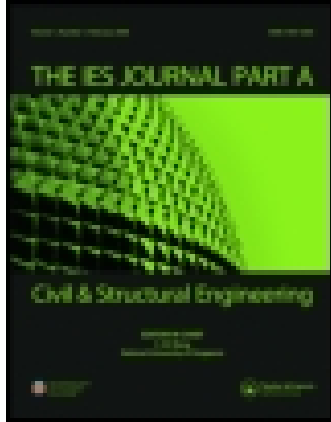


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The IES Journal Part A: Civil & Structural Engineering

Publication details, including instructions for authors and subscription information:

<http://www.tandfonline.com/loi/tiea20>

Flexural buckling of laced column with serpentine lattice

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Published online: 27 Jan 2010.

To cite this article: A.G. Razdolsky (2010) Flexural buckling of laced column with serpentine lattice, The IES Journal Part A: Civil & Structural Engineering, 3:1, 38-49, DOI: [10.1080/19373260903425477](https://doi.org/10.1080/19373260903425477)

To link to this article: <http://dx.doi.org/10.1080/19373260903425477>

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TECHNICAL PAPER

Flexural buckling of laced column with serpentine lattice

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(Received 2 July 2009; final version received 19 October 2009)

Unlike the technique accepted in existing design specifications, the buckling problem of a laced column with a serpentine lattice is formulated as a stability problem of a statically indeterminate structure. The problem is reduced to a two-point boundary value problem for a system of recurrence dependences relating the deformation parameters of column cross-sections passing through the lattice joints. These relations are derived by using the initial value method for solving differential equations of column chord equilibrium. For columns with any degree of static indeterminacy, the critical force is determined as the smallest eigenvalue of the fourth-order system of homogeneous linear algebraic equations. The obtained mode shapes have the form of irregular curves with many points of inflection and disprove a concept that the stability problem of a laced column can be reduced to the analogous problem for an 'equivalent' continuous solid column based on Engesser's assumption. Euler critical forces calculated for a column as a statically indeterminate system are compared with the critical forces from Engesser's equivalent solid column. The phenomenon which is similar to the Boobnov effect can occur for the serpentine column: it can lose stability so that panels of one of the chords are buckled as isolated simply supported bars. This type of buckling is possible when the lattice rigidity of the column exceeds a specific limit. For columns with identical chords, the critical force is a function of the number of sub-panels and the special lattice rigidity parameter. The relationships between the critical force and the lattice rigidity parameter for columns with a varied number of sub-panels can be applied in designing steel-laced columns.

Keywords: buckling; laced serpentine column; statically indeterminate structure

1. Introduction

In design practice, determination of the nominal critical force for a laced column in the direction parallel to the lattices (Ballio and Mazzolani 1983, Gaylord et al. 1992, Galambos 1998, American Institute of Steel Construction 2005) is reduced to calculation of Euler's critical force for the 'equivalent' solid pin-ended column. To take into account the sensitivity of laced columns to shear deformation, the component depending on lattice parameters is introduced in the expression of the slenderness ratio for the equivalent column. This approach (Bleich 1952, Timoshenko and Gere 1961) that has been suggested by Engesser (1891) recognises the possibility of the only sinusoidal half-wave buckling mode shape. In actual fact, laced columns are highly redundant systems, and the loss of column stability can occur by various buckling mode shapes depending on correlation between chord rigidity and lattice rigidity. The stability analysis of systems with a large number of static indeterminacy degrees by traditional methods requires determination of the smallest eigenvalue for a linear algebraic equation system of a higher order.

However, a buckling problem for a laced column with any degree of static indeterminacy can be formulated as a two-point boundary value problem for the eighth-order system of linear recurrence relations (Razdolsky 2005, 2008). As a result of this formulation of the problem, a determination of the critical force is reduced to finding the smallest eigenvalue for the fourth-order system of linear algebraic equations. This technique of solving the static stability problem for the laced column with a crosswise lattice is represented in the author's preceding articles: the torsional buckling problem of the column and the static-geometric analogy between the torsional and flexural buckling of the column were considered by Razdolsky (2005); the flexural buckling of the column was studied by Razdolsky (2008). This article is devoted to the flexural buckling problem of laced columns with a serpentine lattice. Euler critical forces and buckling mode shapes are determined for the column as a statically indeterminate system. The obtained critical forces are compared with those following from Engesser's model of the equivalent solid column. Rigidity properties of the serpentine lattice are described by a special

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parameter. The critical force of the column can be represented as a function of this parameter, and the number of sub-panels in which the column chords are divided by the lattice joints. The graphic representation of this relationship for columns with a varied number of sub-panels can be applied in designing steel-laced columns. Similar relationships for laced columns with a crosswise lattice have been derived in (Razdolsky 2008).

2. Relations between deflections/forces at bound cross-sections of chord sub-panels

Consider a laced column, which consists of two longitudinal chords 1 and 2 connected by two mutually parallel lattices in one. Each chord has a solid cross-section with at least a single axis of symmetry. This axis coincides with the axis of symmetry of the whole column cross-section and is parallel to the lattice planes. In actual practice, the serpentine lattice is one of the most frequently employed lattice types. Each lattice brace connects two joint cross-sections located on the opposite chords. We assume that the joint cross-section of the chord is located between the ascending and descending braces arriving at the joint. In design practice, it is usually assumed that the joints of the lattice make up hinges in the lattice plane. Sections of the chords between adjacent joints are named as panels. We shall distinguish identical quantities corresponding to the different chords by means of subscript indices $i = 1, 2$, where $i = 1$ refers to the left chord. For laced column with serpentine lattice, each column cross-section, which passes through the lattice joint located on one of the chords inside the column, divides the corresponding panel of the opposite chord into two sub-panels (Figure 1). The column cross-sections passing through the lattice joints divide each column chord along its length into n sub-panels. These cross-sections are shown in Figure 1 by dotted lines. We shall name a chord cross-section which divides the panel into sub-panels as off-joint cross-section. Introduce an ordinal numeration $k = 0, 1, 2, \dots, n$ for the joint and off-joint cross-sections of each chord. Note that numeration is taken for each chord separately. Assume that both the column lattices start at the lower (initial) cross-section of the left chord. A length of sub-panels can change along the column length. However, we have denoted the sub-panel length as d . Both the chords are supported at the end cross-sections $k = 0$ and $k = n$. Each chord is compressed by an axial force N_i .

A local Cartesian orthogonal coordinate system is adopted for each chord. It is formed by the principal axis of inertia $O_i Y$ of the cross-section and by the longitudinal axis $O_i Z$ (Figure 1). It is assumed that the

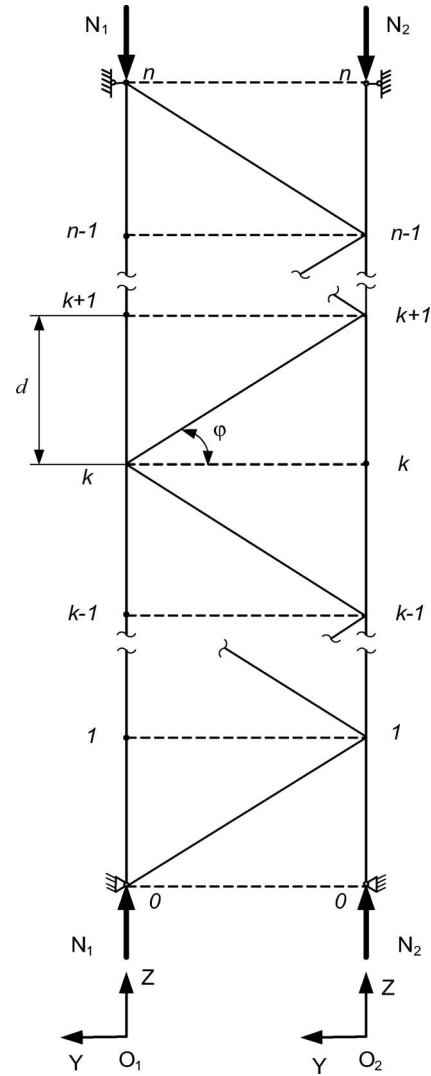


Figure 1. Laced column with serpentine lattice.

column chords can lose stability in the direction of axis $O_i Y$. The virtual displacements of the column chords in the critical state are defined by the well-known differential equation for the bar subjected to axial compression (Brush and Almroth 1975). We denote the displacement, its derivative, the bending moment and the transverse force for the joint and off-joint cross-sections by symbols $y_i(k)$, $y'_i(k)$, $M_i(k)$ and $Q_i(k)$, respectively, where i is a chord number and k is a cross-section number. The coordinate along the axis $O_i Z$ is denoted by the symbol z . The transverse force is supposed to direct normally to the chord axis in the initial unstressed state. In contrast to the common transverse shearing force, the expression for this transverse force includes the component of the axial force that is directed normally to the mentioned chord axis (Rzhanitsyn 1982). The displacement, its

derivative and the bending moment are continuous at the joint cross-sections of the chord. The transverse force takes a constant value inside the sub-panel but undergoes a jump at the joint cross-sections of the chord. The jump is caused by the effect of the braces meeting at the joint. The lateral force along the axis $O_i Y$ applied to the joint cross-section k of the chord i from the two parallel braces connecting this cross-section with the joint cross-section j of the opposite chord (Figure 2) is defined by the displacements of these cross-sections and the rigidity characteristics of the connecting braces (Razdolsky 2005)

$$R_i(k, j) = -2c_i(k, j)[y_i(k) - y_{3-i}(j)] \quad (1)$$

where $y_{3-i}(j)$ is the displacement of the joint cross-section j of the chord $3-i$, which is opposite to the chord i ; $c_i(k, j) = EF_i(k, j) \cos^2 \varphi_i(k, j)/l_i(k, j)$ is the brace rigidity characteristic; E is the Young's modulus; $F_i(k, j)$, $l_i(k, j)$ and $\varphi_i(k, j)$ are, respectively, the cross-sectional area, length and inclination of the brace, connecting the joint cross-section k of the chord i with the joint cross-section j of the opposite chord. A uniformity of expressions including characteristics of both chords at the same time is attained by using notation i and $3-i$ for the chord indices. The braces of the front column lattice only are shown in Figure 2. In the case of a sub-panel located between the joint and off-joint cross-sections of the chord, we shall take that the initial cross-section of the sub-panel immediately following the ascending brace.

Consider a sub-panel of the chord i between the cross-sections k and $k+1$. The values of the

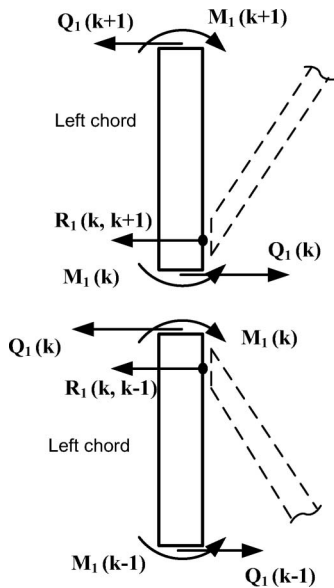


Figure 2. Transverse forces and bending moments at joint cross-sections of left column chord.

displacement, its derivative and the bending moment at the initial cross-section of the sub-panel are

$$y_{0i}(k, k+1) = y_i(k) \quad (2)$$

$$y'_{0i}(k, k+1) = y'_i(k) \quad (3)$$

$$M_{0i}(k, k+1) = M_i(k). \quad (4)$$

When a sub-panel starts from a joint cross-section, the transverse force at the initial cross-section of the sub-panel is given by

$$Q_{0i}(k, k+1) = Q_i(k) - R_i(k, k+1). \quad (5)$$

Recall that the joint cross-section of the chord is located between the ascending and descending braces arriving at the joint. When a sub-panel starts from an off-joint cross-section j , the transverse force at the initial cross-section of the sub-panel is

$$Q_{0i}(j, j+1) = Q_i(j). \quad (6)$$

The transverse force inside of any sub-panel takes the constant value

$$Q_i(k, k+1) = Q_{0i}(k, k+1) \quad (7)$$

and is expressed through the transverse force at the preceding sub-panel. If the cross-sections k and $k-1$ are the joint and off-joint cross-sections, respectively, then

$$\begin{aligned} Q_i(k) &= Q_i(k-1, k) - R_i(k, k-1) \\ &= Q_{0i}(k-1, k) - R_i(k, k-1) \end{aligned} \quad (8)$$

$$Q_{0i}(k-1, k) = Q_i(k-1). \quad (9)$$

The expression of the transverse force at the joint cross-section takes a form

$$Q_i(k) = Q_i(k-1) - R_i(k, k-1). \quad (10)$$

The following non-dimensional notations are used

$$\begin{aligned} \zeta &= \frac{z}{d}, \xi_i = \frac{y_i}{d}, v_i = \frac{d\xi_i}{d\zeta} = \xi'_i, \\ \Gamma_i &= \frac{M_i d}{EJ_i}, \Lambda_i = \frac{Q_i d^2}{EJ_i}, \Omega_i = \sqrt{\frac{N_i}{EJ_i}} d = v_i d \end{aligned} \quad (11)$$

and the non-dimensional lattice rigidity parameter is given by

$$\alpha_i(k, j) = \frac{2c_i(k, j)d^3}{EJ_i}. \quad (12)$$

This parameter can be represented as

$$\alpha_i(k, j) = \frac{F_i(k, j)d^2}{J_i} \sin 2\varphi_i(k, j) \cos \varphi_i(k, j). \quad (13)$$

The non-dimensional deformation parameters ξ_i , v_i and Γ_i at the chord cross-section $k + 1$ can be expressed through parameters $\xi_i(k)$, $v_i(k)$ and $\Gamma_i(k)$ at the cross-section k by using relations of the initial value method (Rzhanitsyn 1982)

$$\begin{aligned} \xi_i(k+1) = & \xi_i(k) + v_i(k) \frac{\sin \Omega_i}{\Omega_i} - \Gamma_i(k) \frac{1 - \cos \Omega_i}{\Omega_i^2} \\ & - \Lambda_{0i}(k, k+1) \frac{\Omega_i - \sin \Omega_i}{\Omega_i^3} \end{aligned} \quad (14)$$

$$\begin{aligned} v_i(k+1) = & v_i(k) \cos \Omega_i - \Gamma_i(k) \frac{\sin \Omega_i}{\Omega_i} \\ & - \Lambda_{0i}(k, k+1) \frac{1 - \cos \Omega_i}{\Omega_i^2} \end{aligned} \quad (15)$$

$$\begin{aligned} \Gamma_i(k+1) = & v_i(k) \Omega_i \sin \Omega_i + \Gamma_i(k) \cos \Omega_i \\ & + \Lambda_{0i}(k, k+1) \frac{\sin \Omega_i}{\Omega_i}. \end{aligned} \quad (16)$$

The non-dimensional transverse forces at the initial cross-section of the sub-panels that start from the joint and off-joint cross-sections, respectively, are

$$\Lambda_{0i}(k, k+1) = \Lambda_i(k) + \alpha_i(k, k+1)[\xi_i(k) - \xi_{3-i}(k+1)] \quad (17)$$

$$\Lambda_{0i}(k-1, k) = \Lambda_i(k-1). \quad (18)$$

The non-dimensional transverse forces at the joint and off-joint cross-sections, respectively, are defined by the following expressions

$$\Lambda_i(k) = \Lambda_i(k-1) + \alpha_i(k, k-1)[\xi_i(k) - \xi_{3-i}(k-1)] \quad (19)$$

$$\Lambda_i(k-1) = \Lambda_{0i}(k-2, k-1). \quad (20)$$

Since the serpentine lattice starts at the initial cross-section of the left chord (Figure 1), the joint cross-sections of the left chord are even-numbered and the joint cross-sections of the right chord are odd-numbered.

3. Reducing buckling problem to a boundary value problem

We introduce the following vector notation

$$\mathbf{V}(k) = \{\xi_1(k), v_1(k), \Gamma_1(k), \Lambda_1(k), \xi_2(k), v_2(k), \Gamma_2(k), \Lambda_2(k)\}^T \quad (21)$$

where superscript 'T' denotes the transposition operation of a vector. The recurrence relations between parameters $\xi_i(k+1)$, $v_i(k+1)$, $\Gamma_i(k+1)$ and $\Lambda_i(k+1)$ at the cross-section $k+1$ of each chord ($i = 1, 2$) and parameters $\xi_i(k)$, $v_i(k)$, $\Gamma_i(k)$ and $\Lambda_i(k)$ at the cross-sections k of both the chords can be derived from the expressions Equations (14)–(20). These relations may be represented in a matrix form

$$\mathbf{V}(k+1) = [B]\mathbf{V}(k) \quad (22)$$

where elements of the matrix $[B]$ are functions of the chord compression parameters Ω_1 and Ω_2 . These recurrence relations represent the eighth-order system of homogeneous difference equations. Such recurrence relations were first derived for a column with a crosswise lattice (Razdolsky 2005, 2008). The form of these relations for such column remains unchanged throughout a column length. For a column with a serpentine lattice, the form of recurrence relations Equation (22) varies along its length at regular $2d$ intervals because the cross-section k of any one of the chords is either the joint cross-section or the off-joint (Figure 1). Four possible forms of recurrence relations Equation (22) for a laced serpentine column are given in the Appendix. The buckling problem consists in the determination of the smallest axial compressive force at which the system Equation (22) has a nontrivial (non-zero) solution satisfying four conditions at the initial point (initial conditions) of the column and four boundary conditions at its endpoint. Thus, the buckling problem is reduced to a two-point boundary value problem for the homogeneous system of difference equations. The complete solution of the boundary value problem can be expressed as the linear combination of four particular solutions $\mathbf{V}_1(k)$, $\mathbf{V}_2(k)$, $\mathbf{V}_3(k)$ and $\mathbf{V}_4(k)$ of the Cauchy problem

$$\mathbf{V}(k) = A_1\mathbf{V}_1(k) + A_2\mathbf{V}_2(k) + A_3\mathbf{V}_3(k) + A_4\mathbf{V}_4(k) \quad (23)$$

where the particular solutions satisfy the given initial conditions. The coefficients A_1 , A_2 , A_3 and A_4 must be chosen so that the complete solution satisfies the given

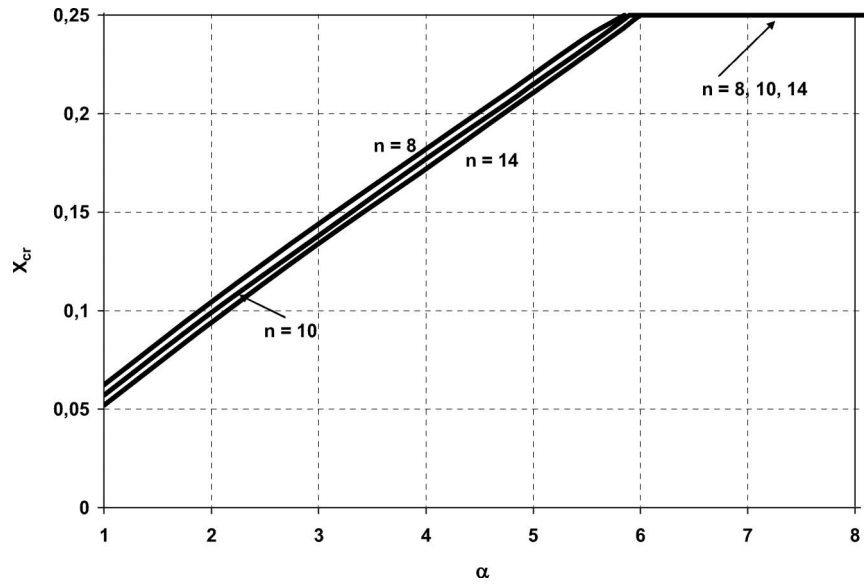


Figure 3. Plots of relative critical force for columns with number of sub-panels $n = 8, 10$.

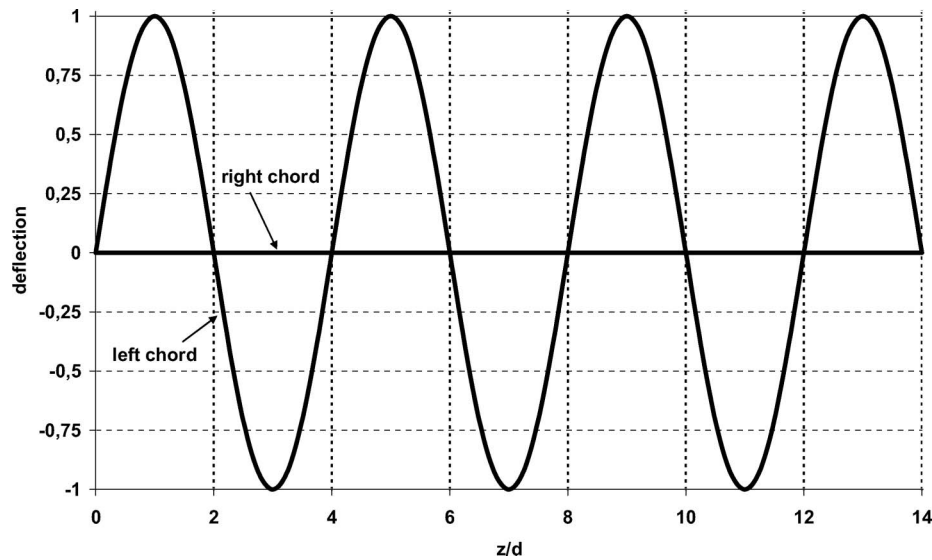


Figure 4. Buckling mode shape of column without displacements of joints (number of sub-panels, $n = 12$; lattice rigidity parameter, $\alpha > 6$, relative critical force $X_{cr} = 0.25$).

boundary conditions at the endpoint (Roberts and Shipman 1972). The values of four linearly independent particular solutions (of vectors $\mathbf{V}_i$) satisfying four given initial conditions are calculated at the column endpoint by a repeated use of the obtained recurrence relations. In order to prevent the loss of linear independence of the particular solutions during calculations, while the current cross-section is moving away from the initial point, a vector re-orthogonalisation must be applied (Godunov 1961, Conte 1966). The substitution of solution (23) in the boundary conditions at the endpoint of the column leads to a system of four homogeneous linear algebraic equations with respect to constants A_1 ,

A_2 , A_3 and A_4 . For a nontrivial solution of this system to exist, the determinant of the coefficients of this system must equal zero. The smallest non-zero value of the axial compressive force is the Euler critical value. The numerical technique for solving the present two-point boundary value problem has been described more fully in the author's previous article (Razdolsky 2005).

4. Critical forces and buckling mode shapes of column

The serpentine lattice does not possess axial symmetry. Therefore, the buckling mode shape is different for the left and right chord of a serpentine column. This fact

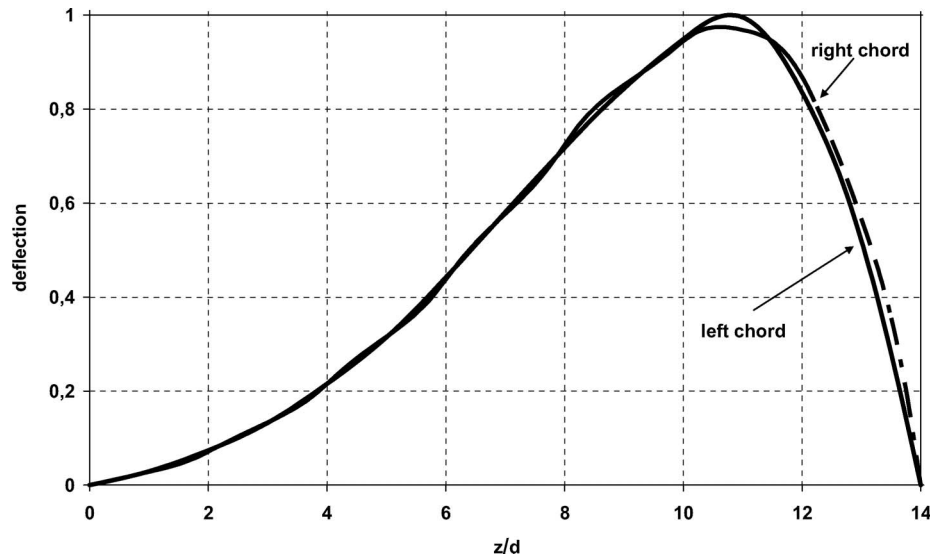


Figure 5. Buckling mode shapes of chords for column with small lattice rigidity (number of sub-panels $n = 14$, lattice rigidity parameter $\alpha = 5$, relative column critical force $X_{cr} = 0.2106$).

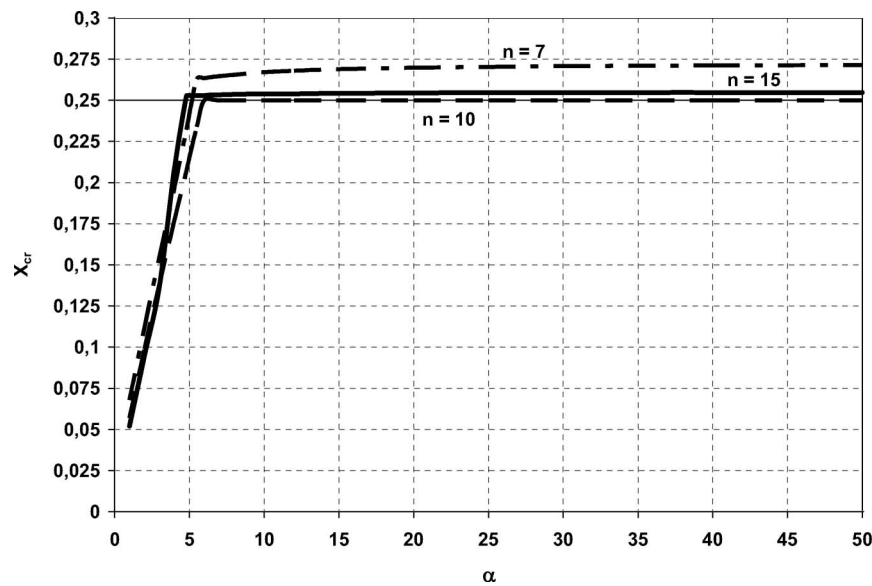


Figure 6. Plots of relative critical force for columns with number of sub-panels $n = 7, 10, 15$.

can be found only by a consideration of column as statically indeterminate system. The solving of the buckling problem for the serpentine laced column is illustrated by using examples with the assumption that column chords are identical and compressed by the same forces. The lattice braces and their inclination are assumed to be the same. The design specifications proceed from the analogous assumption. Because of these assumptions, the lattice rigidity parameter takes a constant value throughout the column length

$$\alpha = \frac{Fd^2}{J} \sin 2\varphi \cos \varphi \quad (24)$$

where the subscript indices for distinguishing the geometric characteristics of the chords and braces may be omitted. Both the chords are assumed to be simply supported at the ends. The critical force for column with any number of sub-panels can be represented as a function of the lattice rigidity parameter. Plots of this function may be used in designing the steel-laced columns. It is appropriate to express the relative critical force through the critical force value N_0 corresponding to flexural buckling of the isolated chord sub-panel

$$X_{cr} = \frac{N_{cr}}{N_0} = \frac{N_{cr}d^2}{\pi^2 EJ}. \quad (25)$$

Recall that each chord at the critical state is compressed by the force N_{cr} . The column chord may contain either entire panels only (the even number of sub-panels), or several entire panels and one or two sub-panels. A figure showing the loss of stability for a column with the even number of sub-panels differs from a figure for a column with the odd number of sub-panels.

Typical plots of the critical force for the column with the even number of chord sub-panels ($n = 8, 10, 14$) are shown in Figure 3. When parameter $\alpha > 6$, the panels of the left chord of the serpentine column lose stability as isolated simply supported bars of length $2d$

(the joint cross-sections are not displaced) (see Figure 4). The right chord remains straight since it has more intermediate lattice joints than the left chord and the arrangement of joints on the chord prevents the division of the chord into panels of length $2d$ without displacements of joints. The relative critical force is defined by the Euler formula Eq. (25) and is equal to $X_{cr} = 0.25$. This phenomenon is analogous to the Boobnov effect for a simply supported compressed solid bar with a number of intermediate elastic supports (Timoshenko and Gere 1961). However, in contrast to the classical Boobnov example, such buckling mode

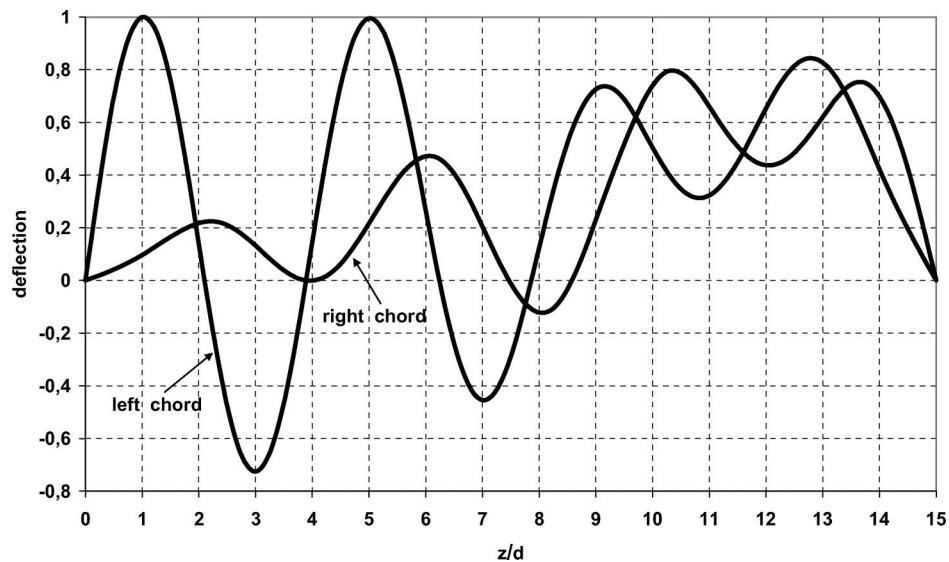


Figure 7. Buckling mode shapes of chords for column with small and medium-size lattice rigidity (number of sub-panels $n = 15$, lattice rigidity parameter $\alpha = 5$, relative column critical force $X_{cr} = 0.2529$).

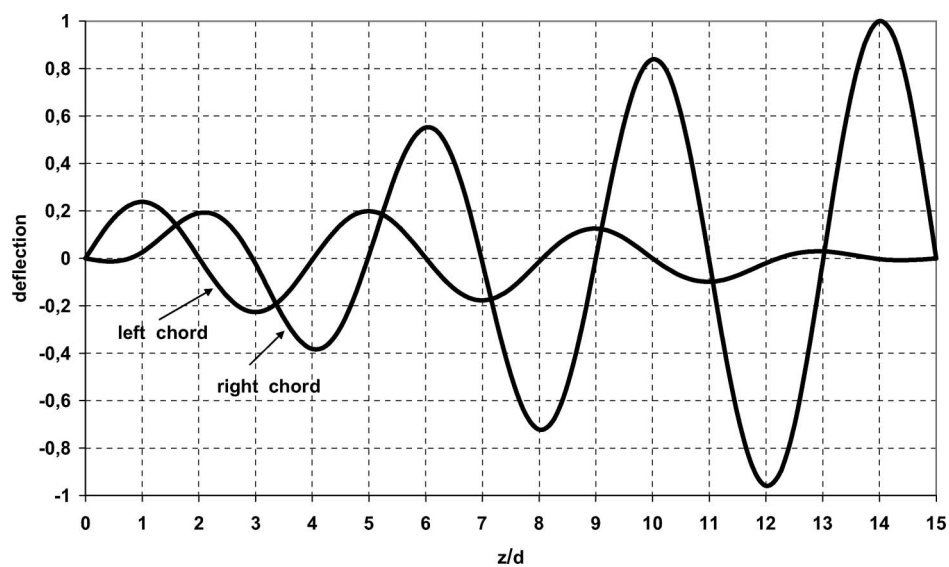


Figure 8. Buckling mode shapes of chords for column with medium-size lattice rigidity (number of sub-panels $n = 15$, lattice rigidity parameter $\alpha = 5$, relative column critical force $X_{cr} = 0.2544$).

shape is due to internal constraints of the structure. When the lattice rigidity parameter is less than 5.7 – 6.0 (Figure 3), the critical force increases with increase of this parameter and the loss of column stability is

Table 1. Sub-panel geometric parameters of column samples.

A (cm ²)	J (cm ⁴)	d (cm)	ϕ (°)	γ
100	1040	100	45	0.00415
118	1360	110	45	0.00380
138	1730	120	45	0.00348

accompanied by displacements of the intermediate lattice joints. The buckling mode shapes of both the column chords with number of sub-panels $n = 14$ at the rigidity parameter $\alpha = 5$ are shown in Figure 5. The relative critical force of the column is $X_{cr} = 0.2106$. The buckling deflections of both the chords are nearly equal in magnitude and directed on the same side.

In the case of an odd number of sub-panels (the column chords contain several entire panels and sub-panel), it is impossible to generate a deflection mode shape of the chord from those of simply supported bars of length $2d$, and the critical force of the column

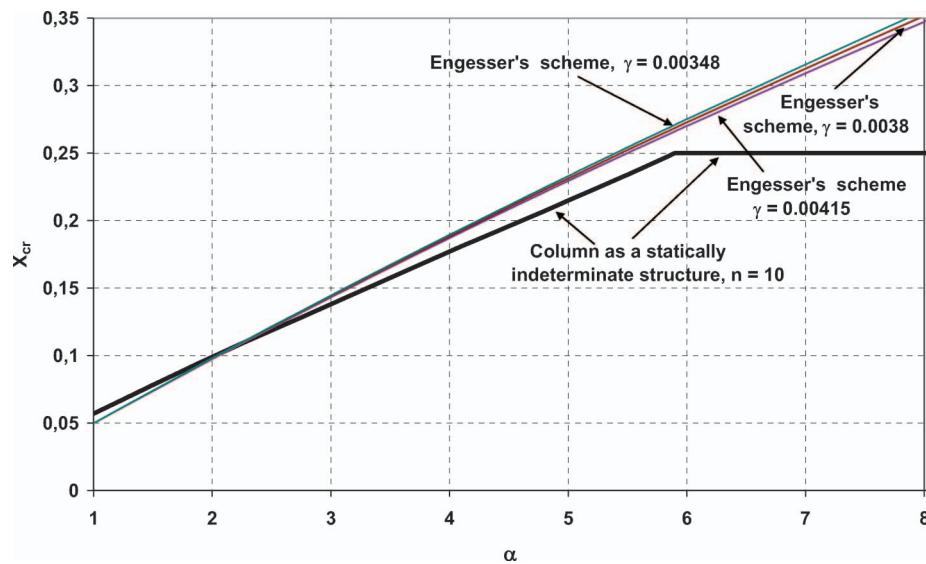


Figure 9. Comparison of critical forces for column as a statically indeterminate structure and Engesser's model of equivalent solid column, number of sub-panels $n = 10$.

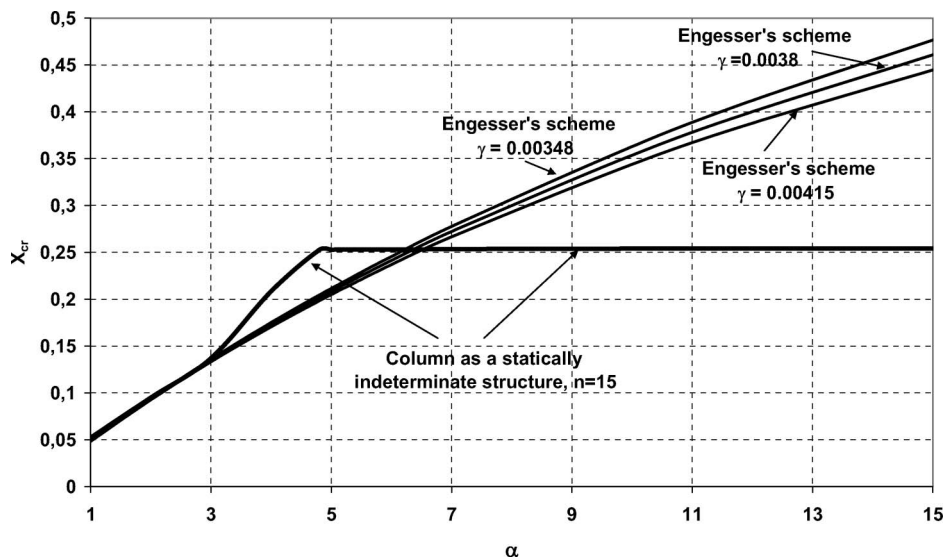


Figure 10. Comparison of critical forces for column as a statically indeterminate structure and Engesser's model of equivalent solid column, number of sub-panels $n = 15$.

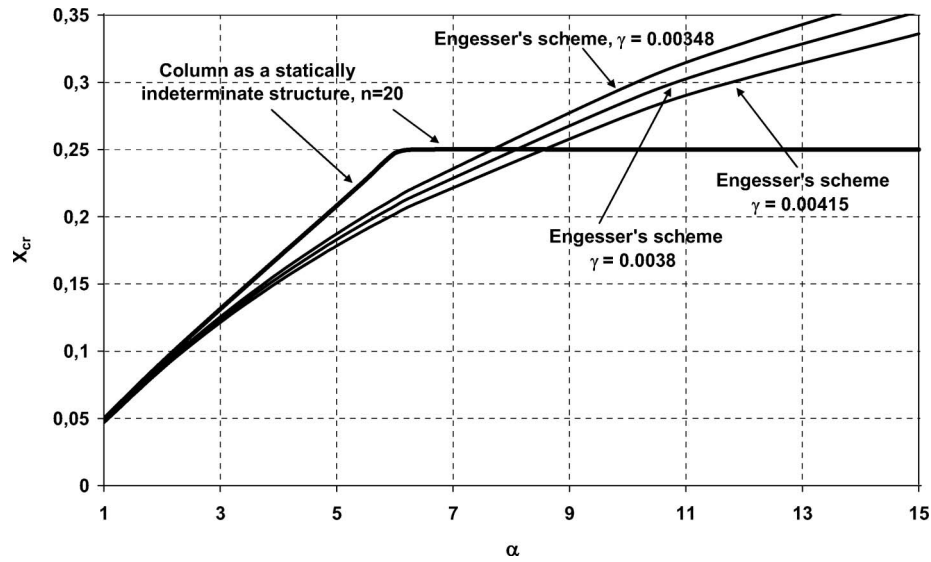


Figure 11. Comparison of critical forces for column as a statically indeterminate structure and Engesser's model of equivalent solid column, number of sub-panels $n = 20$.

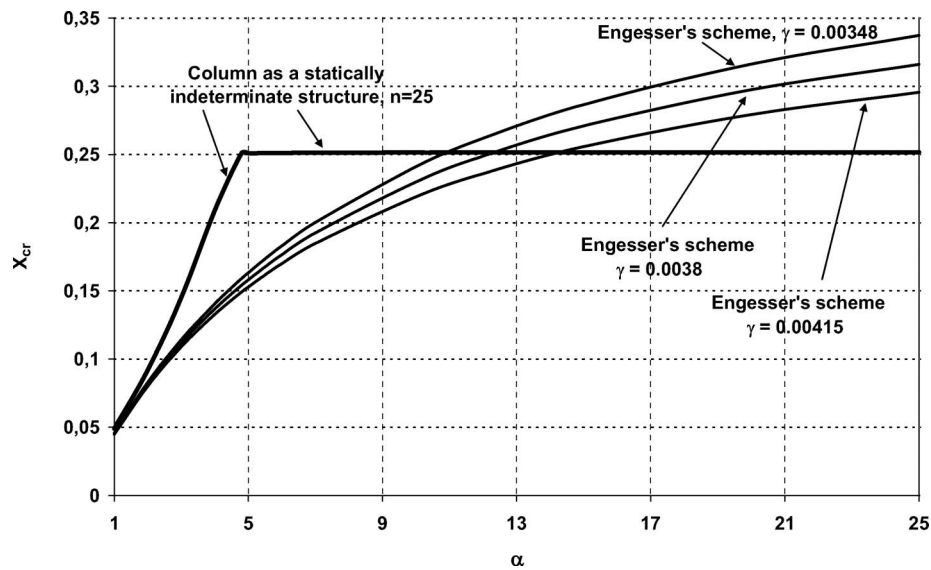


Figure 12. Comparison of critical forces for column as a statically indeterminate structure and Engesser's model of equivalent solid column, number of sub-panels $n = 25$.

can exceed the critical force of the isolated simply supported bar of a length $2d$. The plots of the relative critical force for columns with the number of sub-panels $n = 7, 10, 15$ are shown in Figure 6. The curve of the critical force for column with $n = 15$ is little more than the critical force of the isolated simply supported chord panel of length $2d$, when the lattice rigidity parameter is more than $\alpha = 4.8$. However, the critical force for the column with $n = 7$ can more significantly exceed this force value and reaches $X_{cr} = 0.2715$ when the lattice rigidity parameter

$\alpha = 50$. The buckling mode shapes of both column chords with number of sub-panels $n = 15$ for the columns with small and medium-size lattice rigidity are shown in Figures 7 and 8. In the case of the weak lattice ($\alpha = 5$, $X_{cr} = 0.2529$), the buckling mode shapes of both chords have a tendency to deflection in the same side (Figure 7). A picture of the column loss stability which is close to the Boobnov effect occurs when the lattice rigidity increases: intermediate joint cross-sections of both chords are not displaced except for those closest to the chord ends. A typical

example of such buckling is shown in Figure 8 for column with $\alpha = 15$; the relative critical force is $X_{cr} = 0.2544$.

We correlate the critical forces calculated for the column as a statically indeterminate structure with the critical forces corresponding to Engesser's model. A relative value of the Engesser critical force acting upon the chord of column is described by the formula (Timoshenko and Gere 1961, Section 2.18)

$$\frac{N_*}{N_0} = \frac{I}{2n^2J} \left[1 + \frac{\pi^2}{2n^2 \sin \varphi \cos^2 \varphi} \frac{I}{Fd^2} \right]^{-1} = \left(n^2 \gamma + \frac{2\pi^2}{\alpha} \right)^{-1} \quad (26)$$

where $I = 2J + 0.5A (d/\text{tg } \varphi)^2$ is a moment of inertia of the whole column cross-section, A is an area of the chord cross-section, and γ is a non-dimensional geometric parameter of column sub-panels,

$$\gamma = \left[1 + \frac{A}{J} \left(\frac{d}{2\text{tg } \varphi} \right)^2 \right]^{-1}. \quad (27)$$

We shall carry out the comparison of the critical forces for three values of parameter γ that correspond to column samples generated by the chords in the form of rolled steel I-beams. The parameters of these columns are represented in Table 1. The plots of the critical forces for the columns with the number of sub-panels $n = 10, 15, 20$ and 25 are shown in Figures 9–12. Analysis of these plots shows that the loss of stability of column as a statically indeterminate structure precedes the loss of stability of Engesser's model with the exception of the small-sized region of small values of the rigidity parameter α . Over the most part of a region of the lattice rigidity parameter, Engesser's approach gives a physically inadmissible values of the critical force which significantly exceed the critical force of the isolated chord panel being considered as a simply supported bar.

5. Conclusions

In contrast to the technique accepted in the design specifications now in force, the flexural buckling problem of a laced column with a serpentine lattice is formulated as a stability problem of a statically indeterminate structure. The problem is reduced to a two-point boundary value problem for a system of recurrence dependences relating the deformation parameters of column cross-sections passing through the lattice joints. The critical force for column with any degree of static indeterminacy is determined as

the smallest eigenvalue of the system of linear algebraic equations. In the general case, the buckling mode shapes for column as a statically indeterminate structure is non-sinusoidal, irregular and contain points of inflection. This fact disproves a concept that the stability problem of the laced column can be reduced to the analogous problem for a continuous solid column. The buckling mode shapes of the column chords depend on the number and location of intermediate lattice joints on the chord and the lattice rigidity of the column. The column chords may contain either the entire panels only, or entire panels and sub-panels. A picture of the loss of stability for a column with the even number of sub-panels differs from a picture for a column with the odd number of sub-panels. The column can lose stability so that panels of one of the chords are deflected as isolated simply supported bars while the other chord remains straight. This phenomenon is analogous to the Boobnov effect for a simply supported compressed solid bar with a number of intermediate elastic supports. However, in contrast to the classical Boobnov example, such buckling mode shape is due to internal constraints of the structure and is possible when the lattice rigidity of the column exceeds a specific limit. For columns with the identical chords, the critical force is a function of the number of sub-panels and the special lattice rigidity parameter. Euler critical forces for columns as statically indeterminate structures are compared with the critical forces following from Engesser's model of the equivalent solid column. The loss of stability of column as a statically indeterminate structure precedes the loss of stability of Engesser's model with the exception of a small-sized region of small values of the lattice rigidity parameter. The obtained relationships between the critical force and the lattice rigidity parameter for columns with a varied number of sub-panels can be validly applied in designing steel-laced columns.

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Appendix. System of recurrence relations (Equation (22)) between deflections/forces at bound cross-sections of chord sub-panels of column

- (a) Relations for even cross-sections ($k = 0, 2, 4 \dots$) of the left column chord:

$$\begin{aligned} \xi_1(k+1) &= \xi_1(k) \left[1 - \alpha_1(k, k+1) \frac{\Omega_1 - \sin \Omega_1}{\Omega_1^3} \right] \\ &+ v_1(k) \frac{\sin \Omega_1}{\Omega_1} + \Gamma_1(k) \frac{1 - \cos \Omega_1}{\Omega_1^2} + \Lambda_1(k) \frac{\Omega_1 - \sin \Omega_1}{\Omega_1^3} \\ &+ \alpha_1(k, k+1) \frac{\Omega_1 - \sin \Omega_1}{\Omega_1^3} \\ &\times \left[\xi_2(k) + v_2(k) \frac{\sin \Omega_2}{\Omega_2} + \Gamma_2(k) \frac{1 - \cos \Omega_2}{\Omega_2^2} + \Lambda_2(k) \frac{\Omega_2 - \sin \Omega_2}{\Omega_2^3} \right] \end{aligned} \quad (A1)$$

$$\begin{aligned} v_1(k+1) &= -\xi_1(k) \alpha_1(k, k+1) \frac{1 - \cos \Omega_1}{\Omega_1^2} + v_1(k) \cos \Omega_1 \\ &+ \Gamma_1(k) \frac{\sin \Omega_1}{\Omega_1} + \Lambda_1(k) \frac{1 - \cos \Omega_1}{\Omega_1^2} + \alpha_1(k, k+1) \frac{1 - \cos \Omega_1}{\Omega_1^2} \\ &\times \left[\xi_2(k) + v_2(k) \frac{\sin \Omega_2}{\Omega_2} + \Gamma_2(k) \frac{1 - \cos \Omega_2}{\Omega_2^2} + \Lambda_2(k) \frac{\Omega_2 - \sin \Omega_2}{\Omega_2^3} \right] \end{aligned} \quad (A2)$$

$$\begin{aligned} \Gamma_1(k+1) &= -\xi_1(k) \alpha_1(k, k+1) \frac{\sin \Omega_1}{\Omega_1} - v_1(k) \Omega_1 \sin \Omega_1 \\ &+ \Gamma_1(k) \cos \Omega_1 + \Lambda_1(k) \frac{\sin \Omega_1}{\Omega_1} + \alpha_1(k, k+1) \frac{\sin \Omega_1}{\Omega_1} \\ &\times \left[\xi_2(k) + v_2(k) \frac{\sin \Omega_2}{\Omega_2} + \Gamma_2(k) \frac{1 - \cos \Omega_2}{\Omega_2^2} + \Lambda_2(k) \frac{\Omega_2 - \sin \Omega_2}{\Omega_2^3} \right] \end{aligned} \quad (A3)$$

$$\begin{aligned} \Lambda_1(k+1) &= \Lambda_1(k) + \alpha_1(k, k+1) \\ &\times \left[\xi_2(k) + v_2(k) \frac{\sin \Omega_2}{\Omega_2} + \Gamma_2(k) \frac{1 - \cos \Omega_2}{\Omega_2^2} \right. \\ &\left. + \Lambda_2(k) \frac{\Omega_2 - \sin \Omega_2}{\Omega_2^3} - \xi_1(k) \right]. \end{aligned} \quad (A4)$$

- (b) Relations for odd cross sections ($k = 1, 3, 5 \dots$) of the left column chord:

$$\begin{aligned} \xi_1(k+1) &= \xi_1(k) + v_1(k) \frac{\sin \Omega_1}{\Omega_1} + \Gamma_1(k) \frac{1 - \cos \Omega_1}{\Omega_1^2} \\ &+ \Lambda_1(k) \frac{\Omega_1 - \sin \Omega_1}{\Omega_1^3} \end{aligned} \quad (A5)$$

$$v_1(k+1) = v_1(k) \cos \Omega_1 + \Gamma_1(k) \frac{\sin \Omega_1}{\Omega_1} + \Lambda_1(k) \frac{1 - \cos \Omega_1}{\Omega_1^2} \quad (A6)$$

$$\Gamma_1(k+1) = -v_1(k) \Omega_1 \sin \Omega_1 + \Gamma_1(k) \cos \Omega_1 + \Lambda_1(k) \frac{\sin \Omega_1}{\Omega_1} \quad (A7)$$

$$\begin{aligned} \Lambda_1(k+1) &= \Lambda_1(k) - \alpha_1(k+1, k) \\ &\times \left[\xi_1(k) + v_1(k) \frac{\sin \Omega_1}{\Omega_1} + \Gamma_1(k) \frac{1 - \cos \Omega_1}{\Omega_1^2} \right. \\ &\left. + \Lambda_1(k) \frac{\Omega_1 - \sin \Omega_1}{\Omega_1^3} - \xi_2(k) \right]. \end{aligned} \quad (A8)$$

- (c) Relations for even cross sections ($k = 0, 2, 4 \dots$) of the right column chord:

$$\begin{aligned} \xi_2(k+1) &= \xi_2(k) + v_2(k) \frac{\sin \Omega_2}{\Omega_2} + \Gamma_2(k) \frac{1 - \cos \Omega_2}{\Omega_2^2} \\ &+ \Lambda_2(k) \frac{\Omega_2 - \sin \Omega_2}{\Omega_2^3} \end{aligned} \quad (A9)$$

$$v_2(k+1) = v_2(k) \cos \Omega_2 + \Gamma_2(k) \frac{\sin \Omega_2}{\Omega_2} + \Lambda_2(k) \frac{1 - \cos \Omega_2}{\Omega_2^2} \quad (A10)$$

$$\Gamma_2(k+1) = -v_2(k) \Omega_2 \sin \Omega_2 + \Gamma_2(k) \cos \Omega_2 + \Lambda_2(k) \frac{\sin \Omega_2}{\Omega_2} \quad (A11)$$

$$\begin{aligned} \Lambda_2(k+1) &= \Lambda_2(k) - \alpha_2(k+1, k) \\ &\times \left[\xi_2(k) + v_2(k) \frac{\sin \Omega_2}{\Omega_2} + \Gamma_2(k) \frac{1 - \cos \Omega_2}{\Omega_2^2} \right. \\ &\left. + \Lambda_2(k) \frac{\Omega_2 - \sin \Omega_2}{\Omega_2^3} - \xi_1(k) \right]. \end{aligned} \quad (A12)$$

(d) Relations for odd cross sections ($k = 1, 3, 5 \dots$) of the right column chord:

$$\begin{aligned} & \xi_2(k+1) \\ &= \xi_2(k) \left[1 - \alpha_2(k, k+1) \frac{\Omega_2 - \sin \Omega_2}{\Omega_i^3} \right] + v_2(k) \frac{\sin \Omega_2}{\Omega_2} \\ &+ \Gamma_2(k) \frac{1 - \cos \Omega_2}{\Omega_2^2} + \Lambda_2(k) \frac{\Omega_2 - \sin \Omega_2}{\Omega_i^3} + \alpha_2(k, k+1) \frac{\Omega_2 - \sin \Omega_2}{\Omega_i^3} \\ &\times \left[\xi_1(k) + v_1(k) \frac{\sin \Omega_1}{\Omega_1} + \Gamma_1(k) \frac{1 - \cos \Omega_1}{\Omega_1^2} + \Lambda_1(k) \frac{\Omega_1 - \sin \Omega_1}{\Omega_1^3} \right] \end{aligned} \quad (\text{A13})$$

$$\begin{aligned} & v_2(k+1) \\ &= -\xi_2(k) \alpha_2(k, k+1) \frac{1 - \cos \Omega_2}{\Omega_2^2} + v_2(k) \cos \Omega_2 \\ &+ \Gamma_2(k) \frac{\sin \Omega_2}{\Omega_2} + \Lambda_2(k) \frac{1 - \cos \Omega_2}{\Omega_2^2} + \alpha_2(k, k+1) \frac{1 - \cos \Omega_2}{\Omega_2^2} \\ &\times \left[\xi_1(k) + v_1(k) \frac{\sin \Omega_1}{\Omega_1} + \Gamma_1(k) \frac{1 - \cos \Omega_1}{\Omega_1^2} + \Lambda_1(k) \frac{\Omega_1 - \sin \Omega_1}{\Omega_1^3} \right] \end{aligned} \quad (\text{A14})$$

$$\begin{aligned} & \Gamma_2(k+1) \\ &= -\xi_2(k) \alpha_2(k, k+1) \frac{\sin \Omega_2}{\Omega_2} - v_2(k) \Omega_2 \sin \Omega_2 \\ &+ \Gamma_2(k) \cos \Omega_2 + \Lambda_2(k) \frac{\sin \Omega_2}{\Omega_2} + \alpha_2(k, k+1) \frac{\sin \Omega_2}{\Omega_2} \\ &\times \left[\xi_1(k) + v_1(k) \frac{\sin \Omega_1}{\Omega_1} + \Gamma_1(k) \frac{1 - \cos \Omega_1}{\Omega_1^2} + \Lambda_1(k) \frac{\Omega_1 - \sin \Omega_1}{\Omega_1^3} \right] \end{aligned} \quad (\text{A15})$$

$$\begin{aligned} & \Lambda_2(k+1) \\ &= \Lambda_2(k) + \alpha_2(k, k+1) \\ &\times \left[\xi_1(k) + v_1(k) \frac{\sin \Omega_1}{\Omega_1} + \Gamma_1(k) \frac{1 - \cos \Omega_1}{\Omega_1^2} \right. \\ &\quad \left. + \Lambda_1(k) \frac{\Omega_1 - \sin \Omega_1}{\Omega_1^3} - \xi_2(k) \right]. \end{aligned} \quad (\text{A16})$$