Forward Kinematic Model of a New Spherical Parallel Manipulator Used as a Master Device

H. Saafi, M.A. Laribi, M. Arsicault and S. Zeghloul

Abstract The paper discusses the Forward Kinematic Model (FKM) of a special Spherical Parallel Manipulator (SPM). The special SPM is obtained by modifying one leg of a classic SPM. This new architecture eliminates the singularity from the workspace. The SPM is used as master device for medical tele-operation system. The FKM of the new SPM is calculated using the equation of spherical four-bar mechanism. A method to improve the FKM calculation using extra sensor is proposed in this paper.

Keywords Spherical parallel manipulator · Master device · Forward kinematic model · Singularity

1 Introduction

Nowadays, parallel manipulators are widely popular. Thanks to their high load capacity, their stiffness, their low weight and their precision, parallel manipulators are used in many fields such as medicine, where, many master devices have been developed with parallel architecture [1, 2].

In previous works [3], new master device was developed to control a surgical robot. This device has a spherical parallel architecture. The master architecture was

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chosen to have a mechanism able to provide three pure rotations around a fixed point. The geometric parameters of this structure were optimized to meet a prescribed workspace. However, the main problem of the optimized structure is the presence of parallel singularity in the workspace. To cope with the presence of the singularity, in [4], we proposed to change the architecture of one leg. Then, geometric parameters of the new SPM were optimized in order to have a singular free workspace.

The FKM of the master device is needed to control the surgical robot. The FKM calculates the orientation of the moving platform of the SPM. In this paper, the FKM of the new SPM is presented. The calculation time of the FKM is reduced using an extra sensor. This makes the FKM ready to work in real time.

2 Master Device for a Medical Tele-Operation System

The developed master device is shown in Fig. 1. This device controls a surgical robot. The target application is Minimally Invasive Surgery (MIS). In MIS, the instruments enter to the patient body throw tiny incisions. This limits the motion of the instrument to three rotations around the incision and one translation along the instrument axis. For this reason, the spherical parallel architecture was chosen for the master device since it is able to produce the similar motions.

Fig. 1 Master device of a tele-operation system

The translation motion is not taken into account in this paper because it is uncoupled in the model of the SPM.

The classical SPM has three identical legs, each leg is made of two links and three revolute joints (Fig. 2a). All axes of the revolute joints are intersecting in one common point, called CoR (Center of Rotation). Each link is characterized by the angle between its two revolute joints, as shown in Fig. 2b. This angle is constant and it represents the dimension of the link. The actuated joint axes are located along an orthogonal frame. The orientation of the SPM is described by the ZXZ Euler angles (ψ, θ, and φ).

The Jacobian matrix of the SPM can be written as follows:

$$\mathbf{J}{=}\mathbf{A}^{-1}\mathbf{B} \tag{1}$$

where,

$$\mathbf{A}{=}\left[Z_{2A}\times Z_{3A} \quad Z_{2B}\times Z_{3B} \quad Z_{2C}\times Z_{3C}\right]^{\mathrm{T}} \tag{2}$$

$$\mathbf{B}=\begin{bmatrix} Z_{1A}(Z_{2A}\times Z_{3A}) & 0 & 0 \\ 0 & Z_{1B}(Z_{2B}\times Z_{3B}) & 0 \\ 0 & 0 & Z_{1C}(Z_{2C}\times Z_{3C}) \end{bmatrix} \tag{3}$$

Dexterity is used to identify the presence of singularity in the workspace of the SPM. Dexterity $\eta(\mathbf{J})$ is given by the inverse of the condition number of the Jacobian matrix as follows:

$$\eta(\mathbf{J}){=}\ 1/\kappa(\mathbf{J})\ \text{with}\ \kappa(\mathbf{J}){=}\|\,\mathbf{J}\,\|\ \left\|\,\mathbf{J}^{-1}\right\| \tag{4}$$

Figure 3 shows the distribution of the dexterity for a self-rotation φ = 50°. It shows the presence of parallel singularity inside the workspace (dotted line).

To cope with the problem of the singularity, we proposed in [4] to change the architecture of the leg A. The next Section presents the architecture of the new SPM.

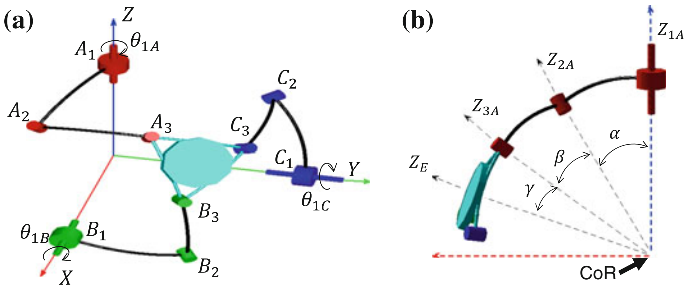


Fig. 2 **a** Classic spherical parallel manipulator. **b** Kinematic of leg A

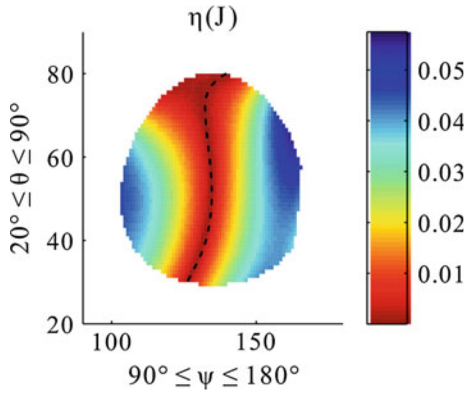


Fig. 3 The dexterity distribution for φ = 50°

3 New Architecture of the Master Device

The new structure of the SPM is shown in Fig. 4a The RRR kinematic structure of leg A was replaced by URU structure as illustrated in Fig. 4b (R for Revolute joint and U for Universal joint). The two links of the new leg have the same length, *L*. The legs B and C are not changed. This new leg generates similar motions to those obtained by the RRR leg.

The matrices **A** and **B** of the new SPM become as follows:

$$\mathbf{A} = \begin{bmatrix} Z_{4A} \times Z_{5A} & Z_{2B} \times Z_{3B} & Z_{2C} \times Z_{3C} \end{bmatrix}^{\mathrm{T}} \tag{5}$$

$$\mathbf{B} = \begin{bmatrix} Z_{1A}(Z_{4A} \times Z_{5A}) & 0 & 0 \\ 0 & Z_{1B}(Z_{2B} \times Z_{3B}) & 0 \\ 0 & 0 & Z_{1C}(Z_{2C} \times Z_{3C}) \end{bmatrix} \tag{6}$$

Figure 5 shows the distribution of the dexterity for the self-rotation φ = 50°. It shows that parallel singularities are outside the useful area of the workspace (dotted lines).

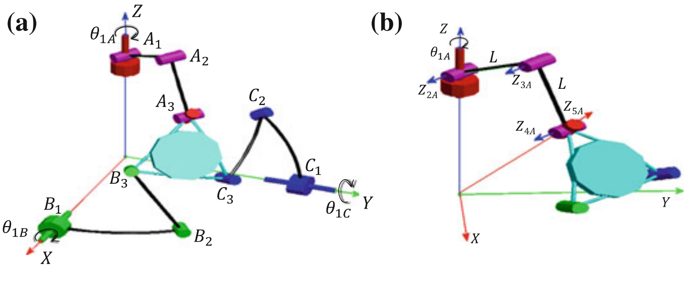


Fig. 4 **a** Kinematic structure of the new SPM. **b** New kinematic of leg A

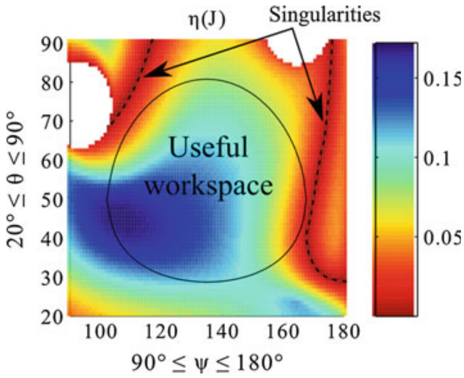


Fig. 5 The dexterity distribution for φ = 50°

4 Forward Kinematic Model

The FKM calculates the orientation (ψ, θ, and φ) of the SPM in a function of active joint angles (θ_{1A}, θ_{1B}, and θ_{1C}). The FKM is solved using a method presented in [5]. This method is based on the input/output equations of spherical four-bar mechanisms. Figure 6 shows a spherical four-bar mechanism. the input and output angles are σ and ξ, respectively.

The input/output equation of the spherical four-bar mechanism is as follows:

$$L(\xi)\cos\sigma + M(\xi)\sin\sigma + N(\xi) = 0 \tag{7}$$

With L, M and N are variables that depend on cos(ξ) and sin(ξ). Their expressions are detailed in [5].

For the SPM, two four-bar mechanisms are considered $A_2A_3B_3B_2$ and $A_2A_3C_3C_2$ (Fig. 7).

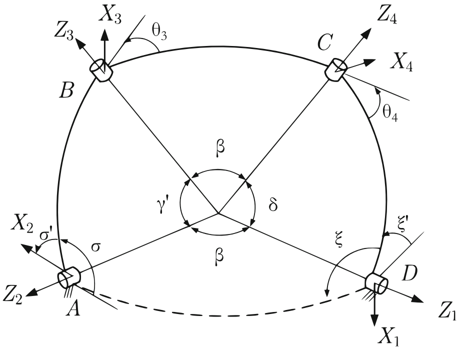


Fig. 6 Spherical four-bar mechanism

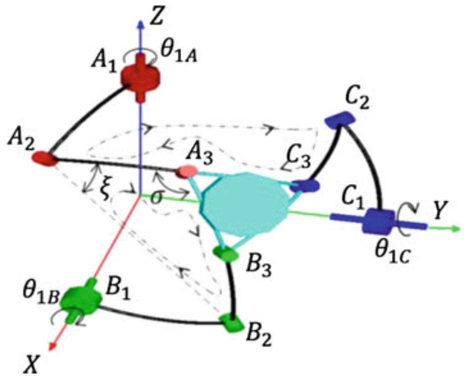


Fig. 7 The two considered spherical four-bar mechanisms

The two input/output equations are as follows:

$$\begin{cases} \mathrm{L}_1\,(\xi)\,\cos\sigma + \mathrm{M}_1(\xi)\,\sin\sigma + \mathrm{N}_1(\xi) = 0 \\ \mathrm{L}_2(\xi)\,\cos\sigma + \mathrm{M}_2(\xi)\,\sin\sigma + \mathrm{N}_2(\xi) = 0 \end{cases} \tag{8}$$

cos(σ) and sin(σ) can be expressed as follows:

$$\cos\sigma = \frac{\mathrm{M}_1\mathrm{N}_2 - \mathrm{M}_2\mathrm{N}_1}{\mathrm{L}_1\mathrm{M}_2 - \mathrm{L}_2\mathrm{M}_1}, \qquad \sin\sigma = \frac{\mathrm{L}_1\mathrm{N}_2 - \mathrm{L}_2\mathrm{N}_1}{\mathrm{L}_1\mathrm{M}_2 - \mathrm{L}_2\mathrm{M}_1} \tag{9}$$

All the possible solutions of angle, ξ, can be found by solving the Eq. (10), obtained by calculating the square sum of cos(σ) and sin(σ), defined bellow:

$$\begin{aligned} N_2^2L_2^2 + 2L_1M_1L_2M_2 - 2L_1N_1L_2N_2 + N_2^2M_1^2 - L_2^2M_1^2 \\ -2M_1N_2M_2N_2 - M_2^2L_1^2 + N_1^2L_2^2 + N_1^2M_2^2 = 0 \end{aligned} \tag{10}$$

The obtained solutions for angle, ξ, are then used to calculate the values of the angle, σ, using Eq. (9). The platform orientation (ψ, θ, and φ) can be calculated using the forward kinematic of leg A.

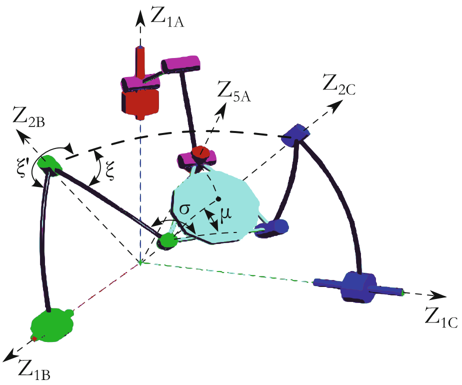
For the new SPM, only one four-bar mechanism can be considered (Fig. 8). The input/output equation is as follows:

$$\mathrm{L}_1(\xi)\,\cos\sigma + \mathrm{M}_1(\xi)\,\sin\sigma + \mathrm{N}_1(\xi) = 0 \tag{11}$$

The second equation is obtained by expressing $\mathrm{Z}_{5\mathrm{A}}$ in function of ξ and σ by using the forward kinematic of leg B as follows:

$$\begin{aligned} Z_{5A} = R_{0B}R_Z(\theta_{1A})R_X(\alpha)R_Z(\xi + \xi')R_X(\beta)R_Z(\sigma - \mu) \\ R_X(\gamma)R_Z\left(\frac{2\pi}{3}\right)R_X(-\gamma)\,Z_{1A} \end{aligned} \tag{12}$$

Fig. 8 The considered spherical four-bar mechanism



Y_{1A} is perpendicular to Z_{5A}, this condition leads to the following equation after the arrangement:

$$L_2(\xi)\cos\sigma + M_2(\xi)\sin\sigma + N_2(\xi) = 0 \tag{13}$$

The system composed of Eqs. (11) and (13) can be solved using the same procedure as the classic SPM. This method gives accurate solutions of the FKM. However, due to the complexity of Eq. (10), the calculation time is more than 100 ms on a PC computer with a processor running at 2.2 GHz. An improvement of the forward kinematic model is presented in the next section.

5 Improvement of the Forward Kinematic Model

In [6], we proposed a solution to improve the FKM calculation. This solution is based on the use of an extra sensor. The extra sensor improves the FKM accuracy and reduces the complexity of the model and the calculation time to less than 50 µs. The same solution is used here. An extra sensor is added in the joint with axis Z_{3B}. This extra sensor leads to a system with only one unknown, since the angle σ is now given directly by the fourth sensor.

Equations (11) and (13) can be arranged as follows:

$$\begin{cases} \overline{L}_1\cos\xi + \overline{M}_1\sin\xi + \overline{N}_1 = 0 \\ \overline{L}_2\cos\xi + \overline{M}_2\sin\xi + \overline{N}_2 = 0 \end{cases} \tag{14}$$

With, $\overline{L}_i$, $\overline{M}_i$ and $\overline{N}_i (i = 1, 2)$ are variables that depend on cos (σ) and sin (σ). The unique solution of ξ is calculated as follows:

$$\xi = a\tan 2(\sin\xi, \cos\xi) \tag{15}$$

With,

$$\cos\xi = \frac{\overline{M}_1\overline{N}_2 - \overline{M}_2\overline{N}_1}{\overline{L}_1\overline{M}_2 - \overline{L}_2\overline{M}_1}, \qquad \sin\xi = \frac{\overline{L}_1\overline{N}_2 - \overline{L}_2\overline{N}_1}{\overline{L}_1\overline{M}_2 - \overline{L}_2\overline{M}_{11}} \tag{16}$$

The Euler angles ψ, θ and φ can be obtained using the same approach presented in Sect. 3 using the forward kinematic of leg B.

The use of an extra sensor makes the FKM a linear problem with a unique solution. The calculation time of the improved FKM is less than 50 μs on a PC computer with a processor running at 2.2 GHz.

6 Conclusions

In this paper, a new master device for a tele-operation system has been presented. This device has a special spherical parallel architecture. The forward kinematic model of the SPM has been studied and then has been improved to work in real time.

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