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Calculation of slenderness ratio for laced columns with serpentine and crosswise lattices

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ABSTRACT

In contrast to the technique accepted in existing design specifications, a slenderness ratio for laced columns with serpentine or crosswise lattices is determined as a result of consideration of the laced column as a statically indeterminate structure. Recent results of solving the buckling problem for laced columns, on the one hand, and the well known relationship between the slenderness ratio of the compressed bar and its elastic critical force, on the other hand, enable representation of the slenderness ratio of the laced column as a function of the special lattice rigidity parameter and the number panels into which the lattice joints divide the column chords. The obtained curves of the slenderness ratio for columns with a different number of panels are slightly distinguished one from another. As a consequence the single dependence between the modified slenderness ratio of the column and the lattice rigidity parameter can be accepted for columns regardless of the number of panels. This dependence is constructed by enveloping at the top the curves corresponding to fixed numbers of panels. The obtained plots of the modified slenderness ratio for columns with serpentine and crosswise lattices can be applied in designing steel-laced columns.

1. Introduction

Design recommendations [1–5] for calculation of the elastic critical force for a laced column are based on the conception of the equivalent slenderness ratio proposed by Engesser in 1891 [6,7]: the buckling problem for a laced column is reduced to calculation of Euler's critical force for the "equivalent" solid pin-ended compressed bar. This approach recognizes the possibility of the only sinusoidal half-wave buckling mode shape for the laced column. In actual fact, laced columns are highly redundant systems and the loss of column stability can occur by various buckling mode shapes depending on a correlation between the chord rigidity and the lattice rigidity. Euler's critical force for a laced column as a statically indeterminate structure can be calculated as a result of solving a two-point boundary value problem for a system of recurrence dependences between the deformation parameters of column cross-sections passing through the lattice joints [8–11]. Recent results of solving the buckling problem for laced columns as a statically indeterminate structure [10,11] disprove Engesser's assumption that the stability problem of the laced column can be reduced to the analogous problem for a continuous solid pin-ended column. In the general case, buckling mode shapes obtained for the column as a statically indeterminate structure take the form of the irregular curve consisting of several half-waves with unequal amplitudes. The stability analysis of a column as a statically indeterminate structure shows that Euler's critical force is a function of the special rigidity parameter of the column lattice and the number of panels into which the lattice joints divide the column chords. In contrast to the technique accepted in existing design specifications, the equivalent slenderness ratio for laced columns is suggested to determine according to the statically indeterminate scheme. The equivalent slenderness ratio for a laced column is defined as a slenderness ratio of the equivalent solid bar. The pin-ended solid bar is equivalent to the given laced column if the Euler's critical force for the bar equals this for the laced column [6,7,1]. Consequently, the equivalent slenderness ratio for the laced column can also be represented as a function of the mentioned lattice rigidity parameter and of the panel number of the column chords.

2. Relations between Euler's critical force and equivalent slenderness ratio for laced columns

Consider laced columns that consist of two identical longitudinal chords linked by braces forming two mutually parallel lattices. In the following, we will discuss two types of lattice, serpentine (Fig. 1(a)) and crosswise (Fig. 1(b)). Each chord has a solid cross-section with at least a single axis of symmetry. This axis coincides with the axis of symmetry of the whole column cross-section, and is parallel to the lattice planes. The column cross-sections passing

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Notation

A	Cross-sectional area of the column chord
A_d	Cross-sectional area of the lattice brace
a	Distance between the column cross-sections passing through the adjacent lattice joints (length of the chord sub-panel for the column with the serpentine lattice (Fig. 1(a)); length of the chord panel for the column with the crosswise lattice (Fig. 1(b)))
n	Number of the chord sub-panels for the column with the serpentine lattice or the chord panel for the column with the crosswise lattice
E	Young's modulus
I	Second moment of inertia of the chord cross-section about its principal axis normal to the lattices
I_0	Second moment of inertia of the whole column cross-section about its principal axis normal to the lattices
N_{cr}	Euler's critical value of the axial compressed force applied to the column chord
N_a	Euler's critical force for the simply supported bar identical in the static-geometry features with the chord panel/sub-panel
N_*	Engesser's critical value of the axial compressed force applied to the column chord
α	Lattice rigidity parameter
φ	Inclination angle of the lattice brace to the column cross-section (Fig. 1)
λ_a	Slenderness ratio of the chord panel/sub-panel
λ_{eq}	Equivalent slenderness ratio of the laced column
λ_m	Modified slenderness ratio of the laced column

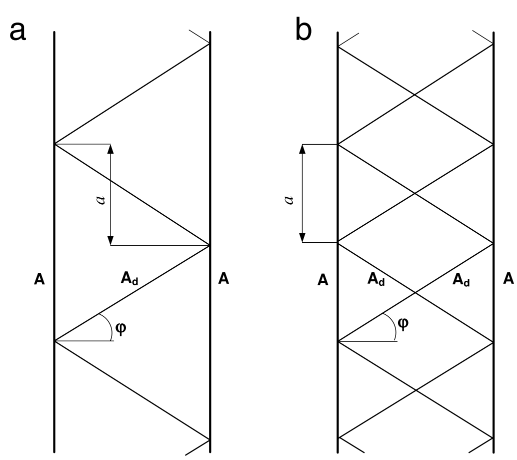

Fig. 1. Laced columns with serpentine and crosswise lattices.

through the lattice joints divide each column chord along its length into panels and sub-panels respectively for crosswise and serpentine lattices. Each chord is compressed by an axial force N.

The equivalent slenderness ratio for the laced column is expressed through Euler's critical value N_{cr} of the axial force

$$\lambda_{eq}^2 = \frac{\pi^2 EA}{N_{cr}} \tag{1}$$

where E = Young's modulus and A = the cross-sectional area of the column chord. We will relate the critical force N_{cr} to Euler's force for a simply supported bar identical in the static-geometry features with the panel or sub-panel of the chord

$$N_a = \frac{\pi^2 EA}{\lambda_a^2}. \tag{2}$$

The slenderness ratio of this bar is

$$\lambda_a = a\sqrt{A/I} \tag{3}$$

where a is a length of the chord panel in the case of the crosswise lattice or the chord sub-panel in the case of the serpentine lattice (Fig. 1). The analysis of elastic stability of a laced column as a static indeterminate structure shows that the relative critical force of the column can be represented as a function of the special non-dimensional lattice rigidity parameter α and the number of chord panels or sub-panels [9–11]

$$\frac{N_{cr}}{N_a} = f(\alpha, n). \tag{4}$$

The lattice rigidity parameter depends on the cross-sectional area of lattice braces and the moment of inertia of the chord cross section (Eq. (12) in Razdolsky [10] and Eq. (24) in Razdolsky [11])

$$\alpha = \frac{A_d a^2}{I}\sin 2\varphi \cos\varphi. \tag{5}$$

It is evident from Eqs. (1)–(2) that the equivalent slenderness ratio of the laced column is defined by the relative critical force of column

$$\lambda_{eq} = \lambda_a\sqrt{\frac{N_a}{N_{cr}}}. \tag{6}$$

The lattice rigidity parameter can be also represented as follows

$$\alpha = \frac{2\tan\varphi}{\left(1+\tan^2\varphi\right)^{3/2}}\frac{A_d}{A}\lambda_a^2. \tag{7}$$

The ratio of the brace cross-sectional area to the chord cross-sectional area can be also expressed through the rigidity parameter α and the slenderness ratio λ_a

$$\frac{A_d}{A} = \frac{\left(1+\tan^2\varphi\right)^{3/2}}{2\tan\varphi}\frac{\alpha}{\lambda_a^2}. \tag{8}$$

We compare the equivalent slenderness ratio calculated for the laced column as a statically indeterminate structure with the slenderness ratio that follows from the Engesser's approach. The Engesser's critical force of the column chord is described by the formula [7, Section 2.18]

$$N_* = \frac{\pi^2 EI_0}{2(na)^2}\left[1 + \frac{\pi^2 I_0}{(na)^2}\frac{1}{mA_d\sin\varphi\cos^2\varphi}\right]^{-1} \tag{9}$$

where $I_0 = 2I + 0.5A(a/tg\varphi)^2$ is the second moment of inertia of the whole column cross-section and m is a factor depending on the lattice scheme ($m = 2$ for serpentine lattice and $m = 4$ for crosswise lattice). We neglect the moment of inertia $2I$ of the individual chords in comparison with the second term in the expression of I_0 and express the brace cross-sectional area through the rigidity parameter α according to Eq. (8). The formula Eq. (9) takes the form

$$N_* = \frac{\pi^2 EA}{4n^2\tan^2\varphi}\left[1 + \frac{\pi^2}{n^2\tan^2\varphi}\frac{\lambda_a^2}{m\alpha}\right]^{-1}. \tag{10}$$

The expression of the equivalent slenderness ratio corresponding to Engesser's approach follows from Eq. (10)

$$\lambda_{eq} = 2\sqrt{\frac{\pi^2\lambda_a^2}{m\alpha} + n^2\tan^2\varphi}. \tag{11}$$

3. Column with a serpentine lattice

Calculation of the equivalent slenderness ratio for laced columns as statically indeterminate structures is based on the relation

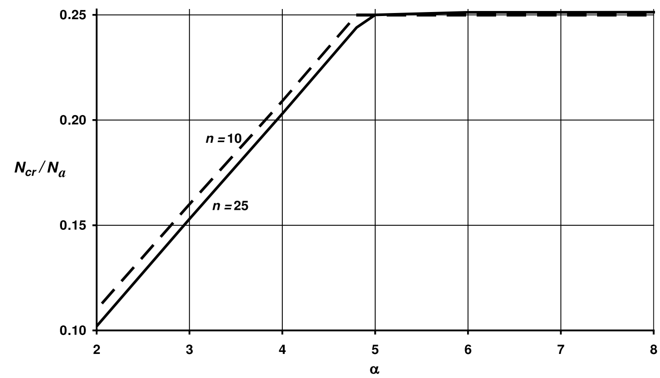

Fig. 2. Relation between Euler's critical force and lattice rigidity parameter for columns with serpentine lattice (number of sub-panels $n = 10, 25$).

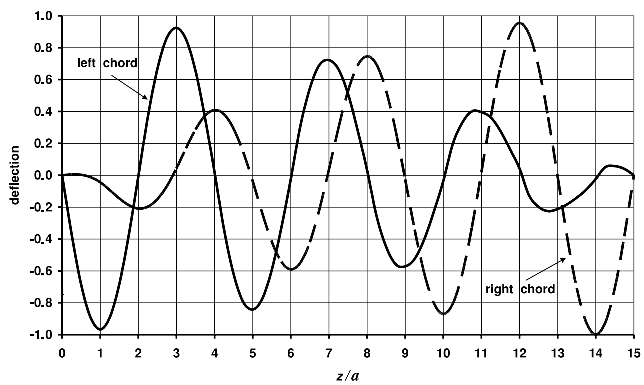

Fig. 3. Buckling mode shape of column with serpentine lattice (number of sub-panels $n = 15$, lattice rigidity parameter $\alpha = 10$, relative critical force $N_{cr}/N_a = 0.254$).

Eq. (4) between the critical force and the lattice rigidity parameter of the column. Such relations are established as a result of solving the two-point boundary value problem for a system of specific difference equations. A way of solving this problem for a column with a serpentine lattice is presented in [11]. Fig. 2 illustrates typical relations between the critical force and the lattice rigidity parameter for columns with a serpentine lattice. The plots in the figure are drawn for columns with the sub-panel number $n = 10$ and $n = 25$. The Euler's critical force remains constant or nearly constant once the lattice rigidity parameter will exceed a specific quantity which we will denote as α_B. This phenomenon is analogous to Boobnov's effect for a simply supported compressed solid bar with a number of intermediate elastic supports [7]. Therefore we will name the value α_B as the Boobnov value of the lattice rigidity parameter. In contrast to the classical Boobnov example, the established property of the laced column is due to lattice braces, i.e. internal constraints of the structure. A column with an even number of sub-panels can lose stability so that joint cross-sections are not displaced and the critical force of the column is equal to the critical force of the isolated chord panel, i.e. the critical force for a simply supported bar with a length $2a$ and with a cross-section identical to the chord cross-section

$$N_{cr} = \frac{N_a}{4} = \frac{\pi^2 EA}{4\lambda_a^2}. \tag{12}$$

The critical force for a column with an odd number of sub-panels can exceed this value by no more than several percent. The buckling mode shape of a column with a number of sub-panels $n = 15$ at a value of the rigidity parameter $\alpha = 10$ is shown in Fig. 3. We assume that column lattices start at the lower (initial) cross-section of the left chord. The buckling mode shapes of chords take the form of a curve consisting of eight non-sinusoidal half-waves with unequal amplitudes. The critical force of the column exceeds only slightly the critical force of the isolated chord panel Eq. (12)

$$N_{cr} = 0.254N_a. \tag{13}$$

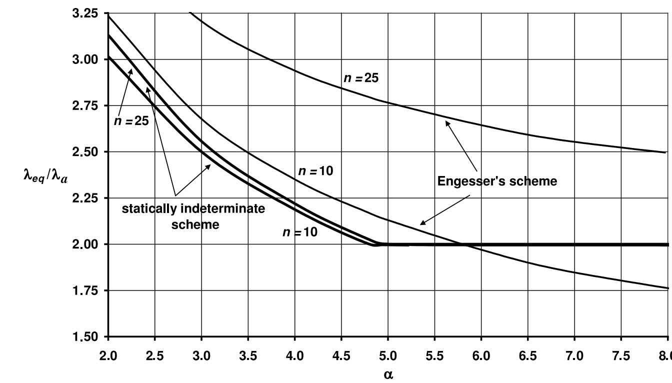

Fig. 4. Comparison of equivalent slenderness ratios calculated according to the statically indeterminate scheme and the Engesser's model for columns with a serpentine lattice (number of sub-panels $n = 10, 25$).

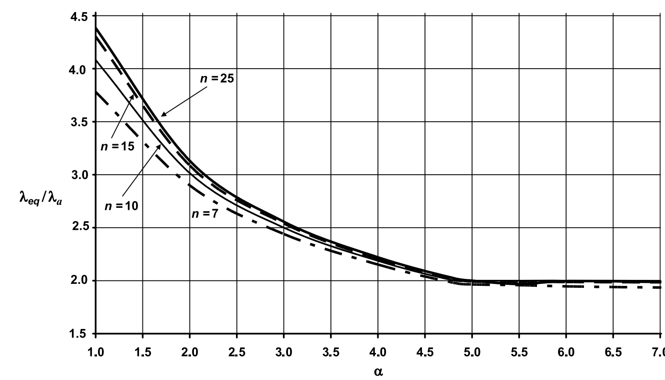

Fig. 5. Relation between equivalent slenderness ratio and lattice rigidity parameter for columns with a serpentine lattice (number of sub-panels $n = 7, 10, 15, 25$).

The dependence Eq. (4) between the critical force and the lattice rigidity parameter enables one to represent the equivalent slenderness ratio for columns with any number of sub-panels as a function of the lattice rigidity parameter

$$\lambda_{eq}/\lambda_a = g(\alpha, n). \tag{14}$$

Fig. 4 illustrates the relation λ_{eq}/λ_a corresponding to the curves of a critical force shown in Fig. 2 ($n = 10, 25$). The Engesser values of the equivalent slenderness ratio Eq. (11), calculated on the assumption that $\lambda_a = 15$ and $\varphi = 30°$, are also shown in Fig. 4. The Engesser's model does not reveal the Boobnov's effect and can give an unrealistic value of the equivalent slenderness ratio that is smaller than a slenderness ratio for the isolated chord panel. Plots of the equivalent slenderness ratio for columns with the sub-panel number $n = 7$, 10, 15 and 25 are displayed in Fig. 5. The curves are slightly distinguished one from another and coincide very closely at the value of the lattice rigidity parameter $\alpha > 4.5$. Consequently, it is possible to construct a single curve for calculation of the column modified slenderness ratio λ_m regardless of the number of sub-panels

$$\lambda_m = F(\alpha)\lambda_a. \tag{15}$$

The ratio λ_m/λ_a can be constructed by enveloping at the top the plots corresponding to fixed numbers of sub-panels in Fig. 5. The curve of this ratio is shown in Fig. 6. The Boobnov value of the lattice rigidity parameter in Fig. 6 is $\alpha_B = 4.9$. A cross-sectional area

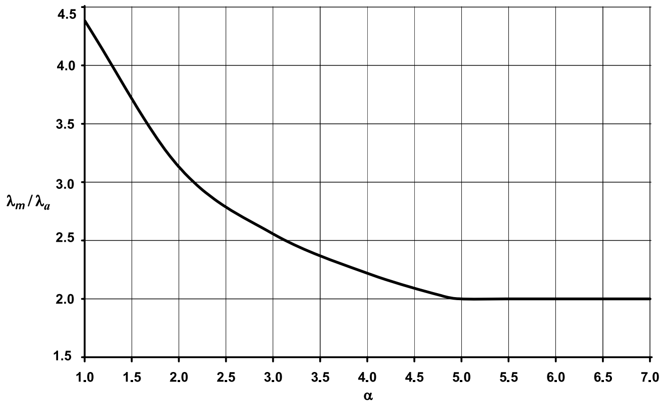

Fig. 6. Curve of modified slenderness ratio being suggested for columns with a serpentine lattice.

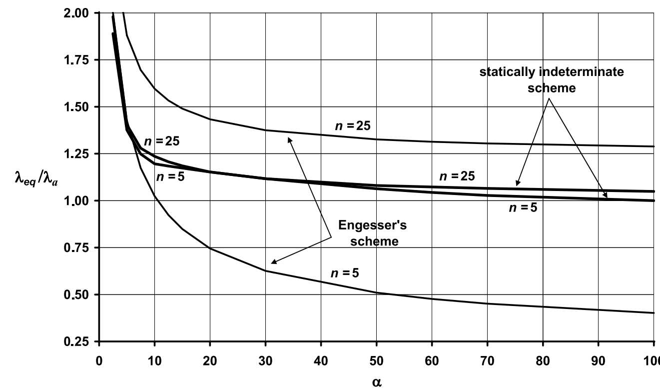

Fig. 9. Comparison of equivalent slenderness ratios calculated according to the statically indeterminate scheme and Engesser's model for columns with a crosswise lattice (number of panels $n = 5, 25$).

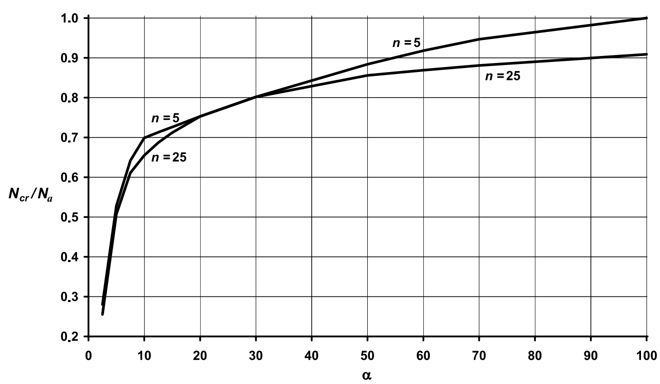

Fig. 7. Relation between Euler's critical force and lattice rigidity parameter for columns with a crosswise lattice (number of panels $n = 5, 25$).

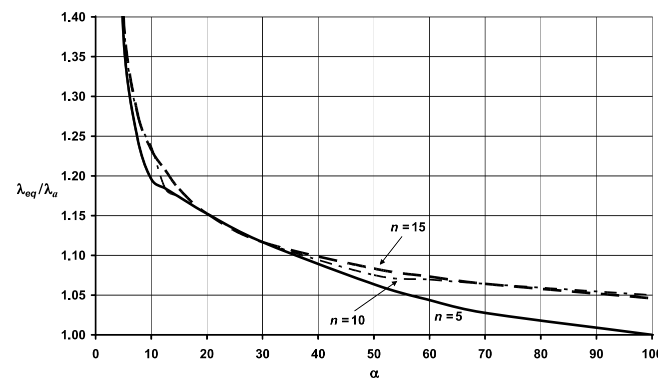

Fig. 10. Relation between equivalent slenderness ratio and lattice rigidity parameter for columns with a crosswise lattice (number of panels $n = 5, 10, 15$).

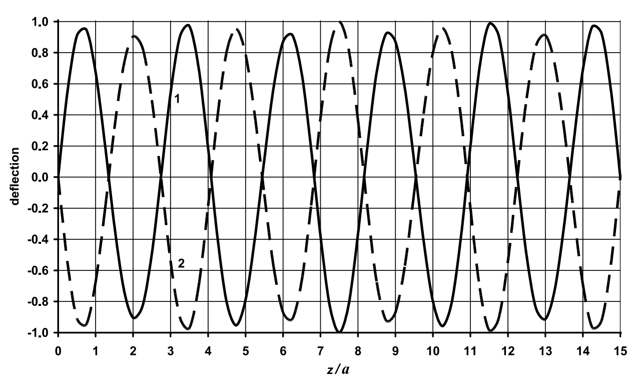

Fig. 8. Buckling mode shape of a column with a crosswise lattice (number of panels $n = 15$, lattice rigidity parameter $\alpha = 10$, relative critical force $N_{cr}/N_a = 0.658$).

of the lattice brace is defined by the relationship Eq. (8). For a column with parameters $\lambda_a = 15$ and $\varphi = 30°$, a cross-sectional area of the brace corresponding to the $\alpha_B = 4.9$ is $A_d = 0.029A$. The plot in Fig. 6 can be applied in designing steel-laced columns with a serpentine lattice.

4. Column with a crosswise lattice

A way of solving the buckling problem of a laced column with a crosswise lattice as a statically indeterminate structure is presented in [9,10]. Fig. 7 illustrates typical relations between The Euler critical force and the lattice rigidity parameter for such columns. The plots in the figure are drawn for columns with a panel number $n = 5$ and $n = 25$. Increasing the lattice rigidity parameter in the range $\alpha \leq 100$ leads to an increase in the critical force. The Boobnov effect appears for the five-panel column when the lattice rigidity parameter reaches value $\alpha = 100$. The buckling mode shape of a column with a number of panels $n = 15$ and with the rigidity parameter $\alpha = 10$ are shown in Fig. 8. The critical state is achieved on the force value $N_{cr} = 0.658N_a$. The buckling deflections of both the chords are symmetric about the longitudinal axis of the column and consist of 11 half-waves with near-equal amplitudes. The dependence between the equivalent slenderness ratio and the lattice rigidity parameter for columns with any number of panels follows from Eq. (4). Plots of equivalent slenderness ratio corresponding to the curves of a critical force displayed in Fig. 7 are shown in Fig. 9. The slenderness ratios for columns with the panel number $n = 5$ and $n = 25$ differ little in magnitude over all given range of the lattice rigidity parameter. The Engesser values of the equivalent slenderness ratio, calculated from Eq. (11) on the assumption that $\lambda_a = 40$ and $\varphi = 45°$, are also shown in Fig. 9. The Engesser scheme gives the slenderness ratios significantly different from those of the statically indeterminate scheme. Plots of the equivalent slenderness ratio for columns with the panel number $n = 5$, 10 and 15 are displayed in Fig. 10. The curves are slightly distinguished one from another. Consequently, a single dependence of the modified slenderness ratio λ_m can be accepted for columns regardless of the number of panels. The ratio λ_m/λ_a can be constructed by enveloping at the top the plots corresponding to fixed numbers of panels in Fig. 10. The curve of this ratio is shown in Fig. 11. The slenderness λ_m decreases with an increase of the lattice rigidity parameter. However the slenderness ratio decreases only slightly when the lattice rigidity parameter exceeds the value $\alpha = 40$. The cross-sectional area of the lattice brace corresponding to this value of the lattice rigidity parameter for a column with parameters $\lambda_a = 40$ and $\varphi = 45°$ is $A_d = 0.035A$. The plot in Fig. 11 can be applied in designing steel-laced columns with a crosswise lattice.

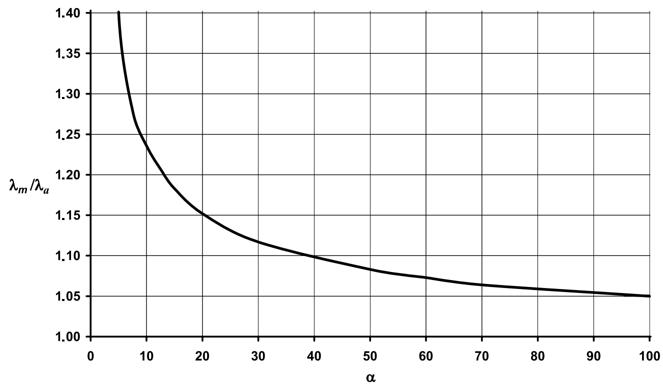

Fig. 11. Curve of modified slenderness ratio being suggested for columns with crosswise lattice.

5. Conclusions

Determination of the modified slenderness ratio for laced columns with a serpentine or crosswise lattice is based on the results of solving the flexural buckling problem of the laced column as a statically indeterminate structure. The well known relationship between the slenderness ratio of the compressed bar and its elastic critical force enables one to express the slenderness ratio of a laced column through the lattice rigidity parameter and the slenderness ratio of the column chord panel. The curves describing a ratio of the column slenderness ratio to the slenderness ratio of the column chord panel for columns with a different number of panels are slightly distinguished one from another and in consequence the single dependence of the modified slenderness ratio can be accepted for columns regardless of the number of panels. This dependence is constructed by enveloping at the top the curves corresponding to fixed numbers of panels.

The slenderness ratio of columns decreases with an increase of the lattice rigidity parameter. However for columns with a serpentine lattice the slenderness ratio remains unchanged when it achieves a magnitude equal to the slenderness ratio of the isolated chord panel. For columns with a crosswise lattice the ratio of the column slenderness ratio to the slenderness of the isolated chord panel ratio only decreases from 1.1 to 1.05 when the lattice rigidity parameter increases from 40 to 100. The ratio between the cross-sectional areas of the lattice brace and the chord cross-section is also expressed through the lattice rigidity parameter and the slenderness ratio of the chord panel. The obtained plots of the modified slenderness ratio for columns with serpentine and crosswise lattices can be applied in designing steel-laced columns.

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