ELSEVIER Journal of Banking & Finance 19 (1995) 1091–1108

Journal of BANKING & FINANCE

Deposit insurance and bank interest rate risk: Pricing and regulatory implications

Jin-Chuan Duan, Arthur F. Moreau, C.W. Sealey *

Faculty of Management, McGill University, Samuel Bronfman Building, 1001 Sherbrooke Street West, Montreal, Quebec H3A 1G5, Canada

Received August 1992; final version received January 1994

Abstract

The linkage between the interest rate risk exposure of banks and the liabilities of a deposit insuring agency is not well understood. In this paper, a model is developed to evaluate the interest rate risk exposure of both deposit taking institutions and deposit insuring agents when bank equity has limited liability and interest rates are stochastic. Based on a sample of U.S. banks, empirical results are presented for the interest rate risk exposure of banks and its impact on the liabilities of the FDIC.

Keywords: Deposit insurance; Interest rate risk; Bank regulation; Contingent claims

JEL classification: G13; G21; G22; G28

1. Introduction

In recent years, the problem of interest rate risk plays a prominent role in discussions of both the management and regulation of depository financial institutions (hereafter referred to as banks). Such prominence is understandable since it is argued that interest rate risk exposure was a major factor in precipitating the

---

* Corresponding author.

SSDI 0378-4266(95)00073-5

savings and loan debacle.[1] In recognition of the importance of interest rate risk, the FDIC Improvement Act of 1991 (FDICIA) instructs regulators to take into account interest rate risk exposure when determining a bank's capital adequacy, and both the Federal Reserve Board and the Office of Thrift Supervision have put forth proposals to implement this legislation.[2]

A major goal in regulating interest rate risk, implicit in FDICIA, is to limit the liability of the FDIC resulting from adverse interest rate movements. Unfortunately, little is known about the linkage between a bank's interest rate risk exposure and the FDIC's insurance liability. Perhaps the primary reason for this lack of understanding is that deposit insurance pricing and interest rate risk management of banks are modelled with two different, and largely unrelated, methodologies; viz., the former is based on limited liability[3] whereas the latter is based on tools adopted from the management of fixed income securities.[4]

In this paper, a model of the interest rate risk exposure of both banks and the FDIC are developed from a common framework based on limited liability and stochastic interest rates. In contrast to existing work,[5] the model takes as a starting point the work of Rabinovitch (1989) on the pricing of contingent claims in the presence of stochastic interest rates. He incorporates the bond pricing model of Vasicek and thus allows for an explicit treatment of term structure effects. Rabinovitch's model is recast in this paper, however, to allow a more tractable analysis, both theoretically and empirically, by decomposing banks' risk into credit and interest rate sources. A distinct characteristic of this approach is that it allows a specific interpretation of the pricing effects of a bank's interest rate sensitivity. The theoretical results are then used to generalize the empirical model of Ronn and Verma (1986) and obtain direct estimates of these pricing effects.

The remainder of the paper is organized as follows: In Section 2, the stochastic process for interest rates is specified and limited liability valuation equations for bank equity and deposit insurance are presented. Based on the valuation results, the interest rate risk exposure of both banks and the FDIC are derived. Section 3 presents the empirical results and Section 4 contains the conclusion.

---

[1] See, for example, Green and Shoven (1986, p. 41), Baer (1990), Benston (1985), Benston et al. (1986), and Kane (1985).

[2] See Office of Thrift Supervision (1991) and Board of Governors of the Federal Reserve System (1992).

[3] Deposit insurance pricing with limited liability began with Merton (1977). See also, for example, Marcus and Shaked (1984) Ronn and Verma (1986), Giammarino et al. (1989), and Duan et al. (1992).

[4] See, for example, Bierwag and Kaufman (1985, 1992), Bierwag (1987), Kaufman (1984), Toevs (1983), and Dermine (1985).

[5] For deposit insurance pricing with stochastic interest rates, see for example McCulloch (1981, 1985), Ronn and Verma (1986), Pennacchi (1987a, 1987b), and Crouhy and Galai (1986, 1991).

2. A model of interest rate risk for banks and the deposit insurer

2.1. A model of bank equity valuation

Consider a bank with a planning horizon that extends over a given time interval $[0, T]$. At time $t = 0$, the bank acquires an asset portfolio (V) and finances these assets with paid-in capital and a homogenous, insured deposit liability which matures at time $t = T$, the face value of which is denoted by F. Because of deposit insurance, deposit liabilities are default-free from the point of view of depositors and earn a fixed, continuously compounded rate of return, denoted by R, that is equal to the rate on a default-free bond with maturity T. At time $t = T$, the bank's liability to depositors is Fe^{RT}. Although deposits are default-free, equity, whose market value is denoted by E, is risky and is assumed to have limited liability in the event of bankruptcy.

As in Vasicek (1977), the instantaneous interest rate is assumed to be governed by a mean-reverting stochastic process. Specifically,

$$dr_t = q(m - r_t)\,dt + v\,dZ_{rt}, \tag{1}$$

where r_t is the instantaneous risk-free rate of interest at time t; m is the long-run mean of the interest rate; v is the volatility of the interest rate; q is a positive constant measuring the magnitude of the mean-reverting force; and Z_{rt} is a Weiner process.

The bank's total asset value is postulated to be governed by the following process:

$$\frac{dV_t}{V_t} = \mu\,dt + \sigma_V\,dZ_{Vt}, \tag{2}$$

where V_t is the value of bank assets at time t; μ is the instantaneous expected return on bank assets, σ_V is the total volatility of bank asset returns, and Z_{Vt} is a Weiner process. The processes Z_{Vt} and Z_{rt} are expected to be correlated.

Eqs. (1) and (2) constitute what is essentially the model employed by Rabinovitch (1989) to value options in the presence of stochastic interest rates. Unlike Rabinovitch, however, the primary goal here is to explicitly examine the interest rate risk exposure of option-type contracts such as bank equity and deposit insurance. Thus, a further decomposition of Eq. (2) is required to provide a direct interpretation of interest rate risk. Specifically, as shown in the Appendix, the total stochastic component of bank assets can be projected onto the interest rate variable to yield

$$\frac{dV_t}{V_t} = [\mu - \phi_V q(m - r_t)]\,dt + \phi_V\,dr_t + \psi\,dW_t, \tag{3}$$

where W_t is a Weiner process that is constructed, as a result of the projection, to be orthogonal to Z_{rt} (see Appendix); i.e., W_t is independent of interest rates. From the form of Eq. (3), it is clear that ϕ_V can be interpreted as the instantaneous interest rate elasticity of bank assets, [6] which is a constant since it is a function of the constants used to define Eqs. (1) and (2), (see Appendix). Finally, ψ is the constant volatility of bank assets caused by all risk that is orthogonal to interest rate risk. Henceforth, ψ is referred to as credit risk. [7] From the definitions of ϕ_V and ψ in the Appendix, it can be shown that total asset risk, σ_V in Eq. (2), can be expressed as $\sqrt{\phi_V^2 v^2 + \psi^2}$. Thus, the decomposition employed above allows the total instantaneous variance of bank assets to be decomposed into both interest rate risk and credit risk.

In a manner similar to Rabinovitch, it is possible to derive a valuation equation for bank equity. Let E_t denote the value of bank equity at time t; $X \equiv Fe^{RT}$ denote the equity holders' terminal obligation to depositors; $P(r_t, t, T)$ denote the time t price of a zero-coupon bond with \$1 payable at T, as derived by Vasicek (1977); and $B(T-t) = [1 - e^{-q(T-t)}]/q$ denote the negative of the instantaneous interest rate elasticity of the default-free, zero-coupon bond with $(T-t)$ to maturity. [8] The equity valuation equation for the bank is

$$E_t = V_t N(h_t) - XP(r_t, t, T) N(h_t - \delta_t), \tag{4}$$

where

$$h_t \equiv \frac{1}{\delta_t} \ln\left[\frac{V_t}{P(r_t, t, T)X}\right] + \frac{\delta_t}{2},$$

$$\delta_t^2 \equiv (\phi_V^2 v^2 + \psi^2)(T-t) + 2\phi_V v^2\left[\frac{(T-1)}{q} + \frac{1}{q^2}(e^{-q(T-1)} - 1)\right]$$
$$+ v^2\left[\frac{(T-1)}{q^2} + \frac{2}{q^3}(e^{-q(T-1)} - 1) + \left(\frac{1 - e^{-2q(T-1)}}{2q^3}\right)\right].$$

Using put-call parity, it is also possible to state the value of the bank's deposit insurance contract. Let I_t denote the actuarial value of a bank's deposit insurance contract at time t. Then,

$$I_t = XP(r_t, t, T)[1 - N(h_t - \delta_t)] - V_t[1 - N(h_t)]. \tag{5}$$

Eq. (5) is in the form of Merton's (1977) deposit insurance pricing formula, but the components are different to reflect the effects of stochastic interest rates.

---

[6] The interest rate elasticity measure used here is the percentage change in value divided by the change in the instantaneous (continuously compounded) interest rate. This definition is equivalent to the conventional measure because the change in the instantaneous rate is the percentage change in one plus the interest rate. This definition is also adopted in Ingersoll (1978).

[7] Credit risk is used here to designate all risk other than interest rate risk. To the extent that defaults on loans are related to interest rates, such risk is considered as part of interest rate risk. Thus, ψ may not strictly correspond to the conventional definition of credit risk.

[8] Note that by Vasicek's (1977) model, $-B(T-t)$ is the interest rate elasticity and is always less than time to maturity, $(T-t)$.

2.2. The bank's interest rate risk

Let ϕ_{E_t} and σ_{E_t} denote the interest rate elasticity of equity and equity volatility at time t, respectively. The specific form of ϕ_{E_t} and σ_{E_t}, under the assumptions introduced above, are as follows:

Proposition 1. Let $\Omega_t \equiv N(h_t)V_t/E_t$. *Then,*

$$\phi_{E_t} = \Omega_t[\phi_V + B(T-t)] - B(T-t), \tag{6}$$

and

$$\sigma_{E_t} = \sqrt{\phi_{E_t}^2 v^2 + \Omega_t^2 \psi^2}\,. \tag{7}$$

Proof: See appendix.

To interpret the elasticity formula in (6), it is useful to reiterate the intuition behind option pricing with stochastic interest rates.[9] The standard approach is to normalize the asset value process using as a numéraire the price of a zero-coupon, default-free bond with $(T-t)$ to maturity. The pricing relevant risk for an option contract, under stochastic interest rates, is the risk exhibited by the normalized asset value process in the numéraire economy. Since the underlying asset and the numéraire bond have interest rate elasticities of ϕ_V and $-B(T-t)$, respectively, the interest rate risk exposure at time t of the normalized asset value process is $[\phi_V + B(T-t)]$. Since Ω is the standard option elasticity, the first term on the RHS of (6) is simply the interest rate risk exposure of equity in the numéraire economy. Adding the interest rate elasticity of the numéraire bond yields the interest rate risk exposure of equity in the original economy.

Since ϕ_V and $-B(T-t)$ are the interest rate elasticities of the bank's assets and liabilities, respectively, the difference of these terms measures the degree of mismatch in the interest rate risk exposure of assets and liabilities. Thus, the term $[\phi_V + B(T-t)]$ is hereafter referred to as the interest rate elasticity gap. Note that the negative of this term is equivalent to the bank's duration gap.

2.3. The interest rate risk exposure of the deposit insurer

Recent proposals dealing with the regulation of interest rate risk concentrate on limiting the FDIC's actuarial liability that results from such risk. There are good

---

[9] An alternative interpretation is possible by rewriting (6) as $\phi_{E_t} = \phi_{V}\Omega_t - (1-\Omega_t)B(T-t)$. From the Black–Scholes (1973) analysis, the return on a call option can be replicated by a portfolio that is long in the underlying asset and short in the risk-free asset. Since bank equity is modeled as a call option on the bank's assets, the replicating portfolio is long in the bank's assets and short in risk-free deposits. Eq. (6) shows that the interest rate elasticity of equity is the weighted average of the elasticities of the individual components of the replicating portfolio.

reasons, however, why regulators should be interested not only in the level of the insurance liability, but in the interest rate sensitivity of the liability as well. First, higher levels of interest rate risk exposure of the FDIC's liability leads to greater potential losses in the event of adverse interest rate movements. Second, as the empirical results of the next section show, the liabilities of the FDIC can be highly sensitive to adverse interest rate movements, and this sensitivity can vary greatly across interest rate regimes. Finally, because of the nature and sheer size of the risk, it is highly unlikely that the FDIC can hedge a significant amount of their exposure to interest rate risk. It would seem that a prudent regulatory course would be for the FDIC to not only limit its actuarial liability, but also the potential for loss resulting from interest rate movements.

Let ϕ_{I_t} denote the interest rate elasticity of the deposit insurance contract for a given bank at time t.

Proposition 2. Let $\Lambda_t \equiv [N(h_t) - 1]V_t/I_t$. *Then,*

$$\phi_{I_t} = \Lambda_t[\phi_V + B(T-t)] - B(T-t). \tag{8}$$

Proof: See Appendix.

Noting that Λ_t is the put option elasticity, the interpretation of the interest rate elasticity of the deposit insurance contract in Eq. (8) is similar to that in equation (6). Since bank equity and deposit insurance are call and put options, respectively, on the same asset, it is evident that banks and the insuring agent will not, in general, share the same level of interest rate risk exposure. Moreover, a bank which completely immunizes its equity holders against interest rate risk, i.e., the bank sets $\phi_E = 0$, cannot at the same time immunize the bank's deposit insurance contract, except for the trivial case where $\phi_V = T = 0$, i.e., where both bank assets and liabilities mature instantaneously. Although this point is a straightforward implication of put-call parity, to the authors' knowledge, it has not been recognized in the literature. The implication here is that the FDIC, in order to limit the potential for large losses from adverse interest rate movements, must be aware of how bank decisions influence that potential. Eq. (8) shows exactly that relationship.

Eq. (8) shows the interest rate elasticity of a single bank's deposit insurance contract. Since a deposit insuring agent such as the FDIC insures the deposits of a large number of banks, the total risk exposure of the insuring agent is the total risk of its portfolio of insurance contracts. Assume that the FDIC insures the deposits of J banks indexed by $j \in J$. Let ϕ_{P_t} and σ_{P_t} denote the interest rate elasticity of total liabilities and the volatility of these liabilities arising from the insuring agent's portfolio of deposit insurance contracts, respectively. Then,

$$\phi_{P_t} = \Sigma_j \omega_{tj} \phi_{I_t j} \tag{9}$$

and

$$\sigma_{P_t} = \left(\phi_{P_t}^2 v^2 + \Sigma_i \Sigma_j \omega_{tj} \omega_{ti} \Lambda_{tj} \Lambda_{ti} \rho_{ij} \psi_j \psi_i \right)^{\frac{1}{2}} \tag{10}$$

where ω_{tj} is the value weight of the deposit insurance contract on the jth bank at time t; ρ_{ij} is the correlation between the credit risks of banks i and j; and $\phi_{I_t j}$ is the time t interest rate elasticity of the deposit insurance contract for the jth bank from Eq. (8). Eq. (9) is immediate since the elasticity of a portfolio is the weighted sum of individual elasticities. To derive (10), simply note that all interest rate risk is perfectly positively correlated, whereas credit risk need not be perfectly correlated. The expression then follows from the usual portfolio variance.

Eq. (10) shows, as expected, that the volatility of the insuring agent's liabilities arises from both interest rate risk and credit risk; however, the two types of risk have somewhat different properties within the portfolio. Considering credit risk first, the risk of the insuring agent from this source depends largely on the degree to which these risks are correlated across banks. In other words, the usual type of diversification is present. For interest rate risk, on the other hand, diversification in the traditional sense is not possible since all banks face a common interest rate movement. Even so, banks' interest rate risks are averagable to the extent that the $\phi_{I_t j}$ terms differ in sign.[10]

3. Empirical estimates

3.1. Empirical methodology and data

Based on the foregoing theoretical results, it is possible to construct an empirical model that consists of three non-linear, simultaneous Equations (4), (6) and (7), that can be solved for the unobservable variables V, ψ, and ϕ_V.[11] The variables on the LHS of this three equation system, E, σ_E, and ϕ_E, are obtained as follows: total equity value, E, is the market price of shares times the number of shares outstanding; the standard deviation of equity, σ_E, for a given quarter, is calculated using the daily returns for the quarter, and is updated quarterly; and the interest rate elasticity of equity, ϕ_E, is calculated using a linear regression of daily equity returns and the change in daily 90-day T-Bill returns over the year prior to the end of the quarter.[12] Then, the value of the deposit insurance contract for each bank can be calculated using Eq. (5), adjusted for dividends, since the deposit

---

[10] This averagability property of the $\phi_{I_t j}$ terms is analogous to beta risk. For example, by adding a negative beta asset to a portfolio with a positive beta, overall systematic risk can be reduced. To the extent that the $\phi_{I_t j}$ terms differ in sign, a similar property holds.

[11] The non-linear system of equations is solved simultaneously to yield exact solutions for the variables V, ψ, and ϕ_V. Like all systems of non-linear equations, unique solutions are not guaranteed; however, the solutions reported in this study seem to be unique within the range of reasonable starting values.

[12] The standard deviation of interest rates, σ, is calculated using the daily returns on 90-day T-Bills over the same period, and the long run estimate of q is approximately 1 (see Footnote 7 of Chan, Karolyi, Longstaff and Sanders (1992)).

Table 1

Estimates of bank risk

The estimates of asset value, V, the interest rate elasticity of assets, ϕ_V, and credit risk, ψ, are obtained from the simultaneous solution to the following system:

$$E = VN(h) - \lambda FN(h - \delta)$$

$$\phi_E = \Omega[\phi_V + B(T)] - B(T)$$

$$\sigma_E = \sqrt{\phi_E^2 v^2 + \Omega^2 \psi^2}$$

where

$$h = \frac{1}{\delta} \ln\left[\frac{V}{\lambda F}\right] + \frac{\delta}{2},$$

$$\delta^2 = (\phi_V^2 v^2 + \psi^2)T + 2\phi_V v^2 \left[\frac{T}{q} + \frac{1}{q^2}(e^{-qT} - 1)\right] + v^2 \left[\frac{T}{q^2} + \frac{2}{q^3}[e^{-qT} - 1] + \left(\frac{1 - e^{-2qT}}{2q^3}\right)\right].$$

$$\Omega = N(h)\frac{V}{E},$$

E is the market value of equity, F is the face value of bank deposits, v is the volatility of interest rates, λ the slack before closure (set to 0.97), T is the time to maturity set equal to one, ϕ_E is the interest rate elasticity of equity, $N(\cdot)$ is the standard cumulative normal distribution and $B(T)$ is the instantaneous duration of a default-free, zero coupon bond of maturity T.

Values for the insurance premium per dollar of insured deposits, IPP, and total asset risk, σ_V, are obtained from the following equations:

$$\sigma_V = \sqrt{\phi_V^2 v^2 + \psi^2}$$

$$IPP = [1 - N(h - \delta)] - \frac{V}{F}[1 - N(h)]$$

The reader is referred to Ronn and Verma (1986) for the methodology used to obtain estimates for the deterministic and stochastic interest rate versions of their model.

Table 1 (continued)

Period	R-V: Deterministic interest rates		R-V: Stochastic interest rates		Elasticity-based model				
	σ_V (%)	IPP × 100	σ_V (%)	IPP × 100	ψ (%)	σ_V (%)	ϕ_I	$[\phi_V + B(1)]$	*IPP* × 100
Small banks									
1975–79	0.888	0.147	0.890	0.147	0.885	1.027	−0.745	0.003	0.198
1980–83	0.966	0.140	0.981	0.141	0.975	1.775	−2.149	−0.079	0.541
1984–89	1.671	0.033	1.674	0.033	1.647	1.761	−2.271	−0.147	0.041
1975–89	1.278	0.092	1.285	0.092	1.269	1.600	−1.888	−0.092	0.229
Medium banks									
1975–79	0.856	0.088	0.859	0.088	0.852	1.014	1.174	−0.024	0.145
1980–83	1.043	0.177	1.055	0.179	1.070	1.769	−3.240	−0.124	0.622
1984–89	1.527	0.068	1.530	0.068	1.490	1.636	−4.132	0.245	0.086
1975–89	1.335	0.093	1.340	0.093	1.316	1.574	−3.532	−0.190	0.201
Large banks									
1975–79	0.873	0.262	0.874	0.263	0.873	1.032	−1.529	−0.032	0.350
1980–83	0.817	0.369	0.832	0.373	0.858	1.805	−5.926	−0.178	1.090
1984–89	1.086	0.089	1.089	0.089	1.024	1.253	7.687	−0.380	0.145
1975–89	0.954	0.215	0.960	0.216	0.938	1.363	−5.662	−0.235	0.476
All banks									
1975–79	0.873	0.215	0.875	0.215	0.872	1.028	−1.339	−0.026	0.292
1980–83	0.883	0.294	0.898	0.297	0.916	1.793	−4.775	−0.151	0.909
1984–89	1.310	0.074	1.314	0.074	1.263	1.449	−5.758	−0.303	0.111
1975–89	1.099	0.165	1.104	0.166	1.083	1.453	−4.531	−0.201	0.369

Note: All numbers are significantly different than zero (at the 5% significance level). In addition, total risk, σ_V, and *IPP* are significantly larger under the proposed model (one tail test, 5% significance level).

insurance contact is not dividend protected. Finally, the values for ϕ_I, ϕ_P, and σ_P are computed using Eqs. (8), (9), and (10), respectively.

Several simplifications can be made to the theoretical model for purposes of empirical tractability. Since these simplifications are dealt with at length in related literature, they are only listed here with references to the relevant sources. First, at the point of each estimation, $t = 0$, since deposits are insured, the present value of the exercise price of the equity option, $XP(r_t, t, T)$, is precisely equal to the face value of the bank's debt, F. [13] Second, the time to expiry of the equity option is set as the interval between bank examinations, i.e., $T = 1$ year. [14] Finally, the FDIC is assumed to enforce bank closures only after they have negative net worth, which is equivalent to setting the exercise price of the equity option at λF, where $\lambda = 0.97$ is used. [15]

The estimates require data on both balance sheet items and market variables. For this study, the balance sheet data are taken from the Quarterly Bank Compustat tapes and market data from the Daily Return CRSP tapes for banks that trade on the NYSE and AMEX, and the Daily Return OTC tapes for those banks that trade over-the-counter. It is possible to match up data between the two return tapes and the Compustat tapes for seventy-two U.S. banks over the fifteen-year period 1975-I–1989-IV. [16]

3.2. Empirical results

Since computations are made for each quarter of the fifteen-year sample period for each of seventy-two banks, to conserve space in presentation, the results are presented in summary form. Three sub-periods are given special attention: 1975–79, the period of pre-deregulation; 1980–83, the period of high interest rate volatility; and 1984–89, the period of more stringent bank regulation as well as

---

[13] This fact has been pointed out by Merton (1977) and Ronn and Verma (1986). Unlike their models, however, in the model here the derivative of the present value of the exercise price is not equal to zero when interest rates are stochastic. F includes all liabilities and preferred stock.

[14] Even though individual bank deposits may have maturities shorter than one year, the timing of deposits and withdrawals of funds occur in such a way that a bank's total deposits remain rather stable, especially with deposit insurance since bank runs are virtually eliminated. Thus, the refinancing decision is in effect made by the FDIC at the audit date rather than by depositors. From the deposit insurer's point of view, it is the audit interval that is important since this is the time when the insurer can force the issue of closure.

[15] See Ronn and Verma (1986), Giammarino et al. (1989) and Duan, Moreau and Sealey (1992).

[16] As in the study by Ronn and Verma (1986), the data for holding company banks include data on the entire holding company operations. The estimates may be biased to the extent that the data do not represent the banking activities of the holding companies. A list of the banks used in this study are available from the authors on request.

Table 2
Estimates of FDIC risk

Estimates of the total sample value of the FDIC's deposit insurance liability, I_P, the interest rate elasticity of the FDIC's total liabilities, ϕ_P, and the total volatility of these liabilities, σ_P, are obtained from the following equations:

$$I_P = \Sigma_j I_j$$

$$\phi_P = \Sigma_j w_j \phi_{Ij}$$

$$\sigma_P = \left\{ \phi_P^2 v^2 + \Sigma_i \Sigma_j w_i w_j \Lambda_i \Lambda_j \psi_i \psi_j \rho_{ij} \right\}^{1/2}$$

where Λ is the deposit insurance put-option elasticity, I_j is the risk-adjusted value of the j^{th} bank's deposit insurance, ϕ_{Ij} is the interest rate elasticity of the j^{th} bank's deposit insurance value, ψ_j is the estimated credit risk of the j^{th} bank, v is the volatility of interest rates, w_j is the portfolio weight of the j^{th} bank's liability to the FDIC, and ρ_{ij} is the correlation of credit risk for banks i and j.

Fraction denotes the value of $[\phi_P^2 v^2 / \sigma_P^2]$, i.e., the fraction of σ_P^2 explained by interest rate risk.

Period	Elasticity-based model					
	v(%)	ϕ_P	I_P($M)	$\phi_P I_P$($M)	σ_P	Fraction
Small banks						
1975–79		0.209	195.60	40.88	0.272	0.023
1980–83		1.229	895.74	1,100.86	0.304	0.172
1984–89		1.019	85.17	86.79	0.468	0.018
1975–89		1.085	361.13	391.83	0.220	0.130
Medium banks						
1975–79		2.290	137.45	314.76	0.349	0.019
1980–83		3.178	1,018.48	3,236.73	0.340	0.194
1984–89		0.901	222.99	200.91	0.552	0.041
1975–89		1.508	423.30	638.34	0.294	0.146
Large banks						
1975–79		2.211	1,356.16	2,998.47	0.339	0.026
1980–83		6.927	6,423.95	44,498.70	0.369	0.416
1984–89		2.439	1,275.10	3,109.97	0.545	0.067
1975–89		4.118	2,802.43	11,540.41	0.313	0.293
All banks						
1975–79	0.707	1.979	1,689.23	3,342.99	0.274	0.035
1980–83	1.968	5.445	8,338.17	45,401.34	0.330	0.462
1984-89	0.638	2.017	1,583.25	3,193.42	0.432	0.072
1975-89	1.050	3.351	3,586.86	12,019.57	0.345	0.329

Note: All numbers are significantly different than zero (at the 5% significance level).

reduced interest rate volatility. For each variable of interest, the individual bank estimates are time series and cross-sectionally averaged using value weights. Tables 1 and 2 present the overall sample averages and the sub-period averages for each variable. Moreover, to determine the statistical significance of these averages, hypotheses tests are presented in each table.

3.2.1. The interest rate risk of banks

The results for banks are presented in Table 1. It was discovered that banks of different sizes can have greatly different interest rate risk exposures; thus, the sample is divided into three categories based on bank size.[17] First, before proceeding to the estimates of banks' interest rate risk, it is interesting to compare the estimates of the model developed here with estimates based on the Ronn and Verma (1986) model. To this end, Table 1 shows sub-period averages and the overall sample averages for asset risk, σ_V, and the deposit insurance premium per dollar of deposits, IPP, for both the deterministic and stochastic interest rate versions of their model. A comparison shows that (1) the Ronn and Verma estimates are virtually identical for both deterministic and stochastic interest rates, a finding that is consistent with Ronn and Verma's own conclusion, and (2) the Ronn and Verma (1986) model, when compared to the model developed here, yields substantially lower estimates of asset risk and the insurance premium, particularly during periods of high interest rate volatility. On average, for the entire sample period, estimates of asset risk and the deposit insurance premium differ across the two models by 32 percent and 123 percent, respectively, and by 115 percent and 205 percent, respectively, for the 1980–83 subperiod.

The basic Ronn and Verma (1986) model assumes deterministic interest rates and consists of only two equations, one each for E and σ_E. When they introduce stochastic interest rates, they introduce an additional assumption in order to maintain their basic two-equation system. Specifically, they assume that the instantaneous variance of the discount bond, and its covariance with bank asset value, are constant (see their Eq. (4)). When pricing derivative contracts, this assumption essentially assumes away the critical difference between interest rate risk and credit risk as elaborated in Section 2 of this paper. By their specification, it is the aggregate measure of risk, not its composition, that is important. As our results indicate, adding an interest rate elasticity equation, i.e., Eq. (6) above, which takes into account the specific nature of interest rate risk, considerably enriches the deposit insurance pricing model.

The estimates of interest rate risk and credit risk for banks are also shown in Table 1. The average equity elasticity for all banks is -4.5. Small and medium size banks have equity elasticities that are considerably lower, in absolute value, than that of large banks. The estimates of portfolio mismatch are presented in terms of the interest rate elasticity gap, $[\phi_V + B(T)]$, discussed earlier. The estimates suggest that small banks have a high degree of "elasticity match" with average values for the elasticity gap of -0.09.[18] Medium and large banks, on the other hand, have average interest rate elasticity gaps of -0.19 and -0.24, respectively. Interestingly, for all three size categories, the elasticity gap (in

---

[17] Bank size is defined as follows: A bank is small if total assets are less than $4 billion for 1975–79, $6.5 billion for 1980–83 and $10 billion for 1984–89. A bank is classified as large if total assets exceed $16 billion for 1975–79, $26 billion for 1980–83 and $40 billion for 1984–89. All other banks are classified as medium in size.

[18] As shown earlier, even if $[\phi_V + B(T)] = 0$, i.e., banks exactly match the interest rate elasticities of assets and liabilities, this does not imply $\phi_E = 0$ or $\phi_I = 0$.

absolute value) increases greatly over the sample period. For all banks, the gap changes from -0.026 in 1975–79 to -0.303 for 1984–89, an eleven-fold increase in magnitude.

Comparing the interest rate risk and the credit risk components of total asset risk for all banks, over the entire period, interest rate risk accounts for a substantial proportion of total asset risk. During the 1980–83 period, a majority of total asset risk can be attributed to interest rate risk, especially for large banks. Interest rate risk is relatively less important during the periods 1975–79 and 1984–89. The fact that total asset risk, σ_V, declines for 1984–89, compared to 1980–83, is largely due to a sharp fall in interest rate volatility. Finally, the estimates suggest that credit risk is only moderately higher in the 1980–83 period than the 1975–79 period; however, for the 1984–89 period, credit risk increases for all size categories and increases almost 40 percent overall. For small banks, credit risk increases by almost 70 percent.

3.2.2. The interest rate risk of the FDIC

The results for the FDIC, along with interest rate volatility figures, are presented in Table 2. [19] As above, the FDIC's portfolio is divided into three categories based on bank size. The results suggest that the FDIC's total sample portfolio, as well as the portfolios for different size categories, have all been exposed to considerable interest rate risk during the sample period. The elasticity of the FDIC's total liability, ϕ_P, is presented in column 3 of Table 2. The sign of ϕ_P, as expected, is positive indicating that increases (decreases) in interest rates increase (decrease) the FDIC's liabilities. Over the sample period, the interest rate elasticity of the FDIC's total liabilities increased by 175 percent in 1980–83 compared to the previous period and has since fallen back to a level only marginally higher than the 1975–79 period. The interest rate risk exposure of the FDIC, measured in dollar terms and shown in column 5, is somewhat different, rising substantially during the 1980–83 period, but declining in recent years to a level that is approximately the same as that of the 1975–79 period.

The behavior of the FDIC's interest rate risk exposure over the sample period can be attributed to two related factors. First, the large increase in FDIC exposure, as measured both by elasticity and dollar value, for 1980–83 is due primarily to the large increase in interest rate volatility accentuated by greater bank mismatching. On average, the interest rate elasticity gap of banks in 1980–83 is almost five times greater in absolute value than in 1975–79. A second observation is that ϕ_P, after increasing almost two-fold in the early 1980s, falls back near its earlier

---

[19] Although this study is based on a sample of 72 banks, representing only a small fraction of total insured banks, these banks hold approximately half of total bank deposits. Thus, the conclusions reached in this section concerning FDIC risk apply to a large, but incomplete, portion of FDIC liabilities and should be interpreted accordingly.

average even though bank mismatching continued to increase. This decline is likely due primarily to the decrease in interest rate volatility. The fall in the dollar value for 1984–89 is attributable to a general reduction in the dollar value of the FDIC's liabilities, brought about by higher capital standards as well as the decline in ϕ_P.

The estimates of the volatility of the FDIC's total liabilities, σ_P, are shown in column 6 of Table 2. [20] Total volatility was lowest during the 1975–79 period, and increases for the two subsequent sub-periods. The movements in σ_P are related to several factors. The increase in 1980–83 is largely due to an increase in interest rate volatility accentuated by an increase in mismatching by banks. In the 1984–89 period, σ_P continues to increase in spite of a sharp decline in interest rate volatility because of further increases in both bank mismatching and higher credit risk. This explanation is supported by the estimates in column 7 of Table 2, which show the percentage contribution of interest rate risk to the total volatility of the FDIC's liabilities.

The results suggest that over the decade of the 1980's, the sources of risk for the FDIC change considerably. During the early years of the decade, a high level of interest rate risk is present. For the latter part of the decade, credit risk is the predominant source of FDIC risk. It is interesting to note that, in spite of increased regulatory surveillance, banks are now exposed to greater levels of both interest rate risk and credit risk than in previous periods. As a result, the volatility of the FDIC's total liabilities is considerably greater than in previous periods. Thus, our results suggest that the FDIC is now exposed to greater risk potential than in earlier periods. The results also suggest that the FDIC remains vulnerable to another period of high interest rate volatility such as 1980–83.

4. Conclusion

In this paper, a model is developed to evaluate the interest rate risk management of banks and its implications for the FDIC. Expressions for the interest rate risk exposure of both banks and the FDIC are derived from a model based on limited liability and stochastic interest rates. These expressions allow for a specific interpretation of the effects of banks' interest rate risk management on the pricing and volatility of FDIC liabilities.

The empirical estimates of the model show that the FDIC has been exposed to considerable interest rate risk during the sample period, particularly during the early 1980's. In general, bank mismatching increases significantly over the sample period and is now higher than in previous periods. Although the importance of

---

[20] The correlation of credit risk across banks is obtained using the orthoganolized value (on the 90-day T-Bill return at the end of the quarter) and is estimated over all quarters.

interest rate risk, as a component of banks' total asset risk, has declined in the last half of the 1980s due to a fall in interest rate volatility, overall, banks are presently more exposed to interest rate risk than at any time over the sample period. A sharp increase in interest rate volatility could present a significant threat to bank capital positions.

For the FDIC, interest rate risk exposure reached a peak in the early 1980s and has since declined as a result of decreasing interest rate volatility. In contrast, the volatility of the FDIC's liabilities has increased steadily over the sample period. Given the greater level of interest rate risk exposure and credit risk exposure of banks and the increased volatility of FDIC liabilities, the results suggest that the FDIC is facing the 1990s in a position that has not improved, and in some ways has deteriorated, over its position at the beginning of the 1980s.

Acknowledgements

We thank M. Berlin, E. Kane, T. Lindley, W. Sackley, two anonymous referees, and the participants at the Conference on Bank Structure and Competition, Chicago, 1992, and the Financial Management Association Meetings, 1992, for valuable comments. Financial assistance from the Social Sciences and Humanities Research Council of Canada, Fonds pour la Formation de Chercheurs et l'Aide à la Recherche du Québec, and the Bank of Montreal is gratefully acknowledged.

References

Baer, H.L., 1990, What we know about the deposit insurance problem. Chicago Fed Letter, July.

Benston, G.J., R. Eisenbeis, P. Horvitz, E. Kane and G. Kaufman, 1986, Perspectives on safe and sound banking: Past, present, and future. (MIT Press, Cambridge).

Benston, G.J., 1985, An analysis of the causes of savings and loan association failures. Monograph Series in Banking and Finance, New York University, New York.

Bierwag, G.O., 1987, Duration analysis: Managing interest rate risk (Ballinger Publishing Co., Cambridge, MA).

Bierwag, G.O. and G.G. Kaufman, 1985, Duration gap for financial institutions, Financial Analysts Journal 68–77.

Bierwag, G.O., and G.G. Kaufman, 1991, Duration gaps with futures and swaps for managing interest rate risk at depository institutions, Journal of Financial Services Research 5, 217–234.

Black, F. and M. Scholes, 1973, The pricing of options and corporate liabilities. Journal of Political Economy 81, 637–659.

Board of Governors of the Federal Reserve System, 1992, Memorandum: Revision to risk-based capital standards, June.

Chan, K.C., G.A. Karolyi, F.A. Longstaff and A.B. Sanders, 1992, Comparison of models of the short-term interest rate, Journal of Finance, 47, 1209–1227.

Crouhy, M., and D. Galai, 1986, An economic assessment of capital requirements in the banking industry. Journal of Banking and Finance 10, 231–241.

Crouhy, M., and D. Galai, 1991, A contingent claim analysis of a regulated depository institution, Journal of Banking and Finance 15, 73–90.

Dermine, J. 1985, The measurement of interest rate risk by financial intermediaries, Journal of Bank Research 86–90.

Duan, J.C., A.F. Moreau and C.W. Sealey, 1992, Fixed-rate deposit insurance and risk-shifting behavior at commercial banks. Journal of Banking and Finance 16, 715–742.

Giammarino, R., E. Schwartz and J. Zechner, 1989, Market valuation of bank assets and deposit insurance in Canada. Canadian Journal of Economics 22, 109–127.

Green, J., and J.B. Shoven, 1986, The effects of interest rates on mortgage payments. Journal of Money, Credit, and Banking 18, 41–59.

Ingersoll, J.E., J. Skelton and R.L. Weil, 1978, Duration forty years later, Journal of Financial and Quantitative Analysis 13, 627–650.

Kane, E.J., 1985, The gathering crisis in federal deposit insurance (MIT Press, Cambridge).

Kaufman, G.G., 1984, Measuring and managing interest rate risks: A primer. Economic perspectives, Federal Reserve Bank of Chicago 16–29.

Marcus, A.J. and I. Shaked, 1984, The valuation of FDIC deposit insurance using option-pricing estimates. Journal of Money, Credit and Banking 16, 446–460.

McCulloch, J.H., 1981 Interest rate risk and capital adequacy for traditional banks and financial intermediaries. In: S.J. Maisel, ed., Risk and capital adequacy in commercial banks. National Bureau of Economic Research Monograph (University of Chicago Press, Chicago) 223–248.

McCulloch, J.H., 1985, Interest-risk sensitive deposit insurance premia. Journal of Banking and Finance, 9, 137–156.

Merton, R.C., 1977, An analytic derivation of the cost of deposit insurance and loan guarantees. Journal of Banking and Finance 1, 3–11.

Office of Thrift Supervision, 1991, The OTS market value model (Capital Markets Division, Washington, DC).

Pennacchi, G.G., 1987a, Alternative forms of deposit insurance: Pricing and bank incentive issues, Journal of Banking and Finance 11, 291–312.

Pennacchi, G.G., 1987b, A reexamination of the over-(or under-) pricing of deposit insurance, Journal of Money, Credit, and Banking 19, 340–360.

Rabinovitch, R., 1989, Pricing stock and bond options when the default-free rate is stochastic, Journal of Financial and Quantitative Analysis 24, 447–457.

Ronn, E.I. and A.K. Verma, 1986, Pricing risk-adjusted deposit insurance: An option-based model, Journal of Finance 41, 871–895.

Toevs, A.L., 1983, Gap management: Managing interest rate risk in banks and thrifts, Economic Review, Federal Reserve Bank of San Francisco, 20–35.

Vasicek, O.A., 1977, An equilibrium characterization of the term structure, Journal of Financial Economics 5, 177–188.

Appendix

Derivation of Eq. (3)

Projecting dZ_{Vt} onto dZ_{rt} yields

$$dZ_{Vt} = \eta dZ_{rt} + (1 - \eta^2)^{1/2} dW_t .$$

where $\eta = \operatorname{cosv}(dZ_{V_t} dZ_{rt})/dt$. As the result of the projection, W_t is orthogonal to Z_{rt} by construction. Substituting this equation into (2) gives rise to

$$\frac{dV_t}{V_t} = \mu dt + \sigma_V \eta dZ_{rt} + \sigma_V (1 - \eta^2)^{1/2} dW_t .$$

Using Eq. (1) in the text, the above expression can be rearranged to yield

$$\begin{aligned} \frac{dV_t}{V_t} &= \mu dt + \frac{\sigma_V \eta}{v} \left[dr_t - q(m - r_t) dt \right] + \sigma_V (1 - \eta^2)^{1/2} dW_t \\ &= \left[\mu - \phi_V q(m - r_t) \right] dt + \phi_V dr_t + \psi dW_t \end{aligned}$$

where $\phi_V \equiv \sigma_V \eta / v$ and $\psi \equiv \sigma_V (1 - \eta^2)^{1/2}$. This completes the derivation. □

Proof of Proposition 1

By (4) and the use of Ito's lemma, it follows that

$$\begin{aligned} dE_t = &\left[\frac{\partial E_t}{\partial t} + \frac{1}{2} \frac{\partial^2 E_t}{\partial r_t^2} v^2 + \frac{1}{2} \frac{\partial^2 E_t}{\partial V_t^2} \sigma_V^2 V_t^2 + \frac{\partial^2 E_t}{\partial V_t \partial r_t} \phi_V v V_t \right] dt \\ &+ \frac{\partial E_t}{\partial r_t} dr_t + \frac{\partial E_t}{\partial V_t} dV_t . \end{aligned}$$

Let C_t denote the term in the square brackets, $\Delta_t = (\partial E_t / \partial r_t)/E_t$ and $\Omega_t = (\partial E_t / \partial V_t)/E_t/V_t)$. Then

$$\begin{aligned} \frac{dE_t}{E_t} &= \frac{C_t}{E_t} dt + \Delta_t dr_t + \Omega_t \frac{dV_t}{V_t} \\ &= \left\{ \frac{C_t}{E_t} + \Omega_t \left[\mu - \phi_V q(m - r_t) \right] \right\} dt + (\Delta_t + \phi_V \Omega_t) dr_t + \Omega_t \psi dW_t \end{aligned}$$

where the last equality follows from (3). Thus,

$$\phi_{E_t} = \phi_V \Omega_t + \Delta_t .$$

By the standard argument, $\Omega_t = N(h_t) V_t / E_t$. To derive an expression for Δ_t, note that

$$\begin{aligned} \frac{\partial E_t}{\partial r_t} &= V_t N'(h_t) \frac{\partial h_t}{\partial r_t} - XP(r_t, t, T) N'(h_t - \delta_t) \frac{\partial (h_t - \delta_t)}{\partial r_t} \\ &\quad - XN(h_t - \delta_t) \frac{\partial P(r_t, t, T)}{\partial r_t} \\ &= XP(r_t, t, T) N(h_t - \delta_t) B(T - t) \\ &= \left[V_t N(h_t) - E_t \right] B(T - t) \end{aligned}$$

The second equality results from the fact that the first two terms cancel out. The third equality is due to Eq. (4). Thus,

$$\begin{aligned}\Delta_t &= \left[\frac{V_t}{E_t}N(h_t) - 1\right]B(T-t)\\ &= (\Omega_t - 1)B(T-t).\end{aligned}$$

These results together yields the equation in (6). For (7), the expression for dE_t/E_t above implies $\sigma_{E_t}^2 = \text{Var}\,(dE_t/E_t) = \phi_{E_t}^2\nu^2 + \Omega_t^2\psi^2$ by the independence of W_t and r_t. □

Proof of Proposition 2

By put-call parity,

$$I_t = E_t - V_t + XP(r_t, t, T)$$

Since interest rate elasticity of a portfolio equals the value-weighted average of its components, the following relationship holds:

$$\phi_{I_t} = \frac{E_t}{I_t}\phi_{E_t} - \frac{V_t}{I_t}\phi_V + \frac{XP(r_t, t, T)}{I_t}[-B(T-t)].$$

Substituting ϕ_{E_t} in Proposition 1 into the above equation yields

$$\begin{aligned}\phi_I &= \frac{E_t}{I_t}\Omega_t[\phi_V + B(T-t)] - \frac{E_t}{I_t}B(T-t) - \frac{V_t}{I_t}\phi_V\\ &\quad - \frac{XP(r_t, t, T)}{I_t}B(T-t)\\ &= [N(h_t) - 1]\frac{V_t}{I_t}[\phi_V + B(T-t)] - \frac{E_t - V_t + XP(r_t, t, T)}{I_t}B(T-t)\\ &= \Delta_t[\phi_V + B(T-t)] - B(T-t)\end{aligned}$$

The last equality results from the application of put-call parity to the numerator of the last term. □