A Novel Method for Online Correction of Amplitude and Phase Imbalances in Sinusoidal Encoders Signals

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***Abstract*—Resolvers and other sinusoidal transducers are angular position and speed sensors that produce supposedly perfect sine and cosine signals in which the angular position is encoded. Suitable converters are normally used to determine the encoded position from the transducer signals. Frequently, the signals present defects, particularly displaying phase and amplitude imbalances. Such defects have detrimental effects on the angle measurement precision. An online method for correcting the transducers signals is presented in this work. This allows elimination of unbalance errors and enable the converters to work properly. The proposed method is based on using three different data points to carry out the correction. Each data point consists of a pair of values of the transducer signals without the need for knowledge of the actual angle itself. A simple algorithm, that may be implemented on a simple microcontroller or processor, performs automatic correction based on the proposed technique. The paper presents details of the theory, algorithm, and simulation results which demonstrate the effectiveness of the proposed method.**

Keywords—Resolvers; Sinusoidal Encoders; Rotational Motion Control; Amplitude-Phase Imbalances; Compensation

I. INTRODUCTION

Rotational position and speed are among the most basic parameters required in industrial motion control applications. Whether in robotics, aerospace, biomedical, military, automation or many other applications, precision of angular position sensors plays a great role in the performance of the system as a whole. Sinusoidal encoders are very popular absolute position and speed transducers. Operating on different principles, including magnetoelectric, inductive, Hall effect or optical techniques, these transducers produce electrical analog quadrature signals that encode the unknown angle allowing an absolute estimation of the angular position [1]-[5]. Resolvers (Fig. 1), for example, are rotating transducers that are used widely due to their reliability and ability to give precise angular measurements in harsh environments. In its basic form, a resolver is similar to an electric motor with the primary winding in the rotor and two secondary mechanically 90^o apart stator windings. Modern resolvers are brushless, a feature that significantly increases their reliability and life-time. A sinusoidal excitation signal is applied to the primary winding. The excitation voltage is also referred to as the reference or the carrier $V_{\mathrm{ref}}(t) = V_m \sin 2\pi f_c t$, where V_m is the peak amplitude, typically a few Volts, and f_c is the frequency of the signal, typically a few kHz and must be considerably higher than the maximum rotational speed of the sensor. This excitation causes the two secondary windings to generate two output voltages that are modulated by the excitation signal. Because of the mechanical phase shift between the windings, one output depends on the sine while the other on the cosine of the rotor angle (i.e. $gV_{\mathrm{ref}} \sin\theta$ and $gV_{\mathrm{ref}} \cos\theta$) where g is a constant that represents the transformation ratio between the primary and secondary windings. The two signals are then demodulated resulting ideally in two sine and cosine signals (SCS) of the form:

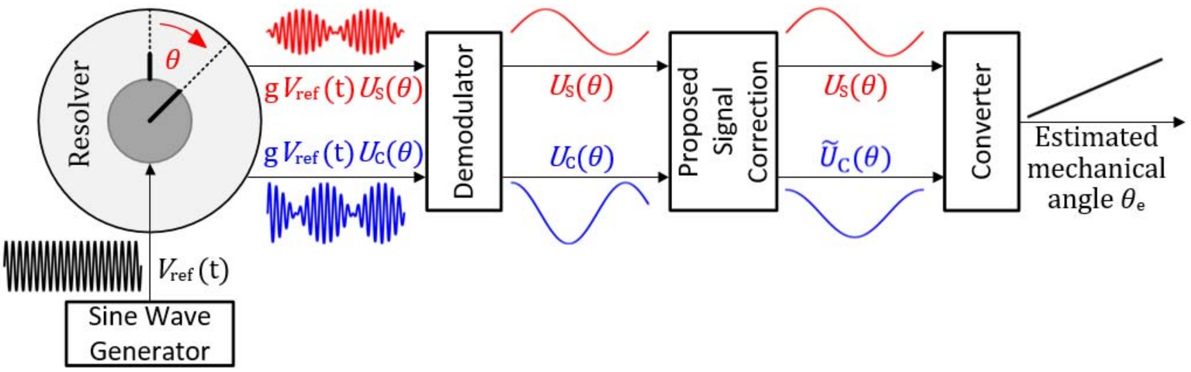


Fig. 1. A Simplified model of a resolver connected to a demodulator, signal correction and converter units.

$$
\begin{aligned}
V_S(\theta) &= A\sin\theta \\
V_C(\theta) &= A\cos\theta
\end{aligned} \tag{1}
$$

where A is the amplitude of the two SCS and depends on the excitation signal amplitude V_m as well as the transformation ratio g and, sometimes, on the demodulation technique. Actually, the signals shown in (1) represent the output of all kinds of sinusoidal encoders not only the resolver. Though, other sinusoidal encoders may not require any demodulation and directly output these signals.

The two SCS then require a suitable conversion method to estimate the unknown angle θ. Many converters have been reported in literature. These include closed-loop converters such as Phase Locked Loop (PLL) converters, which have limited dynamic performance but are more robust [6]-[8]. Open-loop converters, on the other hand, are faster but not as robust as closed-loop ones [9]-[11]. The common thing between all these conversion techniques is that they assume ideal outputs that are shown in (1). In reality, however, the sensors' output signals are not perfectly described by (1), and contain errors from various sources. The main imperfections reported in the literature are: amplitude imbalance, phase imbalance, offsets and harmonics [12]. However, the most significant causes of errors are amplitude and phase imbalances [12]. Figure 2 shows an example of transducers signals that present exaggerated phase and amplitude imbalances; correction of these signals results in balanced versions as shown in the lower part of Fig. 2. To show the significance of amplitude and phase imbalances, either 0.63% amplitude imbalance or 0.18^o phase imbalance is enough to cause 1/2 LSB position error for a 10-bit PLL converter [12]. The other errors are small and have minor impact on the performance of sensors converters [13]. Amplitude imbalance is a result of unequal inductances of the resolver secondary windings [14]. On the other hand, phase imbalance between the two SCS results from errors in the mechanical positioning of these winding (i.e. imperfect quadrature) [12]-[14]. Taking the transducer's sine signal as a reference, these errors can be reflected in the imbalanced signals as follows:

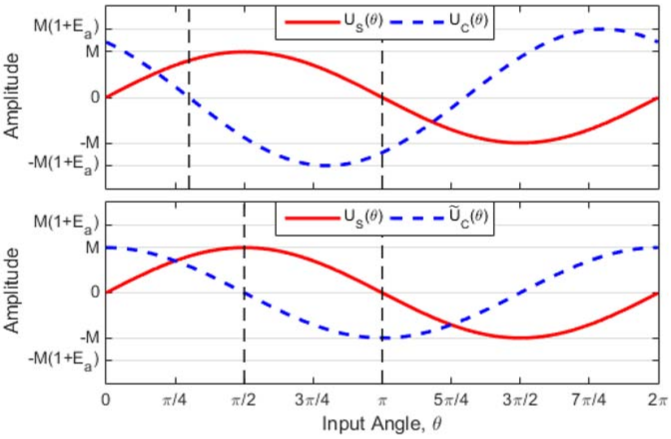


Fig. 2. Example of transducer signals before and after balancing. Upper: amplitude and phase imbalances of $U_C(\theta)$ compared to $U_S(\theta)$. Lower: the corrected cosine $\tilde{U}_C(\theta)$ signal compared to $U_S(\theta)$.

$$
\begin{aligned}
U_S(\theta) &= A\sin\theta \\
U_C(\theta) &= (1+E_a)A\cos(\theta+E_p)
\end{aligned} \tag{2}
$$

where E_a and E_p are the amplitude and phase errors, respectively.

Several methods have been suggested to compensate for these imbalances. In [12], the correction could not be done for the two errors together. Instead, the amplitude imbalance had to be solved manually first, which required examining the outputs of the sensor for the full 360^o interval. After that, the phase imbalance was resolved by replacing $U_S(\theta)$ and $U_C(\theta)$ by their sum and difference respectively, which would eliminate the phase error, but will also impose another amplitude imbalance that has to be corrected again which makes the correction lengthy and inefficient. It is also not easy to be done frequently in order to account for the long term drifts in errors due to aging. Furthermore, the method did not offer amplitude and phase errors estimation. Another method was proposed in [15] that depends on the Lissajous curve of the encoder outputs (where $U_S(\theta)$ is the Y-axis and $U_C(\theta)$ is the X-axis). The areas and radii of the four quadrants of the curve were evaluated to get a fair estimation of the amplitude imbalance and offset error and correct them ignoring, however, the more significant phase error. Also, an interesting algorithm was presented in [16] in which an iterative linear search method known as "steepest descent" is utilized to correct for different non-ideal characteristics of encoders using samples of the $U_S(\theta)$ and $U_C(\theta)$ signals. The main disadvantage, however, is the relatively long time (up to 0.5 seconds) required for the algorithm to converge because of the needed small learning rate, which does not allow online correction at the same time of conversion. Closed-loop schemes were also proposed as in [13] where amplitude and phase errors were corrected using an adaptive PLL with synchronous demodulators, which requires more computational power and increases the time of processing. Moreover, the correction is done continuously with the conversion which adds an unnecessary delay since the errors are not expected to vary continuously. Another closed-loop method was presented in [17] and required evaluation of the square magnitude of the encoder outputs and the mean values over a sufficient number of periods. Because of the feedback required in these methods, they are not valid for certain scenarios such as fast point-to-point positioning where the data are not enough for the loop to correct the errors. Other closed-loop correction techniques were presented in [14], [18] for the specific application of PMSM drives utilizing some parameters from the drive itself. In addition to that, there are methods that utilize multiple encoders to perform correction like in [19] where two resolvers' outputs are compared to estimate the errors and correct them. A method that is, of course, not feasible in many situations.

In the present work, a novel analytical algorithm, that is able to precisely detect and correct both phase and amplitude imbalances, is presented. The algorithm does not require any additional hardware and uses a minimum computational power and processing time allowing it to be implemented on low-cost

16bit microcontrollers and be easily used online during conversion. Furthermore, the proposed correction technique is not tied to any conversion method and can be used without restrictions with both open-loop and closed-loop converters.

II. CORRECTING AND DETECTING AMPLITUDE AND PHASE ERRORS

The idea of the algorithm is to use $U_S(\theta)$ and $U_C(\theta)$ to generate a new cosine signal replacing $U_C(\theta)$ which has phase and amplitude errors. The algorithm requires an accurate knowledge of A, the maximum amplitude of the $U_S(\theta)$ signal, as well as three valid test points at three different angles. A test point is described by a pair of values of the transducer outputs $U_S(\theta)$ and $U_C(\theta)$ at a specific angle θ. The angles at which these test points have been sampled, however, are not needed. The requirements are very simple and easy to satisfy. An accurate knowledge of the maximum amplitude A is a common requirement in most of the reviewed conversion and correction methods. A can be determined by different ways. One of them is to measure the peak-to-peak value of the $U_S(\theta)$ signal and divide it by two. The $U_S(\theta)$ signal is taken as a reference in terms of amplitude and phase. Even if the phase error was in $U_S(\theta)$ not in $U_C(\theta)$, $U_C(\theta)$ will be corrected to be 90 degrees apart from $U_S(\theta)$. In this particular case the mechanical reference angle (the angular position at which the encoder should read a zero degree) will be shifted by the phase error present in $U_S(\theta)$. The following straightforward equation can be easily proven [20]:

$$\gamma\cos(\theta+\alpha)+\beta\sin(\theta)=\begin{aligned}&\sqrt{\gamma^2+\beta^2-2\gamma\beta\sin(\alpha)}\\&\times\cos\left\{\theta+\tan^{-1}\left[\frac{\gamma\sin(\alpha)-\beta}{\gamma\cos(\alpha)}\right]\right\}\end{aligned} \quad (3)$$

where γ and β are scaling factors and α is a phase shift. Using this formula, it is possible to add a scaled version of $U_S(\theta)$ to $U_C(\theta)$ to correct the phase imbalance and obtain a scaled cosine signal as follows:

$$\begin{aligned}&U_C(\theta)+BU_S(\theta)\\&=A\left[(1+E_a)\cos(\theta+E_p)+B\sin(\theta)\right]=AC\cos(\theta)\end{aligned} \quad (4)$$

By comparing (4) to (3), it is clear that B must be chosen such that:

$$\tan^{-1}\left[\frac{(1+E_a)\sin(E_p)-B}{(1+E_a)\cos(E_p)}\right]=0 \quad \therefore B=(1+E_a)\sin(E_p) \quad (5)$$

Then according to (3), (4) and (5), C can be expressed by:

$$\begin{aligned}C=\sqrt{(1+E_a)^2-B^2}&=|1+E_a|\cdot\left|\cos(E_p)\right|\\&=(1+E_a)\cos(E_p)\end{aligned} \quad (6)$$

The absolute values of terms in (6) are equal to the terms themselves because, in practice, E_a is much smaller than 1 and E_p is a small angle around 0. A corrected cosine signal $\widetilde{U}_C(\theta)$ may then be determined as follows (Fig. 2):

$$\widetilde{U}_C(\theta)=A\cos(\theta)=\frac{U_C(\theta)+BU_S(\theta)}{C} \quad (7)$$

The corrected signal requires online determination of suitable values for B and C as shown below. Using B and C, amplitude and phase errors can be quantified from (5) and (6):

$$E_a=\sqrt{B^2+C^2}-1 \quad (8)$$

$$E_p=\tan^{-1}\frac{B}{C} \quad (9)$$

III. THE PROPOSED ALGORITHM

A pseudo-code of the proposed algorithm can be seen in Fig. 3. As mentioned before, three valid test points are needed to find the two unknown correction parameters B and C. Examining and ensuring the validity of the test points will be discussed in the next section. The values of the test points are arranged in two separate vectors: the vector of the $U_S(\theta)$ values $\mathbf{P_S}$ and the vector of the $U_C(\theta)$ values $\mathbf{P_C}$, where $P_S[i]$ and $P_C[i]$ represent the values of $U_S(\theta)$ and $U_C(\theta)$ of the ith test point respectively. The corresponding samples of the absolute value of the corrected cosine signal $\widetilde{U}_C(\theta)=A\cos(\theta)$ are put in a vector named $|\mathbf{P_{AC}}|$, which can be calculated from:

$$|\mathbf{P_{AC}}|=\sqrt{A^2-\mathbf{P_S^2}} \quad (10)$$

```
Require: $\mathbf{P}_S$, $\mathbf{P}_C$, $A$
Ensure: Validity of test points
$|\mathbf{P}_{AC}| \leftarrow \sqrt{A^2-\mathbf{P}_S^2}$
$i \leftarrow [1,2,3]$
$j \leftarrow [2,3,1]$
$k \leftarrow [3,1,2]$
$B \leftarrow 0$
$C \leftarrow 0$
for $n = 1 \rightarrow 3$ do
    $B_1 \leftarrow \frac{|P_{AC}[i[n]]|\cdot P_C[j[n]]-|P_{AC}[j[n]]|\cdot P_C[i[n]]}{|P_{AC}[j[n]]|\cdot P_S[i[n]]-|P_{AC}[i[n]]|\cdot P_S[j[n]]}$
    $B_2 \leftarrow -\frac{|P_{AC}[i[n]]|\cdot P_C[j[n]]+|P_{AC}[j[n]]|\cdot P_C[i[n]]}{|P_{AC}[j[n]]|\cdot P_S[i[n]]+|P_{AC}[i[n]]|\cdot P_S[j[n]]}$
    $C_1 \leftarrow \min(\frac{|P_C[i[n]]+B_1\cdot P_S[i[n]]|}{|P_{AC}[i[n]]|}, \frac{|P_C[j[n]]+B_1\cdot P_S[j[n]]|}{|P_{AC}[j[n]]|})$
    $C_2 \leftarrow \min(\frac{|P_C[i[n]]+B_2\cdot P_S[i[n]]|}{|P_{AC}[i[n]]|}, \frac{|P_C[j[n]]+B_2\cdot P_S[j[n]]|}{|P_{AC}[j[n]]|})$
    $E_1 \leftarrow \left|\left|\frac{P_C[k[n]]+B_1\cdot P_S[k[n]]}{C_1}\right| - |P_{AC}[k[n]]|\right|$
    $E_2 \leftarrow \left|\left|\frac{P_C[k[n]]+B_2\cdot P_S[k[n]]}{C_2}\right| - |P_{AC}[k[n]]|\right|$
    if $E_1 < E_2$ then
        $B \leftarrow B + B_1$
        $C \leftarrow C + C_1$
    else
        $B \leftarrow B + B_2$
        $C \leftarrow C + C_2$
$B \leftarrow B/3$
$C \leftarrow C/3$
$E_a \leftarrow \sqrt{B^2+C^2}-1$
$E_p \leftarrow \tan^{-1}\frac{B}{C}$
```

Fig. 3. A possible implementation of the suggested algorithm in pseudo-code.

It can be noticed from (5) that the phase error E_p can be completely corrected by only controlling the variable B. At the correct value of B, there will be a certain value of C such that:

$$P_C[i] + B \cdot P_S[i] = C \cdot P_{AC}[i] \tag{11}$$

is valid for any test point i. Taking two different test points, i and j:

$$C = \frac{P_C[i]+B \cdot P_S[i]}{P_{AC}[i]} = \frac{P_C[j]+B \cdot P_S[j]}{P_{AC}[j]} \tag{12}$$

which leads to:

$$B = \frac{P_{AC}[i] \cdot P_C[j] - P_{AC}[j] \cdot P_C[i]}{P_{AC}[j] \cdot P_S[i] - P_{AC}[i] \cdot P_S[j]} \tag{13}$$

However, $P_{AC}[i]$ and $P_{AC}[j]$ are not known. What is known is their absolute values from (10). This results in two solutions for (13), the first solution B_1 is true when both $P_{AC}[i]$ and $P_{AC}[j]$ have the same sign, and the second solution B_2 is true when $P_{AC}[i]$ and $P_{AC}[j]$ have different signs:

$$\begin{aligned} B_1 &= \frac{|P_{AC}[i]| \cdot P_C[j] - |P_{AC}[j]| \cdot P_C[i]}{|P_{AC}[j]| \cdot P_S[i] - |P_{AC}[i]| \cdot P_S[j]} \\ B_2 &= -\frac{|P_{AC}[i]| \cdot P_C[j] + |P_{AC}[j]| \cdot P_C[i]}{|P_{AC}[j]| \cdot P_S[i] + |P_{AC}[i]| \cdot P_S[j]} \end{aligned} \tag{14}$$

Each solution will give a corresponding C value, which can be calculated using:

$$\begin{aligned} C_1 &= \min\left(\frac{|P_C[i]+B_1 \cdot P_S[i]|}{|P_{AC}[i]|}, \frac{|P_C[j]+B_1 \cdot P_S[ij]|}{|P_{AC}[j]|}\right) \\ C_2 &= \min\left(\frac{|P_C[i]+B_2 \cdot P_S[i]|}{|P_{AC}[i]|}, \frac{|P_C[j]+B_2 \cdot P_S[j]|}{|P_{AC}[j]|}\right) \end{aligned} \tag{15}$$

In fact, the two terms in the minimum function used to calculate C_1 and C_2 should be equal except in the case when either $P_{AC}[i]$ or $P_{AC}[j]$ is zero, in which one of the terms will go to infinity. This will happen if one of the test points is taken on an angle of 90^o or 270^o. The minimum function is used to account for this case.

Now, unless either $P_{AC}[i]$ or $P_{AC}[j]$ is zero, there will be two different B_1 and B_2 values. One is right and the other is false. For this reason, a third point k is needed to test each solution and determine the correct one. If the B and C values are correct, then according to (11):

$$\left|\frac{P_C[k]+B \cdot P_S[k]}{C}\right| = |P_{AC}[k]| \tag{16}$$

So, the error of each solution can be calculated by:

$$\begin{aligned} E_1 &= \left|\left|\frac{P_C[k]+B_1 \cdot P_S[k]}{C_1}\right| - |P_{AC}[k]|\right| \\ E_2 &= \left|\left|\frac{P_C[k]+B_2 \cdot P_S[k]}{C_2}\right| - |P_{AC}[k]|\right| \end{aligned} \tag{17}$$

The solution that has the minimum error is the correct one. It can be seen that the accuracy of the solution directly depends on the accuracy of the test points. In the ideal case of perfect measurement of test points, the detection and correction will be perfect. In order to increase accuracy, the steps of the algorithm will be repeated three times for all possible combinations of two test points out of three to be used as the ith and jth test points in (14) and (15). The test point left in each time is used as the kth test point in (17). The B and C values of the three solutions will be then averaged to obtain the final one. To do that in software, the indices i, j and k are saved in three 3-elements vectors. The values of first elements of these vectors represent the values of the indices in the first iteration as shown in pseudo-code in Fig. 3.

Using the obtained B and C values, phase and amplitude errors can be estimated using (8) and (9) and can be corrected in run time using (7). Since amplitude and phase errors are not expected to change rapidly under the normal conditions, it is not recommended that the algorithm runs repeatedly during the conversion (i.e., while the converter associated with the transducer is determining the mechanical position θ from the transducer signals). Instead, it could be run every few hours or even days, depending on the anticipated rate of change of the unbalance.

IV. VALIDITY OF THE TEST POINTS

It can be seen from (14) and (15) that the three test points must be different. This is because if two are at the same angle, B_1 would be $0/0$, which is undefined, and B_2 would be equal to $-P_C[i]/P_S[i]$, which will result in $C_2 = 0$. Similarly, no two test points should be taken at angles 180^o apart. In the latter case, because $\sin(\theta + 180) = -\sin\theta$ and also $\cos(\theta + 180) = -\cos\theta$, B_1 would be equal to $-P_C[i]/P_S[i]$ leading to $C_1 = 0$, and B_2 would be undefined. To avoid this scenario, the algorithm performs a very simple check on the three test points. The check basically ensures that the absolute values of $P_S[i]$ and $P_C[i]$ are not equal to the corresponding absolute values of any other test point, which also does not require knowing the angles at which the test points where taken.

V. SIMULATION RESULTS

In order to test the algorithm and verify the theory behind it, a simulation was done using MATLAB. A flowchart of the simulation is shown in Fig. 4. The simulation imposes both amplitude and phase errors to the SCS and then the algorithm is run to estimate the errors and calculate the corrections parameters B and C. Assuming that the amplitude error E_a is less than $\pm 10\%$ of the maximum amplitude A and that the phase error E_p is between $\pm 10^o$, which is a very reasonable assumption, the simulation program iterates over the range of phase error with a small step size. In each iteration, another iteration over the range of amplitude error with a small step size also occurs. For every combination of amplitude and phase errors, three test points are chosen at three random angles. The positions of these angles doesn't affect the calculations. The points are checked for validity and then the algorithm runs to produce the needed correction parameters. Then these parameters are used to correct the unbalanced

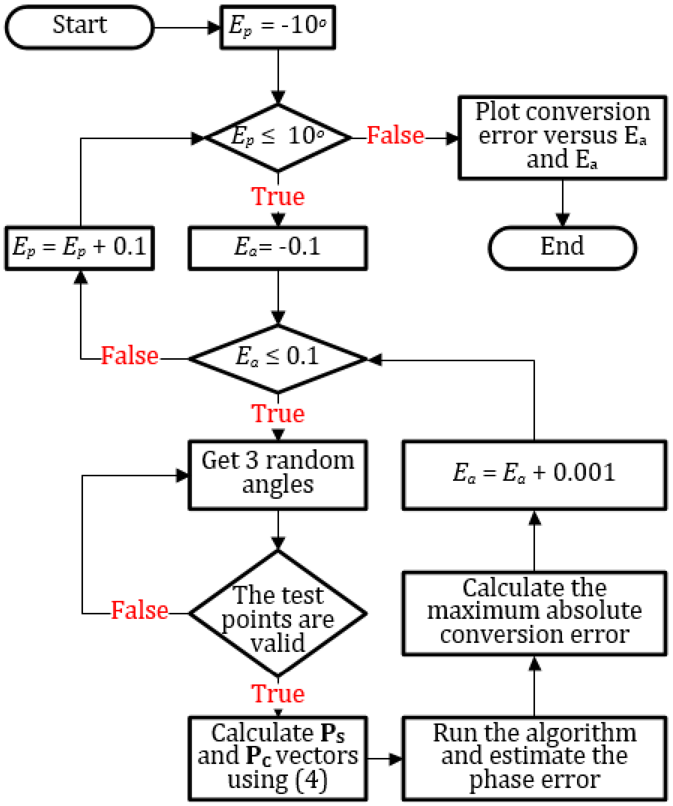


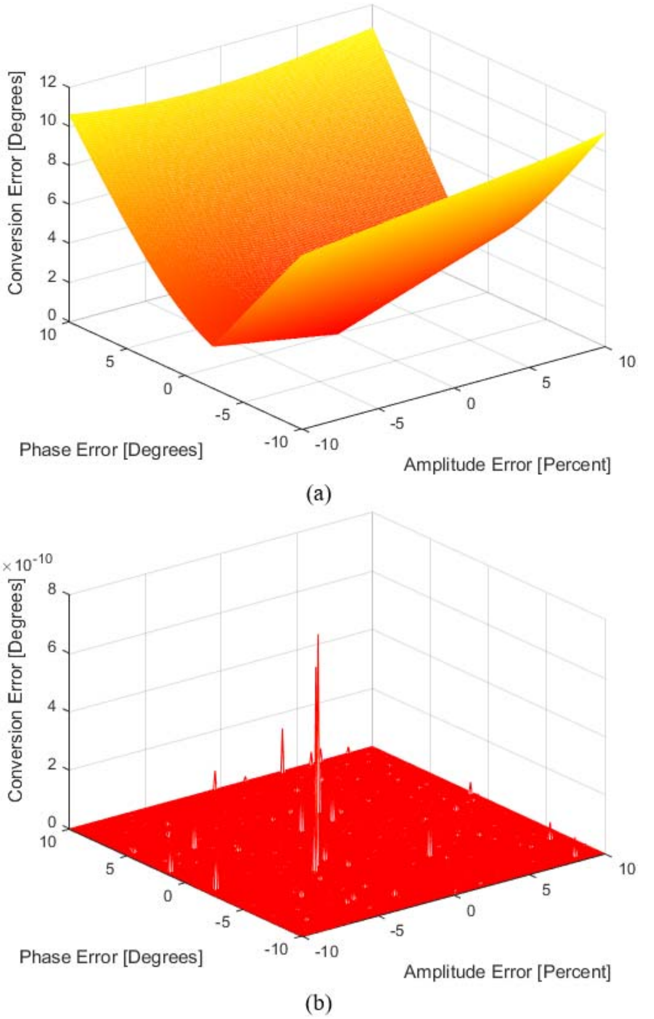
Fig. 4. A flowchart of the simulation program.

signal and give $\tilde{U}_C(\theta)$ as in (7). The corrected signal is entered to a function in MATLAB named "`atan2`" that estimates the angle from its sine and cosine values using arctangent operation combined with logic based on the sign of the two orthogonal components.

The absolute error is then calculated as: $\left|\operatorname{atan2}\left(U_S(\theta), \tilde{U}_C(\theta)\right) - \theta\right|$. This conversion error is calculated for θ from 0 to 360^o and the maximum is taken at each amplitude and phase errors combination. The result of simulation is a three-dimensional graph of the Maximum absolute conversion error after correction as in Fig. 5(b) which shows the error for a total of 40401 amplitude and phase errors combinations. The performance of the algorithm is seen when comparing the conversion error after correction with the error before correction shown in Fig. 5(a). It can be seen that there is almost no error due to the algorithm. The present error can be completely attributed to the digital rounding of the computer processor.

VI. CONCLUSION

A novel method has been presented for correcting phase and amplitude imbalances in signals produced by sinusoidal encoders. The technique is based on using three random but different data points; each point represents values of the sine and cosine signals. A simple algorithm has been developed for online correction of the unbalanced transducer signals that is easy to implement on low-cost microcontrollers. Simulation results using MATLAB confirmed the effectiveness of the proposed method.



(a)

(b)

Fig. 5. Maximum absolute conversion error using arctangent method: (a) before imbalances correction. (b) after imbalance correction

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