

A Novel Resolver-to-360° Linearized Converter

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Abstract—A novel and simple resolver-to-dc converter is presented. It is shown that by appropriate processing of the sine and cosine resolver signals, the proposed converter may produce an output voltage proportional to the shaft angle. A dedicated compensation method is applied to produce an almost perfectly linear output. This enables determination of the angle with reasonable accuracy without a processor and/or a look-up table. The tests carried out under various operating conditions are satisfactory and in good agreement with theory. This paper gives the theoretical analysis, the computer simulation, the full circuit details, and experimental results of the proposed scheme.

Index Terms—Angle measurement, compensation, linearization, resolver converter, resolver-to-dc converter.

I. INTRODUCTION

RESOLVERS are rugged and reliable absolute angle transducers especially suited to hostile environments. These devices are used in various positioning applications including robots, machine tools, aircrafts, radars, and satellite antennas. Resolvers resemble small electric motors and are essentially rotary transformers designed so the coefficient of coupling between rotor and stator windings varies with the shaft angle [1], [2]. Usually, the rotor winding of the resolver is used as primary and is supplied with a sinusoidal excitation voltage

$$V_{ex}(t) = A_{ex} \sin \omega t. \quad (1)$$

As a result, the two windings located, at right angles, in the stator produce amplitude-modulated ac signals, one with the sine and the other with the cosine of shaft angle, Θ . If the angular velocity of the rotor is much smaller than ω , a condition usually fulfilled because of the relatively elevated excitation or carrier frequency (typically a few kilohertz), the stator waveforms of the resolver are given by

$$V_C(t, \Theta) = A \cos \Theta \sin \omega t \quad (2)$$

$$V_S(t, \Theta) = A \sin \Theta \sin \omega t \quad (3)$$

where A is a constant determined by the amplitude of the excitation signal and the transformation ratio, α , between stator and rotor windings ($A = \alpha A_{ex}$).

Various resolver-converters, commonly referred to as resolver-to-digital converters, have been described for the measurement of Θ by appropriate processing of (1)–(3). Most of these schemes require the use of a processor or a look-up

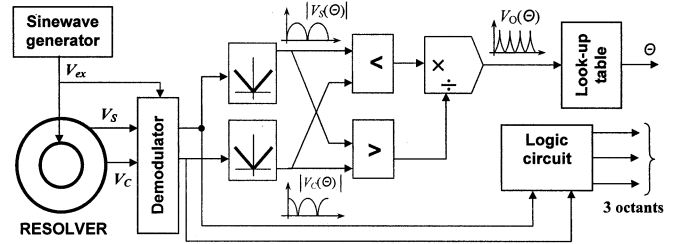


Fig. 1. Simplified diagram of the resolver converter based upon the tangent method.

table technique in order to compute the mechanical angle because of the inherently nonlinear trigonometric resolver signals. Commercial resolver converters are built mainly around the successful tracking method [2]–[7]. This employs a PLL technique based upon the operation in a closed loop in which an estimated angle tracks the shaft angle, Θ . The technique requires the computation of the sine and cosine of the estimated angle, usually from a look-up table addressed with an up/down counter. At steady state the estimated angle matches the real angle Θ .

Other techniques based upon the tangent/cotangent of the shaft angle have been used [8]–[14]. In these schemes, the absolute values of the demodulated sine and cosine signals are determined; by appropriate processing, the smaller of the two values is divided by the greater, providing either the tangent or cotangent of the unknown angle (Fig. 1). Due to the periodic nature of the tangent/cotangent, the full scale of 360° is represented by four identical cycles, each made up of a positive and a negative slope sections. For this reason, three octants are computed in order to identify the interval to which Θ belongs. The value of Θ is then either computed numerically or determined from a look-up-table.

In another mode of operation, while the rotor winding is used as a secondary, both stator windings are used as primaries excited simultaneously with two quadrature waveforms. Thus, the stator and rotor windings exchange roles when compared to the previous two modes of operation. A straightforward analysis shows that the voltage produced by the rotor presents a phase shift dependent upon the rotor shaft angle Θ [15]–[17].

In the present communication, a novel resolver-to-dc converter is presented. The proposed scheme employs the absolute values of the sine and cosine signals just like the arctangent method. However instead of computing the arctangent from the ratio of the absolute values, the proposed scheme uses simply their difference. This difference, $V_0(\Theta)$, is made up of a succession of nearly linear sections. A dedicated linearization technique is developed and applied to $V_0(\Theta)$ in order to produce a linear output. Thus, contrary to the arctangent technique, no

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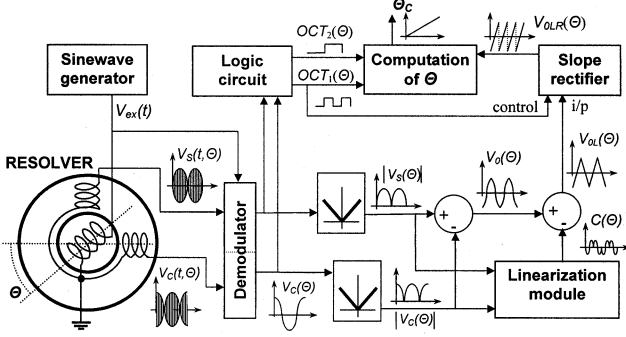
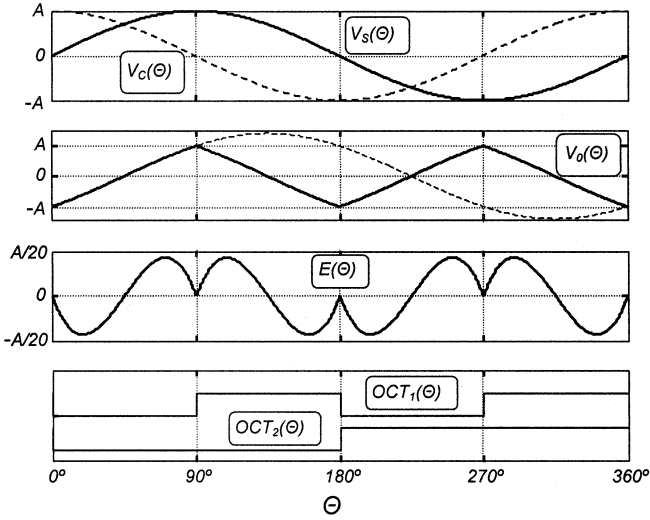


Fig. 2. Schematic diagram of the proposed resolver-to-dc converter.

Fig. 3. Simulation results obtained for $V_0(\theta)$ which is derived from the sine and cosine resolver signals $[V_s(\theta), V_c(\theta)]$. $E(\theta)$ represents the deviation of $V_0(\theta)$ from a perfect triangular wave. Two octants (lower) are required for unambiguous determination of the shaft angle.

processor or look-up table technique is required for the computation of θ .

II. PRINCIPLE OF OPERATION

The simplified diagram of the proposed scheme is shown in Fig. 2. First, the stator signals are demodulated. Then, the difference between the absolute values is determined

$$V_0(\theta) = |V_s(\theta)| - |V_c(\theta)| = A(|\sin \theta| - |\cos \theta|). \quad (4)$$

This voltage $V_0(\theta)$ is made up of the linear sections of four shifted sinewaves [e.g., within the interval $[0, \pi/2]$, $V_0(\theta)$ is equal to $\sqrt{2}A \sin(\theta - (\pi/4))$ which is almost linear from 0 to 90° as shown in Fig. 3). The voltage $V_0(\theta)$ is combined with a linearization signal, $C(\theta)$. This is followed by a slope rectification stage producing a final output voltage, $V_{0LR}(\theta)$, made up of four identical and positive-slope sections (i.e., one section for each $1/4$ revolution).

The simulation results depicted in Fig. 3 show that the obtained $V_0(\theta)$ waveform resemble a perfect triangular wave, $PT(\theta)$. The mathematical analysis and computer simulation given below show that the deviation of $V_0(\theta)$ from $PT(\theta)$ is within $\pm 4.2\%$ of maximum amplitude, A . A linear computation of the shaft angle from $V_0(\theta)$ using a similar equation to (12)

results in an error not exceeding $\pm 1.9^\circ$ over the full range. Evidently, this error may be minimized by using a standard look-up table technique. However, the possibility of linearization of $V_0(\theta)$ is shown to be feasible, thus facilitating the computation of θ without a processor and/or a look-up table.

III. LINEARIZATION

By writing the Fourier expansions of $V_0(\theta)$ and that of a perfect triangular signal of equal amplitude, $PT(\theta)$, it is easy to show that the two signals have the same harmonics and comparable component coefficients

$$\begin{aligned} V_0(\theta) &= \frac{-8A}{\pi} \sum \frac{\cos(n\theta)}{n^2 - 1} \quad n = 2, 6, 10, 14, \dots \\ &= \frac{-8A}{\pi} \left[\frac{\cos(2\theta)}{3} + \frac{\cos(6\theta)}{35} + \frac{\cos(10\theta)}{99} \right. \\ &\quad \left. + \frac{\cos(14\theta)}{195} + \frac{\cos(18\theta)}{323} + \dots \right] \end{aligned} \quad (5)$$

$$\begin{aligned} PT(\theta) &= \frac{-32A}{\pi^2} \sum \frac{\cos(n\theta)}{n^2} \quad n = 2, 6, 10, 14, \dots \\ &= \frac{-8A}{\pi} \left[\frac{\cos(2\theta)}{\pi} + \frac{\cos(6\theta)}{9\pi} + \frac{\cos(10\theta)}{25\pi} \right. \\ &\quad \left. + \frac{\cos(14\theta)}{49\pi} + \frac{\cos(18\theta)}{81\pi} + \dots \right]. \end{aligned} \quad (6)$$

The deviation of $V_0(\theta)$ from $PT(\theta)$ is given by the difference between (5) and (6)

$$\begin{aligned} E(\theta) &= V_0(\theta) - PT(\theta) \\ &= \frac{8A}{\pi} \sum \left(\frac{4}{\pi n^2} - \frac{1}{n^2 - 1} \right) \cos(n\theta) \quad n = 2, 6, 10, 14, \dots \\ &\approx \frac{-10^{-3}A}{\pi} [120.19 \cos(2\theta) - 54.37 \cos(6\theta) \\ &\quad - 21.05 \cos(10\theta) - 10.94 \cos(14\theta) + \dots]. \end{aligned} \quad (7)$$

This predictable error (Fig. 3) is made up of harmonics present in the processed resolver's sine and cosine signals. This suggests the possibility of finding a combination of these signals giving a correction signal ideally having the same equation as (7), but a close approximation could be acceptable. The correction signal, $C(\theta)$, is combined with $V_0(\theta)$ in order to produce a linearized output, $V_{0L}(\theta) = V_0(\theta) - C(\theta) \approx PT(\theta)$, as illustrated in Fig. 2.

By examining carefully the frequency and phase reversal of the error signal, a compensation signal of the following form represents a close fit of the error $E(\theta)$

$$C(\theta) = B \times |\sin \theta| \times |\cos \theta| \times V_0(\theta) \quad (8)$$

where B is a scaling factor estimated by matching the amplitude of $C(\theta)$ in (8) to that of $E(\theta)$ in (7). It can easily be shown that the negative and positive peaks of $E(\theta)$ occur, respectively, for the angle values $(m\pi \pm 0.335)$ and $([m + (1/2)]\pi \pm 0.335)$, where m is an integer. It follows that the amplitude of $E(\theta)$ is $0.042 \times A$. By calculating the corresponding amplitude of $C(\theta)$, the scaling factor may be estimated as $B = 0.221 \times A$. The linearization signal was tested using Matlab/Simulink package, which indicated that the best compensation is obtained

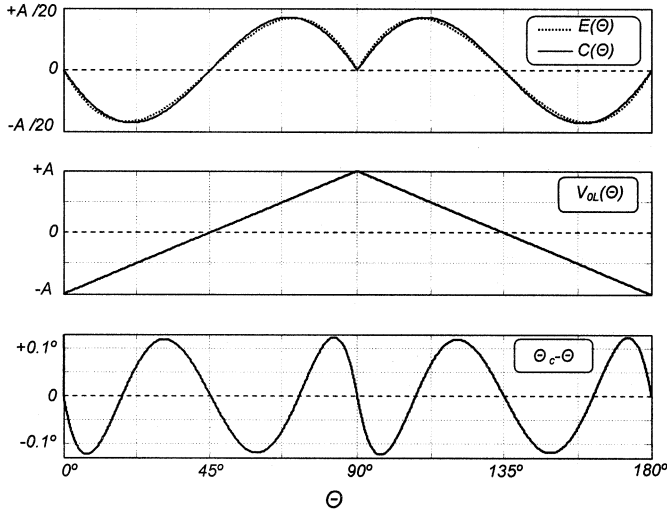


Fig. 4. Simulation results obtained by introducing the compensation signal. Upper: comparison between error and compensation signals. Middle: linearized output of the converter. Lower: residual error of the converter representing the difference between computed (using 12) and true angles.

with a value of $0.223 \times A$ for B . This value for B results in a minimum difference between $C(\Theta)$ and $E(\Theta)$ over the full range of Θ values (see Fig. 4). The minor difference between the estimated and computer-generated values for B is accounted for by the slight mismatch between the peak positions of $C(\Theta)$ and $E(\Theta)$ as illustrated in Fig. 4. In fact, it can easily be shown analytically that the negative and positive peaks of $C(\Theta)$ occur, respectively, at the angle values of $(m\pi \pm 0.365)$ and $((m + 1/2)\pi \pm 0.365)$. This means that the peak positions of the compensation signal are shifted by about 1.7° when compared to those of $E(\Theta)$ determined previously. Finally, (8) is rewritten as

$$\begin{aligned} C(\Theta) &= 0.223A \times |\sin \Theta| \times |\cos \Theta| \times V_0(\Theta) \\ &= 0.223A[-|\sin \Theta| + |\sin \Theta|^2(|\sin \Theta| + |\cos \Theta|)]. \end{aligned} \quad (8a)$$

A lengthy but straightforward mathematical analysis provided the Fourier expansion of $C(\Theta)$

$$\begin{aligned} C(\Theta) &= \frac{0.446A}{\pi} \sum_{n=2,6,10,14,\dots} \left(\frac{1}{n^2-1} + \frac{3}{n^2-3^2} \right) \cos(n\Theta) \\ &\approx \frac{-10^{-3}A}{\pi} [118.93 \cos(2\Theta) - 62.30 \cos(6\Theta) \\ &\quad - 19.21 \cos(10\Theta) - 9.44 \cos(14\Theta) + \dots]. \end{aligned} \quad (9)$$

The above expansion is comparable to that of the error signal (7). Fig. 4 shows the simulation results obtained with the introduction of this compensation signal. The difference between the angle (Θ_C) computed from the converter and actual angle (Θ) is reduced to $\pm 0.12^\circ$ (or ± 7 arc-min) without substantial complication in the practical implementation even using analog circuitry: the implementation of (8a) in its right-hand form requires only a pair of one-quadrant multipliers. The residual error is due to the slight mismatch between the peak positions of $C(\Theta)$ and $E(\Theta)$. Note that even with the residual error the theoretical precision of the converter is better than that obtained with

11 bits digital encoders. Importantly, the accuracy obtained with the compensation method is comparable to the 1 to 30 arc-min accuracy of commercial tracking resolver to digital converters (e.g., AD2S90 from Analog Devices, and 168F200 series from Control Sciences Inc.). In fact, a low-cost resolver itself has a typical accuracy of 10 arc-min [18].

In order to avoid ambiguous determination of the angle from $V_{OL}(\Theta)$, two octants are derived to identify the interval to which Θ belongs, as the 360° scale is divided into four sections. These octants are derived from the signs of the demodulated resolver stator signals using the following boolean equations

$$\text{OCT}_1(\Theta) = \overline{\text{sign}[V_c(\Theta)]} \oplus \overline{\text{sign}[V_s(\Theta)]} \quad (10)$$

$$\text{OCT}_2(\Theta) = \overline{\text{sign}[V_s(\Theta)]}. \quad (11)$$

Similar techniques have been used in [8]–[14]. An additional benefit of deriving (10) is that $V_{OL}(\Theta)$ may be slope-rectified by using a synchronous rectifier controlled by $\text{OCT}_1(\Theta)$. In this way, the computation of the shaft angle from the output of the slope rectifier, $V_{OLR}(\Theta)$, becomes much easier, as the full scale is represented by an output made up of four identical and positive-slope sections

$$\Theta_c = 90 \times \left[\frac{V_{OLR}(\Theta) + A}{2A} + \text{OCT}_1(\Theta) + 2 \times \text{OCT}_2(\Theta) \right]. \quad (12)$$

A linear output is evidently an advantage in instrumentation and in control applications. As a matter of fact, the proposed resolver converter scheme may be used as an inverse trigonometric function calculator in other applications unrelated to resolvers.

IV. EXPERIMENTAL

Fig. 5 shows the simplified electronic circuit diagram of the present converter; the buffering/amplification and demodulating stages are not shown. The amplification at the buffer stage was chosen such that the saw-toothed $V_{OLR}(\theta)$ voltage output varied between -9 and $+9$ V. In this way, the sensitivity of the converter was simply 200 mV/degree. Precision absolute value circuits, built around OA1–OA4, were used. The multiplier and squarer blocks were built around low-cost RC4200 analogue multipliers. The 5-k multiturn trimmer potentiometer at the input of the summing amplifier OA8 is used to set the value of the multiplication factor, B , to $0.223 \times A$ [see (8) and (8a)]. The slope rectifier is built around OA9 and a JFET transistor controlled by $\text{OCT}_1(\Theta)$. Note the circuit does not include reactive components ensuring instantaneous response of the converter.

The setup used for testing the converter included a resolver (Global Drive MDSKA 071-22, 140) driven by a variable speed dc motor. For speeds below 50 rpm, a 12-V dc geared motor (100:1 reduction ratio) incorporating a 1000 pulses-per-revolution encoder was used. The excitation voltage of the resolver (1) was 18-V peak-to-peak with a frequency of 4 kHz. The transformation ratio between stator and rotor windings of the resolver was $\alpha = (1/2)$. As a result, the amplitude of the demodulated stator signals was 4.5 V; thus, the amplification factor required at the buffer stage was approximately 2 in order to achieve 200 mV/degree resolution at the output of the converter.

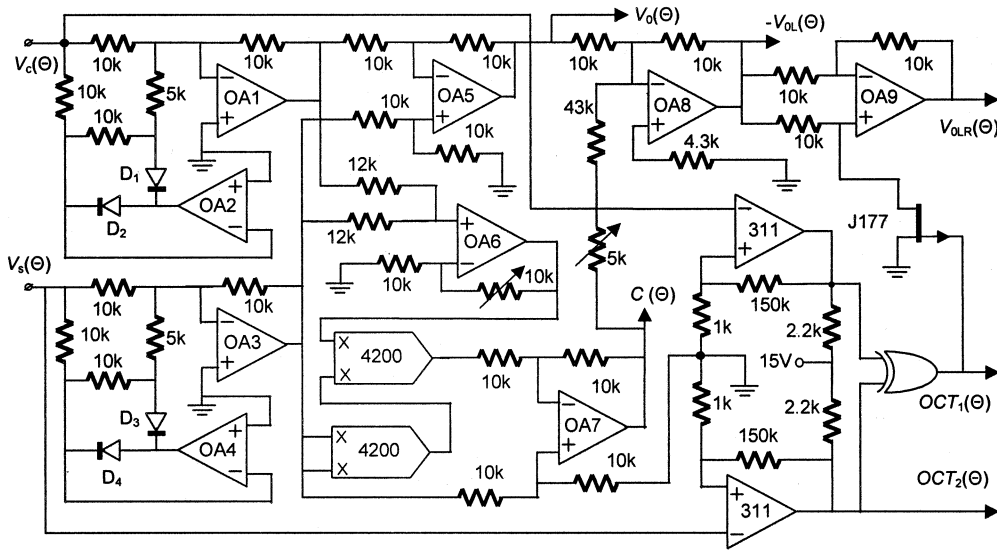


Fig. 5. Circuit diagram of the proposed resolver-to-dc converter. The buffering and demodulation sections are not shown. The circuit was powered from ± 15 V.

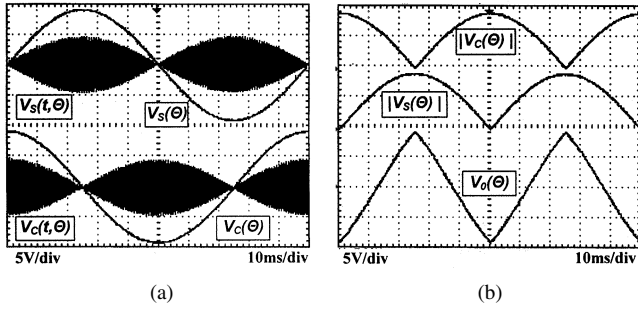


Fig. 6. Experimental results obtained at 600 rpm within a full 360° revolution. (a) Modulated and demodulated/amplified stator signals. (b) Derivation of $V_0(\theta)$.

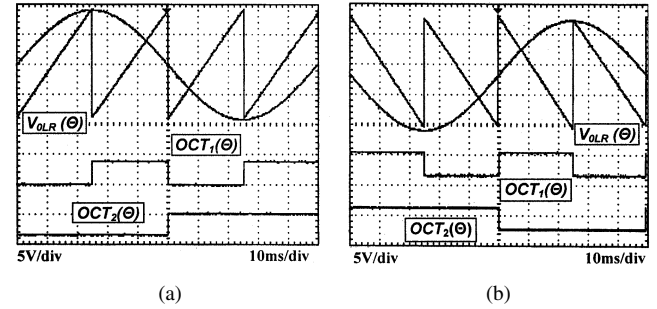


Fig. 8. Experimental results obtained by the slope-rectification module at the output of the converter. The complete shaft revolution is represented by four sections distinguished by the two octants. (a) Forward 600 rpm speed. (b) Reverse 600 rpm speed.

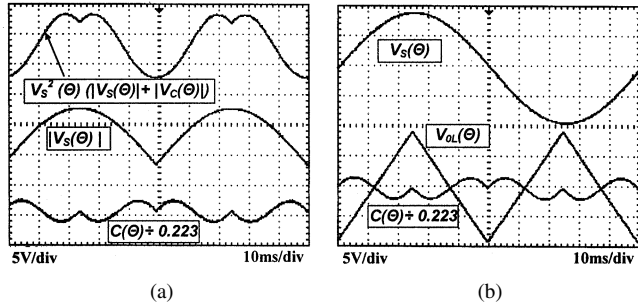


Fig. 7. Experimental results for the linearization of $V_0(\theta)$. (a) Derivation of the compensation signal according to (8a). (b) Effect of the introduction of $C(\theta)$; note the linearity of $V_{0LR}(\theta)$.

The tests were conducted at constant motor excitation voltages. However, the setup did not include means of stabilizing the rotational speed, for example using a flywheel. Unless otherwise specified, the rotational speed was 600 rpm. Fig. 6 shows the various signals of the converter section producing the nonlinearized $V_0(\theta)$ in a full 360° revolution. Fig. 7 shows the derivation of the linearization signal $C(\theta)$ and the effect of its introduction on the output of the converter.

Fig. 8 shows the final output, $V_{0LR}(\theta)$, of the converter together with the octants needed to determine unambiguously the shaft angle. The results are given for forward and reverse

speed of 600 rpm demonstrating the measurement of the absolute angle.

Fig. 9 illustrates the results obtained at various rotational speeds. The converter performed well from standstill to reasonably elevated speeds. Evidently, because of the sampling (i.e., at 4 kHz) in the demodulation section, the lower the speed of rotation the more accurate is the determination of the shaft angle. For example, the operation at 1200 rpm results in a sample taken every step angle of 1.8° , whereas the operation at 1 rpm the step is only 0.0015° . In practice, the operation of the resolver rarely involves high speeds as the resolver is usually used in positioning applications and is normally coupled to the driving motor via a reduction gear.

Fig. 10 compares the results obtained from the proposed converter and those obtained from the 10^5 pulses/rev encoder/gear combination. The value of the shaft angle obtained from the converter was calculated using (12). The plot reveals a good linearity of the converter; the pattern in the range 180° to 360° is similar to that shown in Fig. 10. The difference between the values of the shaft angle measured by the proposed converter/resolver system and those determined from the encoder is small (less than 0.20°) and is comparable to the computer-predicted error of the converter only (0.12°). The difference may be accounted for by 1) the nonideal behavior and errors of the resolver

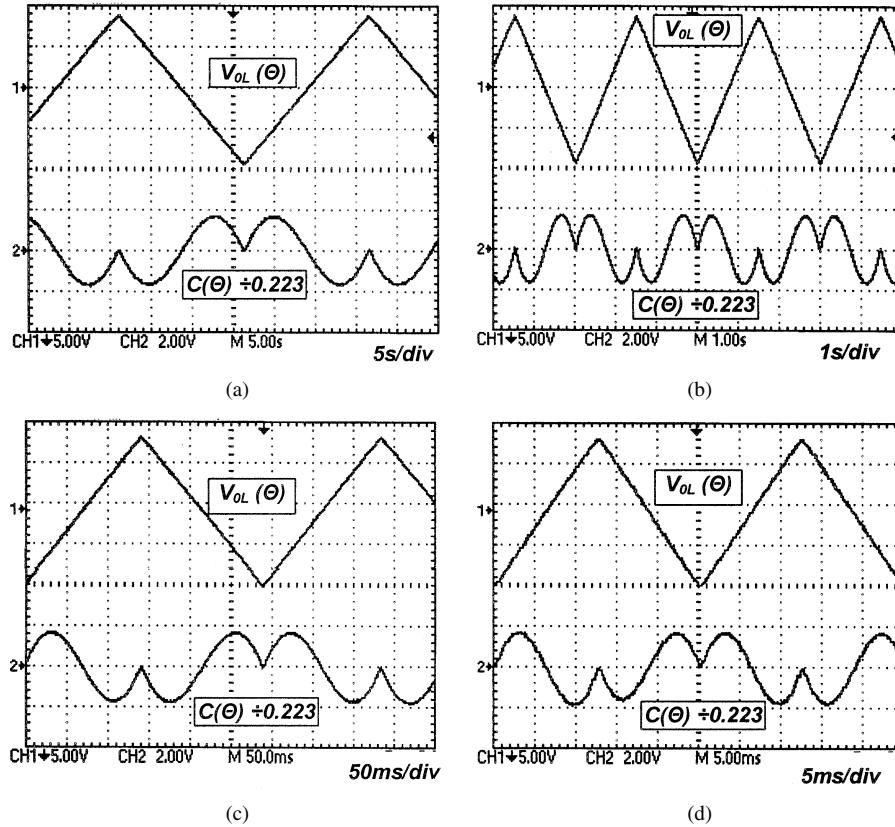


Fig. 9. Experimental results obtained at different speeds: (a) 1 rpm; (b) 10 rpm; (c) 100 rpm; (d) 1200 rpm.

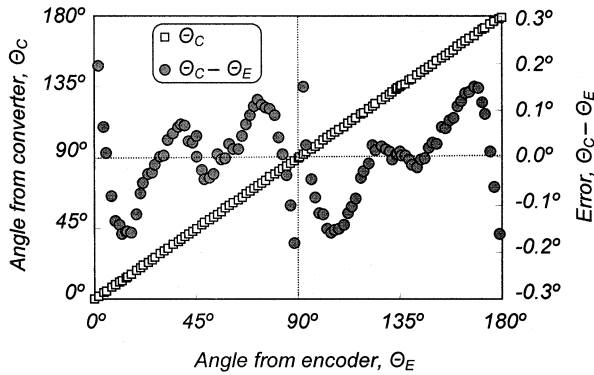


Fig. 10. Experimental plot of the angle determined from the proposed converter (according to 12) versus that estimated from a 10^5 pulses/rev encoder/gear combination in the range 0 to 180° . The error represents the difference between the angle values determined from the proposed converter and the encoder.

itself, 2) the possible nonlinearity/hysteresis of the gear used, and 3) the nonlinearity and offset in the low-cost analogue multipliers used. It is important to note that the residual error of 0.2° (or 12 arc-min), which is the overall error of the proposed converter and resolver system, is still comparable to that of many commercial converters.

V. CONCLUSION

A new scheme of a resolver-to-dc converter for the measurement of mechanical angle is described. The theoretical analysis and computer simulation reveal that by using a dedicated lin-

earization technique, the converter produces an output voltage proportional to the shaft angle, with an error less than 0.12° over the full 360° range. This is a reasonable precision for many resolver applications. The converter has been successfully implemented using analogue circuitry. The experimental results obtained under various operating conditions are in good agreement with theory and simulation. In particular, the introduction of the compensation technique results in good output linearity. The performance of the converter was compared to that of a high-resolution encoder; the results showed satisfactory precision.

REFERENCES

- [1] G. Boyes, Ed., *Synchro and Resolver Conversion*: Memory Devices, Ltd., a division of Analog Devices Inc., Norwood, MA, 1980.
- [2] J. G. Webster, Ed., *The Measurement, Instrumentation, and Sensors Handbook*. Boca Raton, FL: CRC, 1999, pp. 6128–6141.
- [3] G. A. Woolvet, "Digital transducers," *Journal of Physics E: Sci. Instrum.*, vol. 15, no. 12, pp. 1271–1280, Dec. 1982.
- [4] M. L. Gasperi and W. G. Onarheim, "Method and Apparatus for Correcting Resolver Errors," U.S. Patent 4 933 674, 1990.
- [5] C. H. Yim, I. J. Ha, and M. S. Ko, "A resolver-to-digital conversion method for fast tracking," *IEEE Trans. Ind. Electron.*, vol. 39, pp. 369–378, Oct. 1992.
- [6] D. C. Hanselman, "Techniques for improving resolver-to-digital conversion accuracy," *IEEE Trans. Ind. Electron.*, vol. 38, pp. 501–504, Dec. 1991.
- [7] —, "Resolver signal requirements for high accuracy resolver-to-digital conversion," *IEEE Trans. Ind. Electron.*, vol. 37, pp. 556–561, Dec. 1990.
- [8] P. G. Serev and R. M. Bogin, "Programmable Limit Switch System Using a Resolver-to-Digital Angle Converter," U.S. Patent 4 511 884, 1985.
- [9] E. Fletcher, "Procédé de numérisation de la valeur d'un angle défini par ses coordonnées trigonométriques sinus et cosinus," Swiss Patent CH 675 486, in French, 1990.

- [10] P. G. Serev, "Microcontroller Based Resolver-to-Digital Converter," U.S. Patent 4 989 001, 1991.
- [11] S. P. Vlahu, "Variable Reluctance Resolver to Digital Converter," U.S. Patent 5 949 359, 1999.
- [12] J. G. Deppe and J. R. Biel, "AC Encoded Signal to Digital Converter," U.S. Patent 5 034 743, 1991.
- [13] S. P. Vlahu, "Direct Resolver to Digital Converter," U.S. Patent 5 912 638, 1999.
- [14] C. Attaianesi, G. Tomasso, and D. DeBonis, "A low-cost resolver-to-digital converter," in *Proc. IEEE Electric Machines and Drives Conf.*, Cambridge, MA, USA, June 17–20, 2000, pp. 917–921.
- [15] T. Ono, "Resolver-Type Rotational Positioning Arrangement," U.S. Patent 4 529 922, 1985.
- [16] D. C. Alhorn, D. E. Howard, and D. A. Smith, "Resolver to 360° Linear Analog Converter and Method," U.S. Patent 6 104 328, 2000.
- [17] J. J. Duckworth, "Resolver to Incremental Shaft Encoder Converter," U.S. Patent 4 486 845, 1984.
- [18] S. K. Kaul, R. Koul, C. L. Bhat, I. K. Kaul, and A. K. Tickoo, "Use of a 'look-up' table improves the accuracy of a low-cost resolver-based absolute shaft encoder," *Meas. Sci. Technol.*, vol. 8, pp. 329–331, 1997.



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