Cognitive Complexity and Cognitive Flexibility *

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Cognitive complexity is defined as the number of independent dimensions-worth of concepts the individual brings to bear in describing a particular domain of phenomena; it is assessed with a measure of information-yield based on an object-sorting task. Cognitive flexibility is defined as the readiness with which the person's concept system changes selectively in response to appropriate environmental stimuli; it is assessed by inviting the subject to expand the groups he has created on the original sorting task. In general, the greater a subject's cognitive complexity, (a) the greater is the likelihood that he will expand the groups, and (b) the greater is his tendency to gain information (i.e., dimensional complexity) by the expansion. The measure of dimensional complexity was found to be fairly stable over two different lists of objects; moreover, it was found to correlate with independent measures of knowledge about the object-domain.

The research to be reported here has to do with two properties of cognitive structure, dimensional complexity and susceptibility to reorganization. Reference to these as structural properties implies a distinction between cognitive structure and cognitive content, which is not easy to make in definitive terms. However, as a preliminary attempt it may be useful to regard the *content* of cognition as consisting of concepts of objects and their attributes, while the *structure* of cognition refers to relations among these concepts.

The varieties of cognitive content are almost limitless, but they may be subsumed under such rubrics as attitudes, beliefs, values, etc. Meaningful structural properties are probably more limited in number, including characteristics like differentiation, integration, rigidity, flexibility, and so forth. Much of the current thinking about structural properties derives from the pictorial formulations of Lewin,[1] though other mathematical models have been developed more or less independently.[2] Some research by Zajonc [3]

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[1] Kurt Lewin, *Principles of Topological Psychology,* New York: McGraw-Hill, 1936.

[2] See, for example, Robert P. Abelson and Milton J. Rosenberg, "Symbolic Psychologic: A Model of Attitudinal Cognition," *Behavioral Science,* 3 (January, 1958) pp. 1–13; Dorwin Cartwright and Frank Harary, "Structural Balance: A Generalization of Heider's Theory," *Psychological Review,* 63 (September, 1956) pp. 277–293.

[3] Robert B. Zajonc, "The Process of Cognitive Tuning in Communication," *Journal of Abnormal and Social Psychology,* 61 (September, 1960) pp. 159–167.

treats several structural properties of cognition in quantitative fashion; these as well as others have been discussed elsewhere by the present author.[4]

Basic to the conceptualization or measurement of these various complex structural characteristics is the notion of *differentiation,* or the number of distinct concepts entertained by a person with respect to a particular part of his world. Unless one can identify the cognitive elements from which a structure is built, it is difficult to define the structure in other than vague terms. An essential part of the present research consisted in defining what was to be meant by differentiation (it turned out to be dimensional complexity) and in developing a satisfactory way of measuring it. The cognitive domain studied here was the set of nations which the subjects regarded as important in world affairs. Identical structural measures may be applied, however, to any other region of the person's phenomenal world.

THE MEASUREMENT OF DIMENSIONAL COMPLEXITY

It is assumed that the relevant content of cognition for present purposes may be divided into two kinds of concepts: concepts of objects and concepts of attributes. The object-concepts consist of images of things and events which the person comprehends. The attribute-concepts consist of abstract qualities that are seen as inhering in the objects. An attribute may be represented in geometrical fashion as a dimension which (potentially) runs through a number of different object-images, thereby denoting in the objects the presence or absence, or varying amounts, of the quality which the attribute represents. Correspondingly, an object-image can be regarded as the intersection of a multitude of dimensional attributes, each representing a quality which is conceived in the object (or explicitly excluded from it). These two elements of cognitive structure—images and attributes—are intimately interdependent; one constitutes figure for the other's ground, depending on which is the focus of attention.

For a given domain of cognition, one might assess its differentiation simply by asking the subject, in effect, to list its elements.[5] However, this seems to place undue reliance on his introspective powers, and moreover places on him the burden of determining when two elements are different and when they are the same. It is quite possible that a particular subject would use two different words which were nevertheless functionally equivalent in his conceptual space. If, for instance, "communist" and "totalitarian" were used interchangeably by a person, there would be no justification for

[4] William A. Scott, "Cognitive Structure and Social Structure: Some Concepts and Relationships," in Norman F. Washburne, ed., *Decisions, Values, and Groups,* Vol. 2, New York: Pergamon Press, 1962.

[5] This is the method used by Zajonc, *op. cit.*

treating them as distinct attributes. Correspondingly, if the words England, Great Britain, and United Kingdom were invariably used in identical contexts, then there would be no ground for assuming that they referred to distinct images. So in assessing the degree of differentiation of a cognitive structure, it is hazardous to rely completely on the subject's own vocabulary.

Instead, it would seem appropriate to have the subject perform a task in which he actually uses the words applied to a given domain of events, and to let the investigator decide from their pattern of usage just how many distinct concepts are being emphasized.[6] Kelly's Role Constructs Repertory Test,[7] or the modification of it used by Campbell,[8] constitutes a relevant task for describing people, and a variant of it could conceivably have been developed for the present purpose. However, that test is cumbersome to administer and score, and it appeared doubtful that a non-captive population of adults would willingly submit to it. So a task was constructed along the lines of the G. G. W. S. Object-Sorting Test,[9] but different from it in that no fixed set of objects was required, and the "validity" or "goodness" of the groups formed was of no concern.

Applied to the cognitive domain of nations, the sorting task proceeded as follows: the subject was asked, "When you think about the various nations in the world, which ones come to mind as particularly important in world affairs? Any others?" (etc., until *S* ran out of names). "Now, if you were to arrange the nations on this list into groups which belong together, how would you do it? Make as many groups as you want, and in each one put all the nations that have something in common." (As the subject formed each group:) "What is it these nations have in common? Anything else? Are there any other nations that belong in this group? Are there any other groups of nations that have something in common?" (etc., until the subject ran out of meaningful groupings).

From the pattern of groups the subject constructs it is possible to assess the complexity of his cognitive domain as the number of dimensions-worth of information yielded by the grouping system. To the extent that two groups contain identical members, they are deemed to represent, empirically, the same attribute. To the extent that two groups have non-overlapping membership (i.e., no nations in common), they would appear to represent,

[6] See William A. Scott, "Conceptualizing and Measuring Structural Properties of Cognition," in O. J. Harvey, ed., *Cognitive Factors in Motivation and Social Interaction,* New York: Ronald (in press).

[7] George A. Kelly, *The Psychology of Personal Constructs,* New York: Norton, 1955.

[8] Vincent N. Campbell, "Assumed Similarity, Perceived Sociometric Balance, and Social Influence," Ph.D. dissertation, University of Colorado, 1960.

[9] Kurt Goldstein and Martin Scheerer, "Abstract and Concrete Behavior," *Psychological Monographs,* 53, No. 2 (1941).

empirically, either antithetical attributes or simply different categories of a single underlying dimension. Maximum independence of attributes is represented by two groups, each including half the nations on the list, and which have exactly half their members in common. Taking each pair of groups in turn, one could compute a *phi* coefficient to represent their degree of similarity. Since a correlation coefficient may be treated as the cosine of the angle between two vectors representing the attributes, these phi-coefficients could be converted to angles, and the total amount of space utilized by the sum of angles between all pairs would provide a measure of dimensional complexity.

A simpler method of measuring dimensional complexity is made possible by the present sorting task, in which each attribute is dichotomized (i.e., a nation is either included in a group or not). Dimensional complexity is a function of the number of distinctions among the nations provided by the grouping system. A single group would yield two distinctions, representing presence and absence of the attribute; with two independent groups four distinctions can be made; with k independent groups the maximum number of distinctions is 2^k. One can also work backwards from the number of distinctions actually made among the nations, to see how many groups-worth (i.e., dichotomous dimensions-worth) of information they represent. More precisely, the cognitive dimensionality, or the number of groups-worth of information, can be represented as the dispersion of the objects over the set of distinctions yielded by the category system. The measure of dispersion is H, borrowed from information theory.[10] A computational example is provided in Table 1.

Three cautions should be observed in regard to this measure of dimensional complexity. First, it depends on dichotomous attributes, which are certainly not the only kind that people entertain (though we may agree with Kelly [11] that a dichotomous judgment is basic to any more refined discrimination). Second, H probably represents something like a lower bound to the true cognitive dimensionality, since it is difficult to assess any subject's category system exhaustively; moreover, most meaningful attributes do not dichotomize a group of elements at exactly 50 per cent, and this is the only condition under which complete independence of two attributes can be achieved according to this measure. Finally, it should be noted that any tendency toward randomness in assignment of objects to groups will tend to inflate H artificially; if the subject is not paying attention, he may fail to make two groups identical which in his own mind should be. Given these limitations,

[10] Fred Attneave, *Applications of Information Theory to Psychology,* New York: Holt-Dryden, 1959.

[11] George A. Kelly, *op. cit.*

TABLE 1

Sample Grouping of Nations and Computation of Measures of Dimensional Complexity

Group 1	Group 2	Group 3	Group 4	Group 5
China	India	Congo	England	United States
Russia	England	France	India	Russia
United States	France	Egypt	Denmark	
	Egypt	West Germany	Ireland	
	Cuba	Israel	Poland	
	Japan	China	Israel	
	Poland	Cuba		
	Yugoslavia	Korea		
	Israel	Japan		

S's Total List of Nations and Groups in Which They Are Included

China 13	Egypt 23	U. S. 15	Congo 3	W. Germany 3
Russia 15	England 24	Ireland 4	Cuba 23	E. Germany
India 24	France 23	Denmark 4	Japan 23	Poland 24
Yugoslavia 2	Israel 234	Korea 3		

Group Combination	—	2	3	4	13	15	23	24	234	n
Frequency (n_i)	1	1	3	2	1	2	4	3	1	18

Absolute complexity: $H=\Sigma p_i \log_2 \frac{1}{p_i} = \log_2 n - \frac{1}{n}\Sigma n_i \log_2 n_i = 2.98$, where n is the total number of objects; n_i is the number that appear in a particular combination of groups; and $p_i = n_i/n$.

Relative complexity: $R=\frac{H}{\log_2 n}=\frac{2.98}{4.17}=.71$.

H may be treated as an approximate measure of the dimensional complexity of the cognitive domain referring to a particular class of objects. It is a purely structural property, because it does not depend on the contents of the attributes, but on the relations (in this case, similarity or dissimilarity) among them.

An additional measure may be used to correct for varying numbers of objects presented by different subjects. This is R, the index of relative entropy,[12] defined as $R=H/\log_2 n$, where n is the number of nations listed by the subject. While H represents the absolute complexity of the subject's category system, R may be interpreted as the complexity relative to the number of objects to be comprehended. R thus tends to correct downwards the complexity scores of subjects who name a large number of nations, without fully distinguishing among them.

Both of these measures of dimensional complexity, H and R, depend on the particular sorting task by which the relevant data are collected. Alternative measures would be more appropriate to different kinds of tasks—for

[12] Attneave, *op. cit.*

instance, if the attributes were assessed as continuous, rather than dichotomous, variables.[13] Whatever the assessment technique or the structural measure derived from it, it would appear that something akin to the present measure of dimensional complexity represents appropriately the notion of cognitive differentiation. In order for two object-images to be distinguishable, they must appear phenomenally as different combinations of attributes, which in turn requires distinctions among the attributes which define the images.

RELIABILITY OF DIMENSIONAL COMPLEXITY

The test-retest stability of H was assessed in the following fashion: A heterogeneous sample of 107 adults (not college students) were first asked to list and group nations according to the instructions described above. Then they were shown a standard list of 28 countries [14] and asked to group these as well. Instructions for the second task were purposely worded so as neither to encourage nor to discourage repetition of the same sorting categories. ("Now here is another list of nations, some of which you have already mentioned and some of which are new. I wonder if you would arrange these nations into groups that belong together. Make as many groups as you wish, and in each one put all the nations that have something in common. . . ," as before, until the subject ran out of groups. If the subject asked whether he should use the same groups as before, he was told that was all right, but it was also all right to use additional groups, if he thought these were important.)

The correlation between H computed from the individual's own list and H from the standard list was .68, which may be deemed satisfactory, considering that the two lists of nations were invariably different, and the bases for grouping nearly always differed to some extent.

DIMENSIONAL COMPLEXITY AND KNOWLEDGE ABOUT THE COGNITIVE DOMAIN

Cognitive complexity, as conceptualized and measured here, reflects the ability to comprehend a cognitive domain with a variety of independent attributes for describing the objects in it. Presumably a necessary, though not sufficient, basis for this ability is a fund of information about the domain which could provide a source of cognitive differentiation. The reason that

[13] For some other approaches to the measurement of cognitive complexity, see Campbell, *op. cit.;* Kelly, *op. cit.,* and Z. Joseph Ulehla, "Individual Differences in Information Yields of Raters," M.A. thesis, University of Colorado, 1961.

[14] Algeria, Australia, Brazil, China (mainland), Congo, Cuba, Czechoslovakia, Dominican Republic, France, Germany, Ghana, Great Britain, India, Indonesia, Israel, Italy, Japan, Laos, Mexico, Pakistan, Poland, Saudi Arabia, Tunisia, Turkey, Union of South Africa, USSR (Russia), United Arab Republic (Egypt), United States of America.

factual knowledge alone is not sufficient basis for a complex cognitive structure is that it may have been acquired selectively in such a way that all "facts" point toward a stereotyped or undifferentiated view of the domain. This is what one might expect as a consequence of doctrinaire propaganda, in which many negative characteristics are attributed to the bêtes noires, and their opposites to the shining angels; thus, though a great many attribute-concepts may have been acquired by the person, they can be so highly intercorrelated that cognitive dimensionality is minimal.

One would nevertheless expect, on the average, at least some positive correlation between cognitive complexity and level of information about the relevant domain. Two different studies have yielded such a result. One was done by questionnaire administered to a haphazard sample of 167 undergraduate students at a large midwestern university. *H* was assessed from a standard list of 20 nations [15] which subjects grouped in spaces provided on the page. Instructions read: "When you think of the various nations in the world, some of them seem to be more closely related than others. In fact, you can make up groups of nations that share common qualities distinct from others. On this page are six spaces. In each space write the names of several countries which seem similar, or seem to 'belong together.' Choose countries from those listed below. You can put each country in as many groups as you like, but be sure to write down what the group has in common."

Also in the same instrument were 15 factual questions drawn from the "Tulane Data Test"; these assessed knowledge of geography, world politics, and recent events. This information test correlated +.31 with *H*, the measure of dimensional complexity ($\alpha < .001$).

A second study was performed by individual interviews with a haphazard sample of 107 adults (not college students), selected from that university's community in such a way as to yield a heterogeneous group with respect to age, sex, and education level. Cognitive complexity was assessed both from groupings of the subject's own free list of nations and from groupings of a different standard list.[16] The respondent's level of knowledge was measured (roughly) by asking him, following grouping of the standard list, if he knew any of the heads of government of the countries listed. The information score was computed as the number correctly named. It correlated +.37 with *H* calculated from the free list and +.47 with *H* calculated from the standard list (in both cases $\alpha < .001$).

[15] Canada, China (Formosa), China (mainland), Czechoslovakia, Egypt, France, Germany (East), Germany (West), Great Britain, India, Israel, Italy, Japan, Mexico, Pakistan, Poland, Union of South Africa, USSR (Russia), United States of America, Yugoslavia.

[16] See footnote 14.

COGNITIVE FLEXIBILITY

As a first approximation, the flexibility of a cognitive structure may be conceived as the ease with which it permits new views of the object domain to develop in response to appropriate environmental stimuli. Within the structural formulation developed above, this may be interpreted as follows: flexibility consists in the ready alteration of images, by selectively changing the attributes assigned to them; alternatively viewed, it consists in ready alteration of the relations among attributes, so that they can intersect the set of object-images in new ways. By contrast, cognitive rigidity consists both in maintaining fixed images of objects and in maintaining constant correlations among the attributes conceived in the cognitive domain. Within the present formulation, these are two different ways of saying the same thing.

In order to assess the flexibility of the 107 adults' category systems, these were subjected to a mild assault in the following manner: Subjects first listed and grouped their own sets of countries; then they were confronted with the standard list and asked to group that. Following two more interpolated tasks (assessing knowledge of premiers and attitudes toward the countries), their first (free) list was again presented along with the first group made from it, and the interviewer asked: "Now, I wonder if you'd look at this original list and tell me if there are any nations on it that don't appear in your first group, but which might be like them in some way. In other words, are there nations remaining in the total list that could somehow be included with those you have already put in this first group? What are they? Any others? What is it that this new group of nations have in common? Now, would you please look at the second group you made and tell me if there are any other nations on the list that are like these in any way? What are they?" (And so on, through all the groups the subject had originally made.)

There are at least three ways a subject might react to such a demand. He could rigidly adhere to his initial concept system and make no modifications in the groups. Or he could modify them selectively by adding certain nations appropriate to the new categories. Or he could react with a complete breakdown of the original conceptual system and admit nations to all groups indiscriminately. These three types of reactions would have different effects on the dimensional complexity (H) of the category set. Complete rigidity would leave H unaffected (i.e., the same distinctions would be made as before). Selective reorganization would generally yield an increase in H, since this would tend to introduce new distinctions among the nation-objects. Wholesale and indiscriminate reorganization would reduce H, for now there would be a tendency for every nation to appear in every group, thereby eliminating the distinctions among them.

It was predicted that a proclivity toward one or another of these reactions would be a function of the initial complexity of the subject's conceptual structure. The reasoning was as follows: Cognitively simple subjects would tend, primarily, not to reorganize at all, because their low dimensional complexity represents either (1) a paucity of conceived attributes, and hence an inability to entertain alternatives, or (2) such a high correlation among the attributes which they do conceive that a single one of them would be difficult to alter alone. By extension, it was predicted that whatever changes *did* occur in the cognitively simple subjects would be more likely to result in a lowered *H*, since the attributes, being highly correlated, could not be

TABLE 2

Zero-order and Partial Correlations of Cognitive Complexity (H) *with Measures of Cognitive Flexibility*

Flexibility Measure	Zero-Order Correlations	Partial Correlations Holding Contant		n †
		Info.	No. of Groups	
Reorganized groups or not	.21*	.17*	.09	106
Amount of increase in *H* through reorganization	.49**	.45**	.37*	32

* $\alpha < .05$.

** $\alpha < .01$.

† One *S* was omitted from this analysis because she had failed to form any group on the initial sort; the second row of correlations is based only on the 32 subjects who added nations to their original groups. (See footnote 17.)

altered selectively. Cognitively complex subjects, on the other hand, should be more capable of altering their groups in such a way as to gain in information yield (i.e., dimensional complexity), because their relatively independent attributes could be selectively associated with, or dissociated from, the nation-images, thereby increasing (on the average) the number of distinctions made.

The tests of these hypotheses are reported in the first column of Table 2, which shows the relations between *H* and the two measures of flexibility. There is a correlation (point-biserial r) of .21 between the initial *H* and whether or not new nations were added to the groups at all ($\alpha < .05$); among the 32 subjects who did alter their groupings the correlation between initial *H* and magnitude of increase in *H* was .49 ($\alpha < .01$).[17] High complexity sub-

[17] A total of 37 subjects added nations to their original groups. But three of these produced nonsense categories, apparently through failure to understand the instructions. Another subject's data were invalidated through interviewer error, and a fifth was

jects tended to gain information through reorganization; their mean increase in H was .11 ($\alpha < .01$). Low complexity subjects tended to lose in information yield, with a mean decrease in H of .13 ($\alpha < .10$). The difference in these directions of change is opposite to that which would result from regression effects.

The possibility remains, of course, that the obtained correlations between cognitive complexity and flexibility are due to some extraneous variable. Two possible contaminating factors could be controlled statistically in the present study. One was information level, as measured by the number of government heads the respondent could name. With this variable controlled, the partial correlations between H and the two measures of flexibility were .17 and .45 —quite close to the zero-order rs. Another variable one might wish to control is general loquacity; this could be measured, roughly, by the number of groups the subject constructed. The relevant partial correlations here were .09 (NS) and .37 ($\alpha = .05$)—somewhat lower than the zero-order rs.

A considerably better correlation with the first flexibility measure is obtained when R, rather than H is used as the measure of dimensional complexity. The point-biserial r between R and the subject's ability to reorganize his groups was .43 ($\alpha < .0001$); when information level and number of initial groups were held constant statistically, the resulting partial rs were .42 and .40, respectively ($\alpha < .0001$).

Though there may be some other contaminating factor that has not been accounted for, these results appear at present to warrant the conclusion that a particular structural property of cognition, dimensional complexity, is empirically related to the dynamic property, cognitive flexibility. The more adequate a person's conceptual scheme for distinguishing among that group of phenomenal objects which he conceives in a particular cognitive domain, the more readily can he alter the scheme when required, and do so in such a manner as to increase the number of distinctions which it yields. The adequacy of one's distinctions among objects depends on the use of empirically independent concepts for classifying them. The reason why R seems better related to the measure of reorganizing ability, while H is better related to the gain in information through reorganization, is not clear at present. Perhaps some measurement artifact has been overlooked, or perhaps the apparent differences in rs are fortuitous. It may also be that a person with a large number of objects in his phenomenal world needs more dimensions for handling them flexibly than does one who conceives of a smaller number.

eliminated from the present analysis because his initial H was already at maximum value for the number of nations he mentioned (that is, $R = 1.00$), hence it could not increase through reorganization. If all 37 subjects had been included in this analysis, the r in the first column, second row, of Table 2 would have been .46 ($\alpha < .01$), instead of .49.