as executed; that is, by completing the investigation to which this study is meant to be preliminary. We must be content to make some initial allowance on the grounds of what is likely, and what can be discovered of the accuracy achieved in repetition from the model.

Any detailed study of the accuracy of measurement by repetition would have to be based on the re-measurement of a considerable number of buildings, for many students of Greek architecture, including the present author, have tended to regard the variations between theoretically identical parts of a building as insignificant fluctuations about an ideal which is represented by the average measurement. For this reason just one figure tends to be given (with a greater or smaller degree of precision) for the dimensions of each element. The list which follows (TABLE 4) is therefore a random collection of examples. It would appear that in many buildings a variation of about 0·01 m. between 'identical' elements was considered acceptable for the smaller elements, while variations of several centimetres might occur in the larger ones. We have seen that the motive for accuracy in basing the model on the architect's design was less strong than in making the elements of the actual building follow the model, and that the difference in the way the measuring was done in these two processes would also tend to produce greater accuracy in the transfer of measurements from the model to the building than in the creation of the model from the design. We may therefore presume that discrepancies between the model

[170] See Orlandos i. 29 n. 1, ii. 268 n. 3.

TABLE 4. *Variation in theoretically identical measurements*

Element of building	Reference	Extreme values (m.)	Range (m.)	Range (% of mean)
Olympia, temple of Hera				
L	*Olympia* ii. pl. 18	49·99–50·01	0·020	0·04
W	”	18·75–18·75	0·000	—
Selinous, Temple C				
I_L	KP 99	3·75–3·96	0·210	5·5
D_W	”	1·894–1·923	0·029	1·5
D_L	”	1·72–1·894	0·174	9·6
T_W	KP 101	0·94–1·08	0·14	13·9
T_L	”	0·87–0·99	0·12	12·9
S	KP 104	2·03–2·13	0·10	4·8
Selinous, Temple D				
I_W	KP pl. 13	4·34–4·40	0·060	1·4
I_L	”	4·48–4·517	0·037	0·82
Selinous, Temple FS				
I_L	KP 118	4·55–4·65	0·100	2·2
W	KP pl. 16	24·25–24·43	0·18	0·74
Paestum, first temple of Hera ('Basilica')				
I_W	KP pl. 2	2·84–2·89	0·050	1·75
I_L	”	3·07–3·11	0·040	1·3
W	”	24·47–24·525	0·055	0·22
L	”	54·23–54·296	0·066	0·12
Corinth, temple of Apollo				
I_W	*Corinth* i. 1, 120	4·00–4·02	0·020	0·50
Ab	” 121	0·30–0·305	0·005	1·7
A	” ”	1·19–1·215	0·025	2·1
Paestum, temple of Athena ('Ceres')				
I_W	KP pl. 3	2·625–2·635	0·010	0·38
I_L	KP pl. 3	2·605–2·64	0·035	1·3
W over second step	”	14·525–14·527	0·002	0·014
L ”	”	32·875–32·875	0·000	—
Akragas, temple of 'Heracles'				
I_W	KP pl. 21	4·605–4·62	0·15	0·32
I_L	”	4·59–4·64	0·50	1·1
Akragas, temple of 'Juno Lacinia'				
W	KP pl. 24	16·895–16·96	0·065	0·38
L	”	38·13–38·18	0·050	0·13
Cella W	”	9·915–9·93	0·015	0·15
Stylobate block length	”	1·50–1·58	0·080	5·2
S	”	1·475–1·58	0·105	6·9
I_W	KP 168	3·08–3·15	0·070	2·2
I_L	”	3·03–3·13	0·10	3·2
Aigina, temple of Aphaia				
OW	*Aegina* pl. 32	15·49–15·53	0·040	0·26
OL	”	30·50–30·50	0·000	—
S (porches)	”	1·115–1·14	0·025	2·2
Stylobate block length	”	1·30–1·32	0·020	1·5

TABLE 4 (*continued*)

Element of building	Reference	Extreme values (m.)	Range (m.)	Range (% of mean)
Olympia, temple of Zeus				
S_W	*Olympia* ii. pl. 9	2·42–2·43	0·010	0·04
S_L	,,	2·39–2·42	0·030	1·25
S (porches)	,,	1·78–1·80	0·020	1·1
Flank stylobate block length	,,	2·59–2·62	0·030	1·15
Front stylobate block length	,,	2·59–2·61	0·020	0·77
A	*Olympia* ii. 7	1·75–1·77	0·020	1·1
Selinous, temple of Hera (ER)				
I_W	KP pl. 18	4·71–4·735	0·025	0·53
I_L	,,	4·703–4·727	0·024	0·51
Flank stylobate block length	,,	2·34–2·37	0·030	1·3
Selinous, temple A				
I_L	KP pl. 15	2·972–3·016	0·044	1·5
Paestum, second temple of Hera ('Poseidon')				
W	KP pl. 4	24·245–24·285	0·040	0·16
L	,,	59·99–60·005	0·15	0·025
I_W	,,	4·45–4·48	0·030	0·67
I_{WA}	,,	4·26–4·303	0·043	1·0
I_L	,,	4·49–4·51	0·020	0·44
I (int. cella)	,,	3·47–3·51	0·040	1·5
Anta front face	,,	1·677–1·692	0·015	0·81
Anta ext. face	,,	0·93–0·96	0·030	3·2
Akragas, temple of 'Concord'				
W	KP	16·912–16·93	0·018	0·11
L	,,	39·435–39·44	0·005	0·013
I_L	,,	3·19–3·21	0·020	0·62
S	,,	1·60–1·63	0·030	1·9
Stylobate block length	KP 172	1·58–1·615	0·035	2·2
Segesta, temple				
W	KP pl. 19	26·247–26·263	0·016	0·06
L	,,	61·154–61·185	0·031	0·05
I_W	,,	4·348–4·368	0·020	0·45
I_L	,,	4·327–4·365	0·038	0·88
D (unfluted)	,,	1·893–1·995	0·102	5·2
Flank stylobate block length	,,	2·340–2·37	0·030	1·3
Athens, Parthenon				
W	Penrose pl. 4	30·889–30·895	0·006	0·02
L	,,	69·537–69·541	0·004	0·006
I (North)	,,	4·262–4·304	0·042	1·0
I (East)	,,	4·291–4·302	0·011	0·26
I (South)	,,	4·284–4·310	0·026	0·60
I (Opisthodomos)	,,	4·186–4·196	0·010	0·24
Ext. face opisthodomos, anta	,,	0·416–0·423	0·007	1·4
AbW (North)	Penrose pl. 5	2·015–2·056	0·041	2·0
AbW (East)	,,	2·057–2·059	0·002	0·10
AbW (South)	,,	1·997–2·004	0·007	0·33
AbW (West)	,,	2·003–2·005	0·002	0·12
AbW (Opisthodomos)	,,	1·756–1·757	0·001	0·07
T	,,	0·842–0·843	0·001	0·07

TABLE 4 (*continued*)

Element of building	Reference	Extreme values (m.)	Range (m.)	Range (% o
Athens, Propylaia				
I	Penrose pl. 27	3·631–3·633	0·002	0·04
I_A	,,	3·384–3·386	0·002	0·04
Delos, Temple of the Athenians				
Cp	*EAD* xii. 116	0·352–0·357	0·005	1·4
Ab	,, 117	0·139–0·145	0·006	4·2
T	,, 122	0·370–0·372	0·002	0·54
Mutule	,,	0·367–0·380	0·013	3·5
Delos, Great Temple of Apollo				
D	*EAD* xii. 95	0·936–0·97	0·034	3·6
Cp	,,	0·530–0·542	0·012	2·2
Ab	,,	0·200–0·202	0·002	1·0
AbW	,,	1·113–1·116	0·003	0·27
A	*EAD* xii. 96	0·77–0·78	0·010	1·3
F	,,	0·742–0·752	0·010	1·3
Cor	,,	0·287–0·296	0·009	3·1
Toichobate height	,,	0·293–0·298	0·005	1·7
A (pronaos)		0·563–0·564	0·001	0·18
Bassai, temple of Apollo				
A	Roux 29	0·83–0·845	0·015	1·8
F	,,	0·83–0·845	0·015	1·8
Delphi, Tholos				
Stylobate block length	*FdD, Ath. Pron.* 2, 3	1·0575–1·0615	0·004	0·38
S	,,	0·974–0·975	0·001	0·1
Brauron, Stoa				
D	Bouras 35	0·665–0·688	0·023	3·4
I	,, 34	2·77–2·92	0·150	5·3
F	,, 52–3	0·644–0·647	0·003	0·47
TTn	,, 51–2	0·85–0·94	0·009	9·5
Cornice proj.	,, 62–6	0·270–0·276	0·006	2·2
Oropos, Stoa at the Amphiaraion				
D (in flutes)	*BSA* lxiii. 157	0·605–0·625	0·020	3·25
T	,, 159	0·317–0·321	0·004	1·25
Perachora, Stoa by Harbour				
D	*BSA* lxiii. 157	0·58–0·59	0·010	1·7
I	*BSA* lix. 106	2·300–2·303	0·003	0·13

and the design were at least as great as, and perhaps considerably greater than, the variations of 'identical' parts from the mean sizes.

Somewhat different from the error arising from instruments and methods is the occurrence of gross error, that is, major and unusually simple error such as that of accidentally laying out the wrong number of feet, or losing count of the number of rod-lengths laid out. Such error is rare in Greek architecture, but two examples of it may exist at Perachora and at Magnesia on the Maeander. In the Stoa by the Harbour at Perachora the rear walls of the two wings are approximately the same length, but the east wing is 0·32 m. (i.e. about 1 ft.) deeper than the north wing, with the result that the east stylobate is about 0·32 m. longer than the north one. The difference is apparently too great to be accounted for by instrumental error, and there is no

other obvious reason for it.[171] The width of the Agora at Magnesia on the Maeander at the south end is about 4·80 m. less than at the north end. Again, there is no obvious formal or functional reason for the difference, so that some gross error is a possible explanation, although the precise nature of that error is uncertain.[172]

CONCLUSION

Little in this paper can be said to have been proved, but it may be helpful to sum up the probable results which emerge from the evidence considered. This will be done first from the point of view of ancient architectural practice, and then from the point of view of the modern investigator of that practice.

The general character of Greek architecture suggests that temples were consciously designed, and their simple structure and purpose probably allowed the architect to concentrate on the problem of evolving an aesthetically satisfying appearance. Since the restrictions imposed by the site or by financial considerations rarely affected the basic nature of the design, all architects were facing a common problem, and would probably approach it in a common way. From the beginnings of Doric stone architecture, the proportions used were consistent and therefore presumably intentional; there is also good evidence for a preference for round numbers of feet in the design of buildings. In addition, some allowance may be made for designing by eye, within the limits dictated by the rules of proportion, but initially at least this can be regarded as an additional margin of error.

The use of proportion was in most cases probably a method of design, rather than a theoretically justified mathematical procedure. The variation found in Doric architecture, together with the difficulty in finding simple proportional relationships in it, suggests that the system of proportion used was not a modular one, as in Vitruvius' rules for Doric, but a successive system as in his rules for Ionic. It is likely that the rules were formulated in such a way that they could be applied as the building went up, with little detailed designing beforehand. The evidence for commensurable ratios in architecture is stronger than that for incommensurable ones.

Both in formulating rules and in applying them, a Greek architect would almost certainly become involved in fractions. Evidence for the treatment of fractions in the sixth century is lacking, but in the fifth and fourth centuries it would seem that there was no numeral notation for fractions applicable to architecture, and that in their use of verbal expressions the Greeks strongly favoured submultiples, with a further preference for small, or at least non-compound, numbers as denominators. The same preferences are to be found in Vitruvius' rules for architectural design. In the absence of a numeral notation for fractions, and with the use of the abacus as an added incentive in that direction, a considerable amount of rounding out was almost inevitable, in so far as dimensions were calculated beforehand by applying the rules of proportion arithmetically; that could be avoided, however, if the rules were applied geometrically at full scale, so as to derive the size of an element from that of another already in place.

Again, in so far as he calculated his design beforehand, an architect would have to use the conventional system of feet and dactyls, and since orders for blocks of specific size would have to be placed with the quarries, the dimensions of all elements would have to be established with more or less accuracy in feet and dactyls. The extent to which the length of the foot was accurately standardized is uncertain, but the evidence from parallel fields suggests that standards

[171] *BSA* lix (1964) 101. Notice how once this error had occurred, its effect was felt right up the building, with triglyphs of different sizes required for the north and east colonnades (ibid. 108–9).

[172] C. Humann, *Magnesia am Maeander* (1904) 107.

were neither uniform and unchanging nor accurately embodied, so that the contrary viewpoint must be proved, not just assumed.

In setting out the intended dimensions of his building, the architect would necessarily have used instruments which were liable to error, and which are unlikely to have been calibrated more finely than by dactyls. The errors are likely to have affected the setting of a certain number of feet more seriously than the repetition or division of a distance already set out, and the widespread use of *paradeigmata*, full-scale specimens of elaborate parts, was probably intended to overcome these difficulties, since uniformity was visually more important than the precise embodiment of a particular proportion or number of feet. Nevertheless, even in repetition perfect accuracy could never be achieved, and variations of over 2 per cent occur in buildings of the sixth and fifth centuries. Since errors in repetition are likely to have been less serious, errors of at least that amount are likely to have occurred between the design as conceived in terms of proportion and dimension, and the execution of the specimen which the surviving architectire followed.

The general picture of Greek architectural practice which thus emerges seems a reasonably consistent one. Precision was the aim in the vital matters of uniformity and the continuity of visible lines, both matters where it could be achieved by taking pains with simple means; but in other aspects of design it is doubtful whether either the technological or the mathematical means for achieving theoretical precision were available to Greek architects, and this would explain the apparent changes of plan and failures of foresight which appear in many Greek buildings, particularly in unusual ones. Unless very strong evidence for them can be presented, we should probably disregard explanations of Greek architecture which entail complicated mathematics, detailed preliminary planning, measurements in fractions of a dactyl, and so on.

For the scholar who wishes to understand Greek architectural design, the foregoing discussion suggests some steps which may be helpful. If it is accepted that Greek design methods are likely to have been conventional, then it is important to use this presumed continuity to distinguish what was intended from what is simply coincidental—that is, to show that a proposed rule does not just apply to a feature of one building, but explains also the design of the same feature in several other buildings, preferably of the same general area and period. The discussion of accuracy and of the treatment of fractions may show how precisely we are to expect a rule to be realized in the actual building to which it was applied.

The discussion of accuracy also applies, of course, to investigations of the foot unit used in a building, but since we cannot tell *a priori* what parts of a building are most likely to have been designed as round numbers of feet, and since the occurrence of simple proportional relationships may be misleading in an investigation of the unit of measurement, the study of proportion should, theoretically at least, preceed the study of units of measurement. For while it is unlikely that even a thorough study of proportion will unlock all the secrets of Doric design method, such a study should provide some objective evidence for deciding which parts of the building the architect considered in his design, and which relationships result from the application of the normal rules of proportion, rather than the use of a common unit of measurement.

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