

Stability of battened columns with and without taper

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In this investigation built-up columns with a linear variation of depth under various support conditions are analysed as rigid-jointed frameworks. A rigorous analysis for determining critical loads allows formulae to be established for buckling load estimates of practical importance. The individual and coupling effects on the critical loads are assessed for a variety of parameters such as: degree of nonuniformity, number of panels as well as stiffness and length ratios of component members. The proposed solution technique is demonstrated with examples.

Keywords: stability, battened columns, tapered build-up members, support conditions, critical buckling load

Frameworks with built-up members of varying cross section are frequently encountered in engineering practice. The earliest study on this problem was by Engesser¹ who established approximate formulae for the buckling load of battened and laced columns. Bleich² and Timoshenko and Gere³ presented more accurate and understandable stability analyses for nonuniform battened and laced columns. Nevertheless such built-up columns were not considered as frameworks of a high degree of indeterminacy. Indeed, various simplifying assumptions regarding the buckled configuration, the slope continuity of the chords, and the degree of end fixity of the lacing bars were made which permitted approximate formulae for establishing the internal forces of the framework. More accurate studies dealing with the stability analysis^{4,5} and the post-buckling response⁶ of simple frames having latticed members with linear depth variation have been presented.

In this investigation tapered built-up members are analysed as rigid-jointed frameworks with particular emphasis on battened columns. The main objectives of this paper are to use a linear stability analysis to establish critical loads of tapered battened columns; to discuss the effect of various parameters on the critical loads; and to avoid the aforementioned intractable analysis by proposing approximate but reliable formulae for predicting the load-carrying capacity of such members.

Post-buckling effects on the critical load are not accounted for, however, the inclusion of such effects could be justified in view of some residual post-buckling strength in the members under consideration. In this case one can employ a reliable simplified post-buckling analysis^{7,8}.

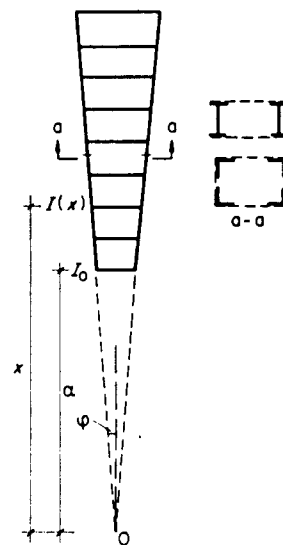


Figure 1 Members with batten plates of varying cross-section

Statement of the problem

The axial load-carrying capacity of tapered members having a linearly varying depth with a constant cross-sectional area along their axis is studied. Such members with batten plates are shown in Figure 1. In this case the moment of inertia of the cross-section varies as a second power of the distance from the point of intersection of the two chord axes, as follows (Figure 2)

$$I(x) = I_0 \left(\frac{x}{\alpha} \right)^2 \quad (1)$$

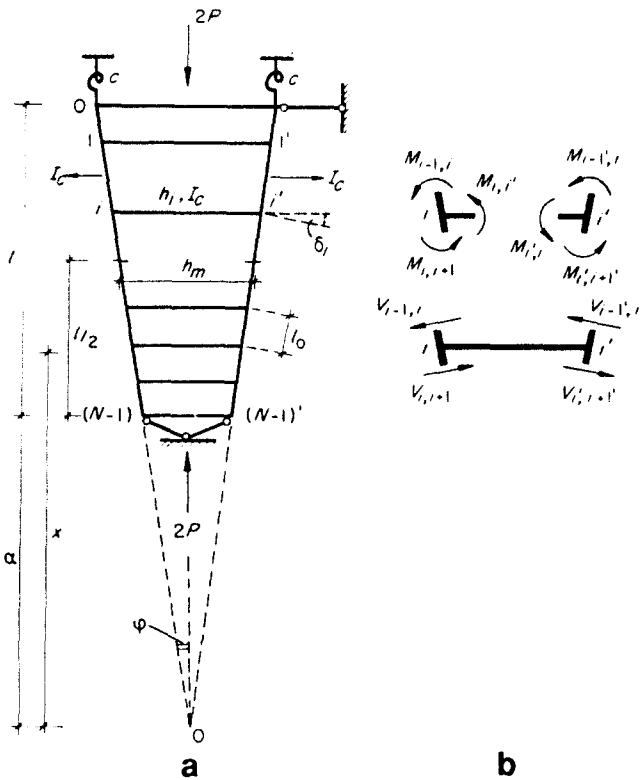


Figure 2 Geometry and sign convention of a partially fixed-hinged no-nuniform column

where α , x are distances from the origin as shown in Figure 2a, and I_0 is the moment of inertia at the bottom of the member ($x = \alpha$). Such a law corresponds with sufficient accuracy to the case of a tapered member consisting of four angles or two channels (or double tees) connected by lacing bars or batten plates. The joints for the latter case are considered to be rigidly connected. Hence the battened member is a highly redundant framework.

The analysis that follows refers to members under an axial compressive load P with the lower end hinged and the upper end partially fixed, as shown in Figure 2. It is also assumed that the batten plates are subjected only to bending since their axial load is negligible before buckling. The axial deformation of the members is not taken into account. The geometrical description, internal forces and sign convention are also shown in Figure 2. The deformed state will be established by using a linear stability analysis in terms of ϑ , and δ_i , where ϑ_i and δ_i are the angle of rotation and the lateral deflection respectively of the i th joint ($i = 0, 1, \dots, N-1$).

The following auxiliary quantities are also used

$$\mu = \frac{I_b}{I_c}, \quad n = \frac{h_m}{l}, \quad l = \frac{l}{\alpha}, \quad \beta^2 = \frac{Pl^2}{EI_c} \quad (2)$$

where I_c and I_b are the moments of inertia of each chord and each batten plate respectively; h_m the depth of the cross-section at the middle of the member; l is the length of the member, and E is the modulus of elasticity.

Mathematical analysis

Consider the built up column shown in Figure 2a which is composed of $N-1$ equal panels of length $l_0 \cos \varphi$. The column, being symmetric with respect to its longitudinal axis, is subjected to an axial compressive load $2P$. The detailed geometry and the sign convention of the internal forces are shown in Figure 2. The slope deflection method which includes the effect of axial forces is used below.

The bending moments and the shearing force corresponding to the $(i-1, i)$ chord segment with length l_0 have the following expressions⁹

$$\left. \begin{aligned} M_{i-1,i} &= \frac{2EI_c}{l_0} \left[\alpha_n \vartheta_{i-1} + \alpha_f \vartheta_i \right. \\ &\quad \left. + (\alpha_n + \alpha_f) \frac{\delta_{i-1} + \delta_i}{l_0} \right] \\ M_{i,i-1} &= \frac{2EI_c}{l_0} \left[\alpha_n \vartheta_i + \alpha_f \vartheta_{i-1} \right. \\ &\quad \left. + (\alpha_n + \alpha_f) \frac{\delta_{i-1} - \delta_i}{l_0} \right] \\ V_{i-1,i} &= \frac{M_{i-1,i} + M_{i,i-1}}{l_0} - \frac{P}{\cos \varphi} \frac{\delta_{i-1} - \delta_i}{l_0} \end{aligned} \right\} \quad (3)$$

where

$$\left. \begin{aligned} \alpha_n &= \frac{\varphi_n}{2(\varphi_n^2 - \varphi_f^2)}, \quad \alpha_f = \frac{\varphi_f}{2(\varphi_n^2 - \varphi_f^2)} \\ \varphi_n &= \frac{1 - \beta_c \cot \beta_c}{\beta_c^2}, \quad \varphi_f = \frac{\frac{\beta_c}{\sin \beta_c} - 1}{\beta_c^2} \\ \cos \varphi &= \frac{1}{\sqrt{1 + \left(\frac{n\bar{l}}{2 + \bar{l}} \right)^2}} \end{aligned} \right\} \quad (4)$$

and $\beta_c^2 = \beta^2 / \cos \varphi$ being constant along the length of each chord. Equilibrium of moments at each joint i' and i yield $2N$ equations, while the equilibrium of shearing forces as the free-body diagram of Figure 2b shows, yields $N-2$ equations.

These equations can be written in the matrix form as follows

$$\begin{bmatrix} A & K \\ K^T & B \end{bmatrix} \begin{bmatrix} \vartheta \\ \delta \end{bmatrix} = 0 \quad (5)$$

where $\delta = \delta/l_0$, while the matrices A (of order $2N \times 2N$), B (of order $(N-2) \times (N-2)$) and K (of order $2N \times (N-2)$) are functions of the quantities α_n , α_f (equations (4)), μ , n , $\bar{l}$ (equations (2)), and K^T is the

transpose of matrix K . The vanishing of the (stability) determinant associated with equation (5) leads to the overall buckling equation of the member from which the successive buckling loads are numerically evaluated.

Using this equation one can treat the case of a simply supported column by setting $c = 0$, as well as the case of the propped cantilever column (fixed-hinged column) by setting $c \rightarrow \infty$ (implying $\vartheta_0 = \vartheta_{0'} = 0$), where c = rotational spring constant (Figure 2a).

Numerical results and discussion

With the aid of numerical evaluation of the buckling equation one can obtain critical (buckling) loads corresponding to various geometrical configurations of simply supported and fixed-hinged columns. A variety of numerical results are given in graphical form.

Figures 3a and b, correspond to a simply supported beam and a proposed beam respectively, they show the variation of the dimensionless critical load β_{cr}^2 versus μ for various values of n ($=0.1, 0.3, 0.5$), $\bar{l}$ ($=0, 10$) and for $N = 10$.

The critical loads corresponding to nonuniform columns ($\bar{l} \neq 0$) are higher than those of the uniform columns ($\bar{l} = 0$), for the segments of the curves β_{cr}^2 versus μ which are located below the dotted lines in both plots.

For the usual case of a simply supported column with $n = 0.1$ and $\mu = 0.5$, the critical load of the uniform column is greater by about 10% than the critical load of the corresponding nonuniform column. Note that such a difference decreases substantially with an increase in n .

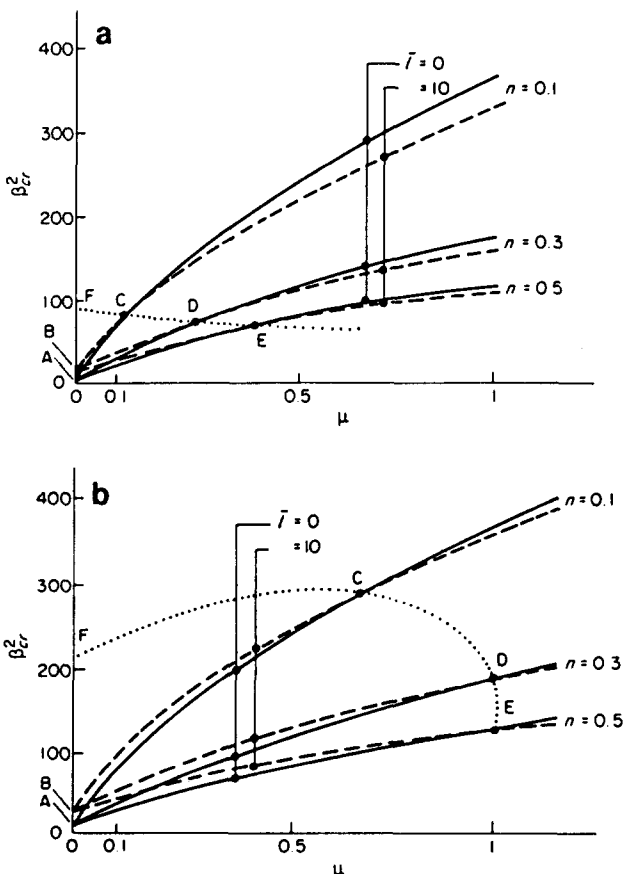


Figure 3 Critical loads β_{cr}^2 for a simply supported (a) and a fixed-hinged (b) column, as functions of μ , n and $\bar{l}$

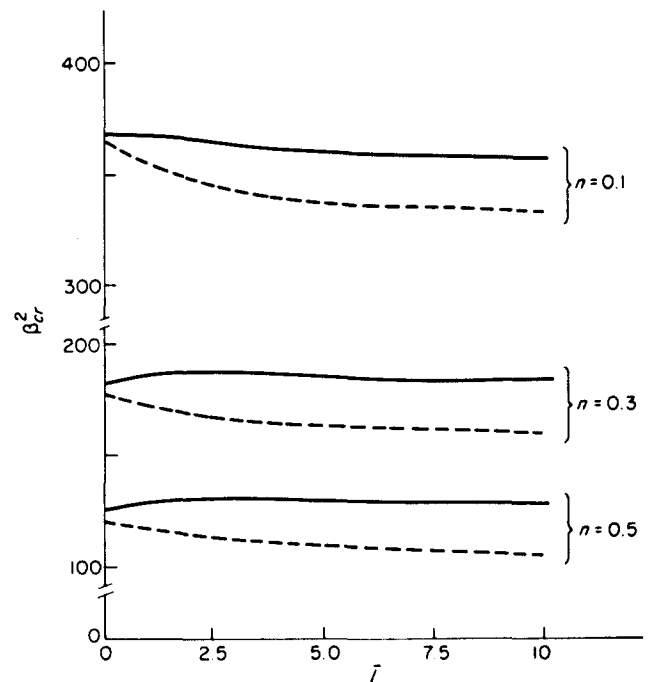


Figure 4 Critical loads β_{cr}^2 versus $\bar{l}$, for $\mu = 1$, $N = 10$ and various values of n . $\bar{l} = 0$ corresponds to a uniform column. (—), fixed-hinged; (---), hinged-hinged

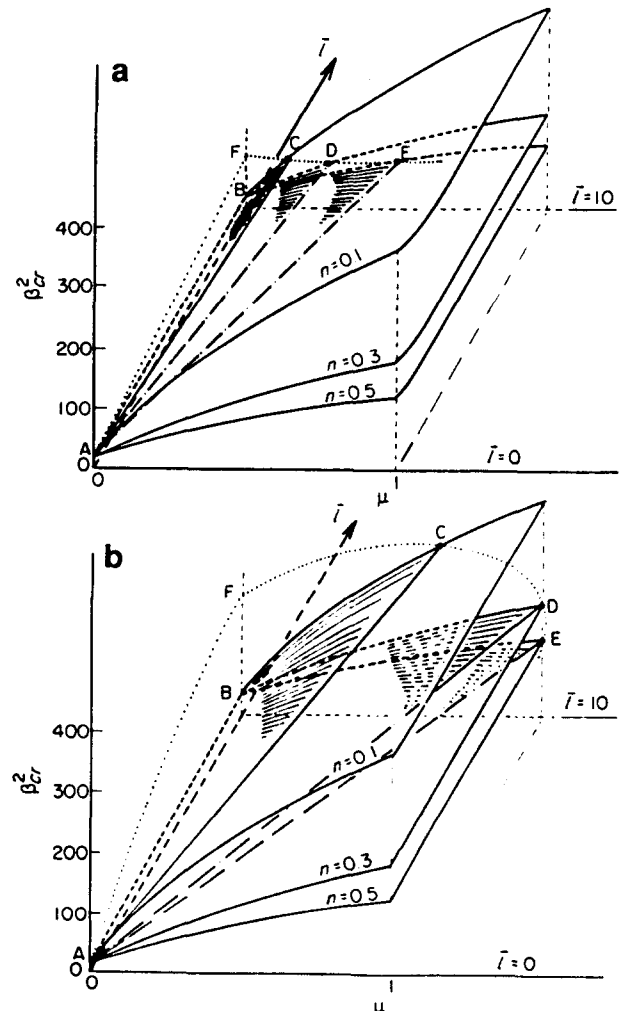


Figure 5 Critical loads β_{cr}^2 for simply supported (a) and fixed-hinged (b) columns, as functions of μ , $\bar{l}$ and n

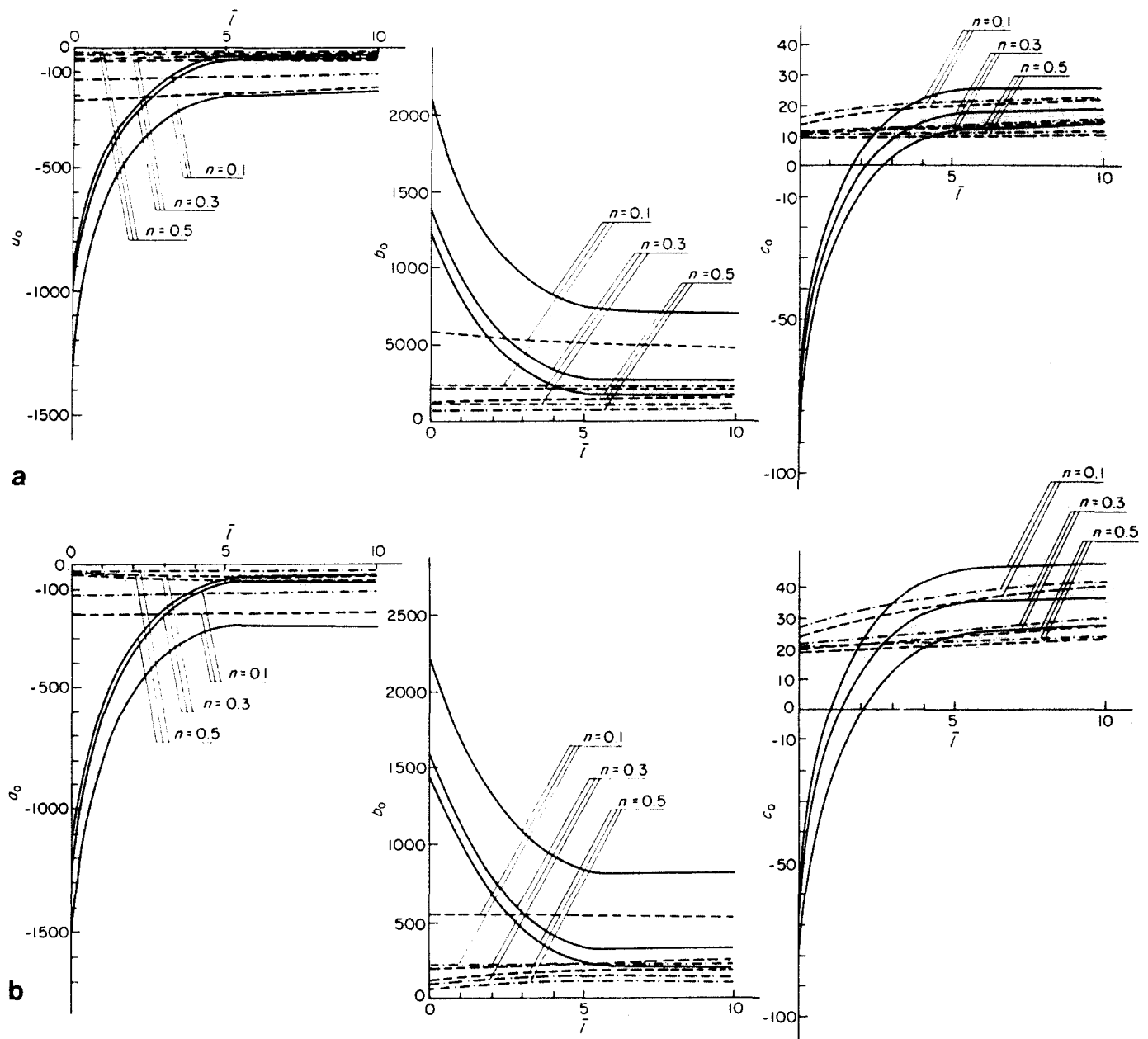


Figure 6 Coefficients a_0 , b_0 and c_0 as functions of $\bar{l}$, n and N corresponding; a, to simply supported columns; and b, to fixed-hinge columns. (· · · · ·), $N = 5$; (— — —), $N = 10$; (—), $N = 15$

For the case of a fixed-hinged column, the maximum difference between the critical load of the nonuniform column and that of the uniform one, occurs for $n = 0.1$ and $\mu = 0.1$ and reaches 20%. Contrary to the previous case, the critical load of the nonuniform column is greater than that of the uniform column.

It is also worth observing that the difference in the critical loads corresponding to a simply supported column and a fixed-hinged column for the same geometrical configurations is much less than the corresponding difference for columns with solid cross-sections. Indeed for solid cross-sections obtained for $\mu \rightarrow \infty$ and $n \rightarrow 0$ (in the case $\bar{l} = 0$) such a difference tends to $1/0.7^2 \approx 2$, while for battened members it is much less as is shown in Figure 4. From this figure, one can see the variation of β_{cr}^2 versus $\bar{l}$ for $N = 10$, $\mu = 1$ and various values of n ($= 0.1, 0.3, 0.5$). It should be

noted that this figure has been taken from Figure 5 in order to show the difference of the critical loads between the cases of a pinned-pinned (Figure 5a) and a pinned-fixed (Figure 5b) column.

A three-dimensional graphical representation of β_{cr}^2 with respect to μ and $\bar{l}$, corresponding to a simply supported column and a fixed-hinged column is given for various values of n ($= 0.1, 0.3, 0.5$), in Figures 5a and 5b respectively. In these plots the projection of the curves (corresponding to $n = 0.1, 0.3, 0.5$) of the plane $\bar{l} = 0$, on the plane $\bar{l} = 10$ gives the intersection points F, C, D and E, as shown in Figure 3. Note that such a projection has not been included in Figures 5a and 5b, in order to avoid any confusion. From these figures it is evident that the effect of nonuniformity on the critical load is more pronounced for the fixed-hinged column than the corresponding simply supported one.

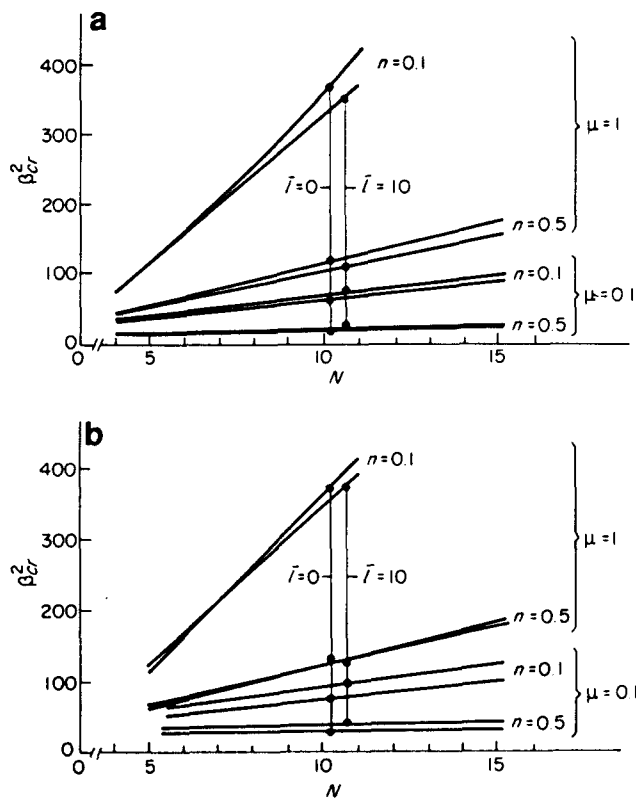


Figure 7 Critical loads β_{cr}^2 , for simply supported (a), and fixed-hinged (b) columns, as functions of N , n and μ

A reliable formula for establishing the critical load β_{cr}^2 of both types of columns is given as a function of the parameters μ , n , $\bar{l}$ and N ($N=5, 10, 15$), as follows

$$\beta_{cr}^2 = a_0 \mu^2 + b_0 \mu + c_0 \quad (6)$$

where the dimensionless coefficients a_0 , b_0 and c_0 are taken from the nomograms presented in Figure 6a (simply supported column) and Figure 6b (fixed-hinged column), for various values of $\bar{l}$ and n .

For instance, for a simply supported column corresponding to $\mu = 0.2$, $\bar{l} = 4$, $n = 0.1$ and $N = 15$, using Figure 6a we find $a_0 = -230$, $b_0 = 800$, $c_0 = 22$. Using equation (6) we obtained $\beta_{cr}^2 = 172.8$ which slightly differs from the value of the exact critical load $\beta_{cr}^2 = 165.4$. Similarly, for a propped column corresponding to $\mu = 0.4$, $\bar{l} = 6$, $n = 0.3$ and $N = 10$, using Figure 6b we find: $a_0 = -80$, $b_0 = 230$, $c_0 = 24$, and therefore $\beta_{cr}^2 = 103.2$ while the exact value is $\beta_{cr}^2 = 106.6$.

Considering columns with given length (l) and moment of inertia (I_c) one can study the dependence of

the critical load β_{cr}^2 versus N , μ , n and $\bar{l}$ for a simply supported column as well as for a fixed-hinged column, as shown in Figures 7a and 7b respectively. In both columns the increase in the number of panels implies an increase in the critical load. Such an increase is more pronounced for slender columns (i.e. $n = 0.1$) than it is for short ones ($n = 0.5$) regardless of the degree of nonuniformity of the cross-section. This is because the latter columns behave like slender frames.

Conclusions

The most important conclusions of this investigation based on two types of columns are as follows. First, there are ranges of variation of the stiffness and length ratio parameters for which the nonuniformity of the cross-section for both types of columns implies an increase in the load-carrying capacity. Such an increase of the load-carrying capacity may reach 10% for the simply supported column and 20% for the fixed-hinged column. Second, it should also be noted that the difference in the critical loads between these two types of columns is much less than the corresponding difference of critical loads of columns with solid cross-sections. Third, for large values of the ratio of the moments of inertia μ and for very small values of the length ratio n both types of nonuniform columns behave as those of solid cross-section. Finally, the cumbersome numerical evaluation for establishing the exact load-carrying capacity of both types of columns can be avoided by using a simple approximate but very reliable formula.

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